Review of Elementary Probability Theory

EE 706: Communication Networks

Saravanan Vijayakumaran
sarva@ee.iitb.ac.in

Department of Electrical Engineering
Indian Institute of Technology Bombay

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Our approach

- Two approaches exist to the study of probability theory
  - Rigorous approach based on tools of measure theory
  - Nonrigorous approach with focus on problem-solving methods
- Our goal is to analyze the performance of network protocols using the tools of probability theory
- For this course, second approach is sufficient

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Outline

What is a sample space?

- We perform an experiment with unpredictable outcome
- All possible outcomes are known

Definition

A sample space is the set of all possible outcomes of an experiment and is denoted by S.

Examples

- Coin toss: $S = \{\text{Heads}, \text{Tails}\}$
- Roll of a die: $S = \{1, 2, 3, 4, 5, 6\}$
- Tossing of two coins: $S = \{(H, H), (T, H), (H, T), (T, T)\}$
- A box contains three balls: one red, one green and one blue. Ball is drawn, replaced and a ball is drawn again. What is S? Without the replacement, what is S?
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- Experiment is measuring a car's lifetime. $S = [0, \infty)$

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What is an event?

Definition

An event is any subset of a sample space.

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- Roll of a die: $S = \{1, 2, 3, 4, 5, 6\}$. $E = \{2, 4, 6\}$ is the event that an even number appears when the die is rolled.
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More about events

Definition

For an event E, E^c is the complement of E. It consists of all outcomes in the sample space S that are not in E.

Examples

- Roll of a die: $S = \{1, 2, 3, 4, 5, 6\}$. $E = \{2, 4, 6\}$. $E^c = \{1, 3, 5\}$.
- In general, $S^c = \phi$ where ϕ denotes the empty set corresponding to the null event consisting of no outcomes.

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For events E and F, $E \cup F$ is called the union of E and F. It consists of all outcomes in the sample space S that are either in E or F or in both E and F.

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- If $E_1, E_2, E_3, \ldots$ are events, $\cup_{n=1}^{\infty} E_i$ is the event that consists of all the outcomes that are in E_n for at least one value of $n = 1, 2, 3, \ldots$.

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- If $E_1, E_2, E_3, \ldots$ are events, $\cap_{n=1}^{\infty} E_i$ is the event that consists of the outcomes that are in every E_n for $n = 1, 2, 3, \ldots$.

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Events E and F are said to be mutually exclusive if $E \cap F = \phi$. Mutually exclusive events have no outcomes in common.

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Probability of an Event

Definition

For each event E of a sample space S, a number $P(E)$ called the probability of the event E is assigned which satisfies the following three conditions:

1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$
3. For any sequence of events $E_1, E_2, \ldots$ that are pairwise mutually exclusive, i.e. $E_n \cap E_m = \phi$ for $n \neq m$, we have

$$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$$

Example

- Coin toss: $S = \{\text{Heads}, \text{Tails}\}$. $P(\{\text{Heads}\}) = P(\{\text{Tails}\}) = \frac{1}{2}$.

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Probabilities of Events

Equally Likely Events

If a sample space is composed of N equally likely mutually exclusive events, probability of each event is $\frac{1}{N}$.

Probability of the complement

$$P(E^c) = 1 - P(E)$$

Probability of the union

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

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For two events E and F, the probability of the occurrence of E given that F has occurred is called the conditional probability of E given F and is denoted by $P(E|F)$. Furthermore, $P(E|F) = \frac{P(E \cap F)}{P(F)}$ whenever $P(F) > 0$.

Example

- Suppose a box has balls numbered one to ten and one ball is drawn from it. $S = \{1, 2, \ldots, 10\}$
- If we are told that the number on the ball is at least five, what is the probability that the number is actually ten?
- $E = \{10\}$ and $F = \{5, 6, \ldots, 10\}$.
- $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{\frac{1}{10}}{\frac{6}{10}} = \frac{1}{6}$
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Remark

Sometimes $P(E|F)$ is given and we want to calculate $P(E \cap F)$.

Example

- Suppose Stewie takes EE706, he will receive an AA grade in it with probability $\frac{1}{2}$.
- Suppose he takes EE708, he will receive an AA grade in it with probability $\frac{1}{3}$.
- He decides to make his choice between the courses based on the flip of a fair coin. What is the probability that he will get an AA in EE708?
- $F =$ Event that Stewie chooses EE708, E is the event he gets an AA in whatever course he chooses. $P(E|F) = \frac{1}{3}$.
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- Suppose Stewie takes EE706, he will receive an AA grade in it with probability $\frac{1}{2}$.
- Suppose he takes EE708, he will receive an AA grade in it with probability $\frac{1}{3}$.
- He decides to make his choice between the courses based on the flip of a fair coin. What is the probability that he will get an AA in EE708?
- F = Event that Stewie chooses EE708, E is the event he gets an AA in whatever course he chooses. $P(E|F) = \frac{1}{3}$.
- $P(E \cap F) = P(E|F)P(F) = \frac{1}{3}\frac{1}{2} = \frac{1}{6}$

Conditional Probability

Remark

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Two events E and F are said to be independent if $P(E \cap F) = P(E)P(F)$

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- Suppose we toss two fair dice. Let E_n denote the event that the sum of the die values is n.
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- $E_6 = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$ and $F_4 = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$.
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Bayes' Theorem

Theorem

Given two events E and F where $P(F) > 0$, Bayes' theorem states that

$$P(E|F) = \frac{P(F|E)P(E)}{P(F)}.$$

Remarks

- The theorem is useful when calculating $P(F|E)$ is easier than calculating $P(E|F)$.
- A useful expansion of the denominator is

$$\begin{aligned} P(E) &= P[(E \cap F) \cup (E \cap F^c)] = P(E \cap F) + P(E \cap F^c) \\ &= P(E|F)P(F) + P(E|F^c)P(F^c) \end{aligned}$$

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What is a random variable?

Definition

A random variable is a real-valued function defined on a sample space.

Examples

- Coin toss: $S = \{\text{Heads}, \text{Tails}\}$. Random variable $X = 1$ if outcome is $\{Heads\}$ and 0 otherwise.
- Tossing of fair two dice: $S = \{(i,j) : 1 \le i, j \le 6\}$.
- Random variable $X =$ Sum of the values in the outcome $= i + j$ for outcome (i,j).
- Since the value of a random variable depends on the outcome of an experiment, we can think of the set of possible values as a new sample space and probabilities can be assigned to subsets of this space.
- $P(X = 4) = P\{(1,3),(2,2),(3,1)\} = \frac{3}{36}$

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Cumulative distribution function

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The cumulative distribution function (cdf) $F(\cdot)$ of a random variable X is defined for any real number a, $-\infty < a < \infty$ by

$$F(a) = P(X \leq a)$$

Properties

- $F(a)$ is a nondecreasing function of a.
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Discrete Random Variable

Definition

A discrete random variable is random variable (RV) whose range is finite or countable, i.e. it takes a finite or countable number of values.

Definition

For a discrete RV, we define the probability mass function $p(a)$ as

$$p(a) = P[X = a]$$

Properties

- If X takes on values $x_1, x_2, x_3, \ldots$, then $p(x_i) > 0, i = 1, 2, \ldots$ and $p(x) = 0$ for all other values x.
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The Bernoulli Random Variable

- Consider an experiment whose outcomes can be classified as either a *success* or a *failure*.
- Let X equal 1 if the outcome is a success and 0 if the outcome is a failure
- A Bernoulli random variable is a random variable whose probability mass function of X is given by

$$\begin{aligned} p(0) &= P[X = 0] = 1 - q \\ p(1) &= P[X = 1] = q \end{aligned}$$

where $q, 0 \leq q \leq 1$ is the probability that the experiment is a success.

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- Suppose that n independent experiments or trials, each of which results in a *success* with probability q and in a failure with probability $1 - q$.
- If X represents the number of successes in the n trials, then X is said to be a binomial random variable.
- The probability mass function of a binomial random variable having parameters (n, q) is given by

$$p(i) = \binom{n}{i} q^i (1-q)^{n-i}, \quad i = 0, 1, 2, \ldots$$

where

$$\binom{n}{i} = \frac{n!}{(n-i)!i!}$$

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Continuous Random Variable

- A continuous random variable is a random variable which can take on an uncountable number of values.
- A random variable X is said to be a continuous random variable if there exists a non-negative function $f(x)$ defined for all real $x \in \{-\infty, \infty\}$, having the property that for any set B of real numbers
$$P[X \in B] = \int_B f(x)\, dx$$
The function $f(x)$ is said to be the probability density function(pdf) of the random variable X.
- $1 = \{X \in (-\infty, \infty)\} = \int_{-\infty}^{\infty} f(x)\, dx$
- $P\{a \leq X \leq b\} = \int_a^b f(x)\, dx$;$P\{X = a\} = \int_a^a f(x)\, dx = 0$
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- $P\{a - \frac{\epsilon}{2} \leq X \leq a + \frac{\epsilon}{2}\} = \int_{a-\frac{\epsilon}{2}}^{a+\frac{\epsilon}{2} b} f(x)\, dx \approx \epsilon f(a)$
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Continuous Random Variable

- A continuous random variable is a random variable which can take on an uncountable number of values.
- A random variable X is said to be a continuous random variable if there exists a non-negative function $f(x)$ defined for all real $x \in \{-\infty, \infty\}$, having the property that for any set B of real numbers

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Continuous Random Variables

- X is a uniform random variable on the interval (a, b) if its pdf is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a < x < b \\ 0, & \text{otherwise} \end{cases}$$

- X is a Gaussian or normal random variable with parameters μ and σ^2 if its pdf is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

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- If X is a discrete random variable with probability mass function $p(x)$, the expected value of X is given by

$$E[X] = \sum_{x:p(x)>0} xp(x).$$

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