Indian Institute of Technology Bombay

Department of Electrical Engineering


**Handout 17** EE 706 Communication Networks

Solutions to Quiz 6 March 9, 2010

1. $\{A(t)|t \geq 0\}$ is a *Poisson process* with rate $\lambda$. So the number of arrivals in any interval of length $\tau$ is Poisson distributed with parameter $\lambda\tau$,

$$\Pr\{A(t+\tau) - A(t) = n\} = e^{-\lambda\tau}\frac{(\lambda\tau)^n}{n!}, \quad n = 0, 1, 2, \ldots$$

   (a) If $\lambda = 1$ arrival per second, what is the the probability of zero arrivals from time $t = 5$s to $t = 10$s? [1 point]

   **Ans.**

   Setting $t = 5$ and $\tau = 5$, we get

$$\Pr\{A(10) - A(5) = 0\} = e^{-5}\frac{(5)^0}{0!} = e^{-5}$$

   (b) If $\lambda = 1$ arrival per second, what is the the probability of zero arrivals from time $t = 5$s to $t = 10$s given that there was one arrival in the interval of time $[0, 5)$? [1 point]

   **Ans.**

   For a Poisson process, the number of arrivals in disjoint intervals are independent random variables. If $E_0$ is the event of zero arrivals in the interval $[5, 10]$ and $E_1$ is the event that one arrival occurs in the interval $[0, 5)$, then $\Pr\{E_0|E_1\} = \Pr\{E_0\} = e^{-5}$ from part (i).

2. The interarrival times of a Poisson process are independent and exponentially distributed. So if $\tau_n$ is the time between the arrivals of the $n$th and $(n-1)$th customers, we have

$$\Pr[\tau_n \leq s] = 1 - e^{-\lambda s}, \text{ for } s \geq 0.$$

   (a) If $\lambda = 1$ arrival per second, what is the the probability that the fourth customer will arrive at least 5 seconds after the third customer arrived? [1 point]

   **Ans.**

   Let $t_n$ be the arrival time of the $n$th customer, then

$$\Pr\{t_4 \geq t_3 + 5\} = \Pr\{t_4 - t_3 \geq 5\} = \Pr\{\tau_4 \geq 5\} = e^{-5}$$

   (b) If $\lambda = 1$ arrival per second, what is the the probability that the fourth customer will arrive at least 5 seconds after the third customer and the fifth customer will arrive at most 10 seconds after the fourth customer? [1 point]

   **Ans.**

   Since the interarrival times are independent, we have

$$\Pr\{(\tau_4 \geq 5) \cap (\tau_5 \leq 10)\} = \Pr\{\tau_4 \geq 5\}\Pr\{\tau_5 \leq 10\} = e^{-5}(1 - e^{-10})$$

3. Suppose a queueing system has no customers in it at time $t = 0$. If customer arrivals occur at times $t = 1, 3, 4$ and customer departures occur at times $t = 2, 5, 6$, plot $\alpha(t)$ = The number of arrivals in the system upto time $t$, $\beta(t)$ = The number of departures in the system upto time $t$ and $N(t)$ = The number of customers in the system at time $t$. [6 points]

**Ans.**