EE 703: Digital Message Transmission (Autumn 2022)
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Assignment 4: 20 points — Date: September 27, 2022

1. [5 points] Let $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ be non-zero signals which are pairwise orthogonal, i.e. $\langle \phi_i, \phi_j \rangle = 0$ for $i \neq j$. Show that $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are linearly independent.

2. [5 points] Let $\phi_1(t), \phi_2(t), \phi_3(t)$ be real unit energy signals which are orthogonal, i.e. $\|\phi_1\|^2 = \|\phi_2\|^2 = \|\phi_3\|^2 = 1$ and $\langle \phi_1, \phi_2 \rangle = \langle \phi_2, \phi_3 \rangle = \langle \phi_3, \phi_1 \rangle = 0$.

   Determine an orthonormal basis for the set of signals $s_1(t), s_2(t), s_3(t)$ which are given by the following equations where $j = \sqrt{-1}$.

$$\begin{aligned} s_1(t) &= \phi_1(t) - \phi_2(t) + j\left[\phi_1(t) + \phi_2(t)\right], \\ s_2(t) &= \phi_1(t) + j\left[\phi_2(t) - \phi_3(t)\right] \\ s_3(t) &= 2\phi_3(t). \end{aligned}$$

3. [10 points] Let $\phi_1(t), \phi_2(t), \phi_3(t)$, and $\phi_4(t)$ be real unit energy signals which are pairwise orthogonal, i.e. $\|\phi_1\|^2 = \|\phi_2\|^2 = \|\phi_3\|^2 = \|\phi_4\|^2 = 1$ and $\langle \phi_i, \phi_j \rangle = 0$ for $i \neq j$. Let $\psi_1(t)$ to $\psi_3(t)$ be given by

$$\begin{aligned} \psi_1(t) &= \phi_1(t) + \phi_2(t) + \phi_3(t) + \phi_4(t), \\ \psi_2(t) &= \phi_1(t) + \phi_2(t) - \phi_3(t) - \phi_4(t), \\ \psi_3(t) &= \phi_1(t) - \phi_2(t) + \phi_3(t) - \phi_4(t). \end{aligned}$$

   Consider the following hypothesis testing problem in AWGN where the hypotheses are equally likely.

$$\begin{aligned} H_1 &: \; y(t) = \psi_1(t) + n(t) \\ H_2 &: \; y(t) = \psi_2(t) + n(t) \\ H_3 &: \; y(t) = \psi_3(t) + n(t) \end{aligned}$$

   Let the observed signal be given by $y(t) = 2\psi_1(t) + \psi_2(t)$. Find the output of the optimal decision rule. You have to show the calculations which lead to your answer.