EE 706: Communication Networks

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Assignment 2 : **20 points** **Due date**: February 13, 2012

1. Suppose random bits are transmitted using polar NRZ line coding at the rate of one bit per second. Suppose the receiver delay locked loop loses timing synchronization if there are no signal transitions for $N + \epsilon$ seconds where N is a positive integer and ϵ is a real number less than 1. Assume that the reception of a message bit begins at $t = 0$ with the receiver having achieved timing synchronization. [5 points]

   (a) What is the probability that the receiver has not lost timing synchronization at $t = K$ where K is a positive integer greater than or equal to N?

   (b) Using Matlab or Scilab, simulate the scenario and confirm your derived expression for different values of N and K. Include a printout of your code with your submission.

2. Suppose a 10-bit random message is bit stuffed to avoid the appearance of the flag bit sequence 01111110 by inserting a zero bit after the appearance of a sequence of five consecutive ones. [5 points]

   (a) What is the average length of the message after bit stuffing?

   (b) Using Matlab or Scilab, simulate the scenario and confirm the calculated value. Include a printout of your code with your submission.

3. Consider a pseudo-random spreading sequence $\{a_l : l = 0, 1, 2, \ldots\}$ having period N_c. The a_l's can be modeled as an independent and identically distributed sequence of random variables taking values $+1$ and -1 with equal probability. [10 points]

   (a) Computed the mean and second moment of the following autocorrelation function as a function of k where $k \in \mathbb{Z}$. Note that the sequence is periodic.

   $$R(k) = \frac{1}{N_c} \sum_{l=0}^{N_c-1} a_l a_{l+k}$$

   (b) In direct sequence spread spectrum systems, the preamble can be represented as multiple copies of the spreading sequence above modulated onto a chip waveform $\psi(t)$ of duration T_c. The following equation represents a signal having three periods of the spreading sequence modulated onto $\psi(t)$.

   $$c(t) = \sum_{l=-N_c}^{2N_c-1} a_l \psi(t - lT_c)$$

The received preamble is correlated with a template signal $s(t)$ which is one period of the spreading sequence modulated onto a chip waveform $\psi(t)$.

$$s(t) = \sum_{l=0}^{N_c-1} a_l \psi(t - lT_c)$$

Using the first part, calculate the mean and second moment of the following crosscorrelation function as a function of a real number $\tau \in [-3T_c, 3T_c]$.

$$R(\tau) = \frac{1}{N_c} \int c(t)s(t-\tau)\ dt$$

You can assume that $\psi(t)$ is a square pulse of unit amplitude and duration T_c starting at $t = 0$. Sketch the second moment of $R(\tau)$ versus τ. *Hint:* Try drawing a picture of the waveforms for various values of τ.