

1. [5 points] Let $\hat{s}_p(t)$ be the Hilbert transform of a passband signal $s_p(t)$. Show that $\langle s_p, \hat{s}_p \rangle = 0$.
2. [5 points] Suppose we define the complex envelope of a passband signal $s_p(t)$ centered at $\pm f_c$ as

$$S(f) = S_p(f + f_c)u(f + f_c)$$

where $S_p(f)$ is the Fourier transform of $s_p(t)$. Derive the following with explanations for each step.

- (a) $s_p(t)$ in terms of $s(t)$
 - (b) $s_p(t)$ in terms of $s_c(t)$ and $s_s(t)$ (the in-phase and quadrature components of $s(t)$)
 - (c) $s(t)$ in terms of $s_p(t)$
 - (d) $S_p(f)$ in terms of $S(f)$
 - (e) The relationship between $\|s\|^2$ and $\|s_p\|^2$.
3. [5 points] Suppose we define the complex envelope of a passband signal $s_p(t)$ centered at $\pm f_c$ as

$$S(f) = 2S_p(f - f_c)u(-f + f_c)$$

where $S_p(f)$ is the Fourier transform of $s_p(t)$. Derive the following with explanations for each step.

- (a) $s_p(t)$ in terms of $s(t)$
 - (b) $s_p(t)$ in terms of $s_c(t)$ and $s_s(t)$ (the in-phase and quadrature components of $s(t)$)
 - (c) $s(t)$ in terms of $s_p(t)$
 - (d) $S_p(f)$ in terms of $S(f)$
 - (e) The relationship between $\|s\|^2$ and $\|s_p\|^2$.
4. [5 points] Consider the passband signals $s_1(t) = \sqrt{2} \cos(2\pi f_1 t)$ and $s_2(t) = \sqrt{2} \cos(2\pi f_2 t)$ where $f_1 \neq f_2$. Calculate the complex baseband representations of these signals for $f_c = f_1$.