Indian Institute of Technology Bombay

Department of Electrical Engineering

Handout 25 EE 706 Communication Networks

Solutions to Assignment 4 March 26, 2010

1. What can you say about the following queueing systems which have been codified in Kendall's notation? [10 points]

   (a) $M/M/2$

   Ans. This queueing system has customers arriving according to a Poisson process where the interarrival times are iid with an exponential distribution. The service times are iid with an exponential distribution. The number of servers is 2.

   (b) $M/M/2/2$

   Ans. This queueing system has customers arriving according to a Poisson process where the interarrival times are iid with an exponential distribution. The service times are iid with an exponential distribution. The number of servers is 2. The storage capacity of the system is 2.

   (c) $M/M/2/2/2$

   Ans. This queueing system has customers arriving according to a Poisson process where the interarrival times are iid with an exponential distribution. The service times are iid with an exponential distribution. The number of servers is 2. The storage capacity of the system is 2. The size of the customer population is 2.

   (d) $G/G/2$

   Ans. This queueing system has customer interarrival times iid with a general distribution. The service times are iid with a general distribution. The number of servers is 2.

   (e) $D/D/2$

   Ans. The customer interarrival times and service times in this queueing system are deterministic. The number of servers is 2.

2. An $M/M/1$ queueing system has arrival rate equal to 10 customers per second and service rate equal to 11 customers per second. [10 points]

   (a) What is the average number of customers in the system?

   Ans. Arrival rate $\lambda = 10$ customers/second and service rate $\mu = 11$ customers/second.

   Average number of customers in the system, $\bar{N} = \frac{\lambda}{\mu-\lambda} = \frac{10}{11-10} = 10$ customers.

   (b) What is the average time spent by a customer in the system?

   Ans.

   Average time spent by a customer in the system, $T = \frac{\bar{N}}{\lambda} = \frac{10}{10} = 1$ second.

(c) What is the average time spent by a customer waiting in the queue?

Ans.

Average time spent by a customer waiting in the queue, $W = \frac{1}{\mu-\lambda} - \frac{1}{\mu} = 1 - \frac{1}{11} = \frac{10}{11} = 0.909$ seconds.

(d) What is the average number of customers in the queue?

Ans.

Average number of customers in the queue, $\bar{N}_q = \lambda W = \frac{100}{11} = 9.09$ customers.

3. A queueing system has customers arriving according to a Poisson process with interarrival times which are independent and exponentially distributed with rate 10 customers per second. The service times of each customer are independent and identically distributed according to a uniform distribution between 0 and 100 milliseconds.

[10 points]

Ans.

This queueing system can be modeled as an $M/G/1$ queueing system. In order to calculate the average statistics of the system we need the first and second moments of the service. The first moment or mean service time is $\overline{X} = 50$ milliseconds for the given uniform distribution. The service rate is $\mu = \frac{1}{\overline{X}} = \frac{1}{50 \times 10^{-3}} = 20$ customer/second. The second moment is $\overline{X^2} = \frac{10^{-2}}{3}$ square seconds. The parameter $\rho = \frac{\lambda}{\mu} = \frac{1}{2}$.

(a) What is the average number of customers in the system?

Ans.

Average number of customers in the system, $\bar{N} = \rho + \frac{\lambda^2 \overline{X^2}}{2(1-\rho)} = 0.5 + \frac{10^2 10^{-2}}{3 \times 2(1-0.5)} = \frac{5}{6}$ customers.

(b) What is the average time spent by a customer in the system?

Ans.

Average time spent by the customer in the system, $T = \frac{\bar{N}}{\lambda} = \frac{5}{60} = \frac{1}{12}$ seconds.

(c) What is the average time spent by a customer waiting in the queue?

Ans.

Average time spent by the customer waiting in queue, $W = T - \overline{X} = \frac{1}{12} - 0.05 = \frac{1}{30}$ seconds.

(d) What is the average number of customers in the queue?

Ans.

Average number of customers in the queue, $\bar{N}_q = \lambda W = \frac{1}{3}$.