EE 703: Digital Message Transmission (Autumn 2021)
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Assignment 4: 20 points | Date: September 28, 2021

1. [5 points] Let $\phi_1(t), \phi_2(t), \phi_3(t)$ be real unit energy signals which are orthogonal, i.e. $\|\phi_1\|^2 = \|\phi_2\|^2 = \|\phi_3\|^2 = 1$ and $\langle \phi_1, \phi_2 \rangle = \langle \phi_2, \phi_3 \rangle = \langle \phi_3, \phi_1 \rangle = 0$.

   Determine an orthonormal basis for the set of signals $s_1(t), s_2(t), s_3(t)$ which are given by the following equations where $j = \sqrt{-1}$.

$$\begin{aligned} s_1(t) &= 2\phi_2(t) - 3\phi_3(t) + 2j\phi_1(t), \\ s_2(t) &= \phi_2(t), \\ s_3(t) &= 2\phi_1(t) + 3j\phi_3(t). \end{aligned}$$

2. [5 points] Let $\phi_1(t)$ and $\phi_2(t)$ be real unit energy signals which are orthogonal, i.e. $\|\phi_1\|^2 = \|\phi_2\|^2 = 1$ and $\langle \phi_1, \phi_2 \rangle = 0$. Let $\psi_1(t)$ and $\psi_2(t)$ be given by

$$\begin{aligned} \psi_1(t) &= \phi_1(t) + \phi_2(t), \\ \psi_2(t) &= \phi_1(t) - \phi_2(t). \end{aligned}$$

   Consider the following hypothesis testing problem in AWGN where the hypotheses are equally likely, $s_1(t) = 3\psi_1(t) + 2\psi_2(t)$, and $s_2(t) = \psi_1(t) + 3\psi_2(t)$.

$$\begin{aligned} H_1 &: \; y(t) = s_1(t) + n(t) \\ H_2 &: \; y(t) = s_2(t) + n(t) \end{aligned}$$

   Let the observed signal be given by $y(t) = \psi_1(t) + 2\psi_2(t)$. Find the output of the optimal decision rule. You have to show the calculations which lead to your answer.

3. [10 points] Let $\psi_1(t), \psi_2(t)$ be a complex orthonormal basis. Let $n(t) = n_c(t) + jn_s(t)$ be complex white Gaussian noise with PSD $2\sigma^2$. Then $n_c(t)$ and $n_s(t)$ are independent real WGN processes with PSD σ^2. Consider the projection of $n(t)$ onto the orthonormal basis given by

$$\mathbf{N} = \begin{bmatrix} \langle n, \psi_1 \rangle \\ \langle n, \psi_2 \rangle \end{bmatrix} = \begin{bmatrix} N_{1,c} + jN_{1,s} \\ N_{2,c} + jN_{2,s} \end{bmatrix}.$$

   (a) Show that $N_{1,c}$ and $N_{2,c}$ are indepedent random variables.

   (b) Show that $N_{1,c}$ and $N_{2,s}$ are indepedent random variables.