EE 703: Digital Message Transmission (Autumn 2021)
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Assignment 6: 20 points — Date: October 14, 2021

1. [10 points] Derive the ML estimator of the parameter p given M independent observations $Y_i \sim \text{Bernoulli}(p)$ where $i = 1, 2, \ldots, M$.

2. [10 points] Suppose X and Y are jointly Gaussian random variables. Let the joint pdf be given by

$$p_{XY}(x, y) = \frac{1}{2\pi |\mathbf{C}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} (\mathbf{s} - \boldsymbol{\mu})^T \mathbf{C}^{-1} (\mathbf{s} - \boldsymbol{\mu}) \right)$$

where $\mathbf{s} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\boldsymbol{\mu} = \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$.

Suppose Y is observed and we want to estimate X. Derive the MMSE estimator of X.