EE 703: Digital Message Transmission (Autumn 2021)
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Assignment 3: 20 points Date: August 25, 2021

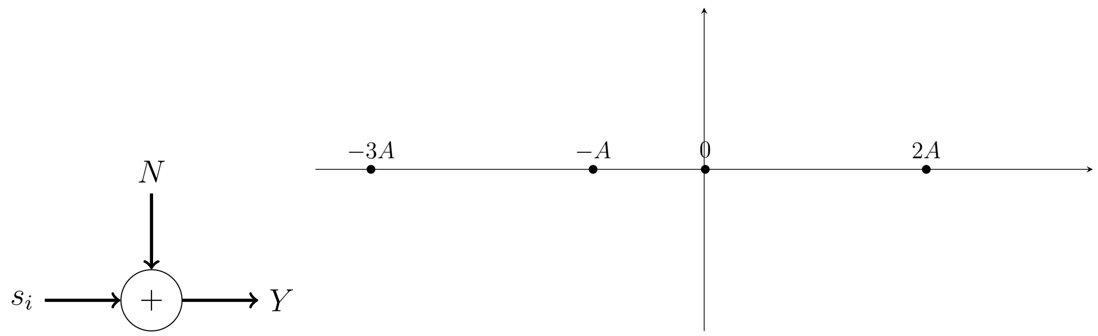
1. Let X be a Gaussian random variable with mean $\mu = -3$ and variance $\sigma^2 = 4$. Express the folllowing probabilities in terms of the Q function with positive arguments.
   (a) [1 point] $P[X > 5]$
   (b) [1 point] $P[X < -1]$
   (c) [1 point] $P[1 < X < 4]$
   (d) [2 points] $P[X^2 + X > 2]$

2. [5 points] Consider the following binary hypothesis testing problem where the hypotheses H_0 and H_1 have prior probabilities π_0 and π_1 respectively. Assume that $\mu_1 > \mu_0$ and $\pi_0 \neq \pi_1$.

$$
\begin{aligned}
H_0 &: Y \sim \mathcal{N}(\mu_0, \sigma^2) \\
H_1 &: Y \sim \mathcal{N}(\mu_1, \sigma^2)
\end{aligned}
$$

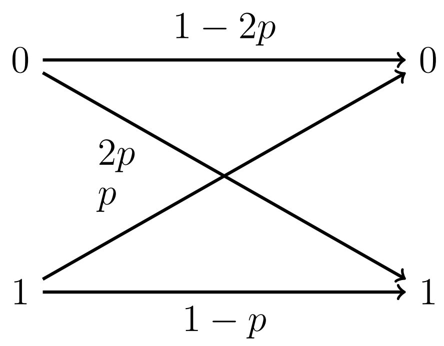
   Derive the optimal decision rule which minimizes the probability of decision error. Show your steps.

3. [5 points] The constellation $s_0 = -3A, s_1 = -A, s_2 = 0, s_3 = 2A$ is corrupted by noise N which is a zero mean Gaussian random variable having variance σ^2. Assume all four constellation points are equally likely to be transmitted.



   (a) Find the optimal decision rule based on the observation Y. Show your steps.
   (b) Find the average probability of decision error for the optimal decision rule. Express your final answer in terms of the Q function. Show your steps.

4. [5 points] Suppose the input to the following binary channel is equally likely to be 0 or 1. The arrow labels signify the transition probabilities. For example, the label $1-2p$ signifies that the probability of seeing a 0 at the channel output conditioned on the channel input being 0 is $1-2p$. Based on the output of the channel, we would like to make a decision about the input bit. Assuming $0 \le p < \frac{1}{2}$, derive the following:



(a) The optimal decision rule which minimizes the probability of decision error.

(b) The minimum probability of decision error of the optimal decision rule as a function of p.