EE 720: An Introduction to Number Theory and Cryptography (Spring 2018)

Lecture 10 — February 9, 2018

Instructor: Saravanan Vijayakumaran *Scribe: Saravanan Vijayakumaran*

1 Lecture Plan

- Recap the construction of CPA-secure encryption scheme
- Define pseudorandom permutations
- Describe block cipher modes
- Give construction of DES block cipher

2 Recap

CPA-Secure Encryption from Pseudorandom Functions

- Let F be a pseudorandom function. Define a private-key encryption scheme for messages of length n as follows:
  - `Gen`: On input 1^n, choose k uniformly from $\{0,1\}^n$.
  - `Enc`: Given $k \in \{0,1\}^n$ and message $m \in \{0,1\}^n$, choose uniform $r \in \{0,1\}^n$ and output the ciphertext
  $$c := \langle r, F_k(r) \oplus m \rangle.$$
  - `Dec`: Given $k \in \{0,1\}^n$ and ciphertext $c = \langle r, s \rangle$, output the plaintext message
  $$m := F_k(r) \oplus s.$$

Theorem (Thereom 3.31 of KL)**.** *If F is a pseudorandom function, then the above construction is a CPA-secure private-key encryption scheme for messages of length n.*

- What is a drawback of this construction?

3 Pseudorandom Permutations and Block Ciphers

- In practice, constructions of pseudorandom permutations are used instead of pseudorandom functions.
- Let $\texttt{Perm}_n$ be the set of all permutations (bijections) on $\{0,1\}^n$. An $f \in \texttt{Perm}_n$ can be seen as a lookup table where any two distinct rows must be different.

- $|\texttt{Perm}_n| = (2^n)!$
- A function $F : \{0,1\}^{l_{key}(n)} \times \{0,1\}^{l_{in}(n)} \to \{0,1\}^{l_{in}(n)}$ is called a *keyed permutation* if for all $k \in \{0,1\}^{l_{key}}(n)$, F_k is a permutation.
- $l_{in}(n)$ is called the *block length* of F.
- F is *length-preserving* if $l_{key}(n) = l_{in}(n) = n$.
- F is said to be *efficient* if both $F_k(x)$ and $F_k^{-1}(y)$ have polynomial-time algorithms for all k, x, y.
- A *pseudorandom permutation* is a permutation which cannot be efficiently distinguished from a random permutation, i.e. a permutation uniformly chosen from $\texttt{Perm}_n$.
- When the blocklength is sufficiently long, a random permutation is indistinguishable from a random function (by birthday problem analysis).
- In practice, constructions of pseudorandom permutations are called *block ciphers*.

3.1 Block Cipher Modes of Operation

3.1.1 Electronic Code Book (ECB) Mode

- **Insecure and should not be used. Included in the exposition as a warning to not use it**.
- Let $m = m_1, m_2, \ldots, m_l$ where $m_i \in \{0,1\}^n$.
- Let F be a block cipher with block length n.
- $c \coloneqq \langle F_k(m_1), F_k(m_2), \ldots, F_K(m_l) \rangle$
- ECB is deterministic and cannot be CPA-secure.

3.1.2 Cipher Block Chaining (CBC) Mode

- Let $m = m_1, m_2, \ldots, m_l$ where $m_i \in \{0,1\}^n$.
- Let F be a length-preserving block cipher with block length n.
- A uniform *initialization vector (IV)* of length n is first chosen.
- $c_0 = IV$. For $i = 1, \ldots, l$, $c_i \coloneqq F_k(c_{i-1} \oplus m_i)$.
- For $i = 1, 2, \ldots, l$, $m_i \coloneqq F_k^{-1}(c_i) \oplus c_i$.
- This mode has a ciphertext which is larger than the plaintext by n bits.
- Decryption is much faster than encryption.

3.1.3 Counter (CTR) Mode

- Let $m = m_1, m_2, \ldots, m_l$ where $m_i \in \{0,1\}^n$.
- Let F be a length-preserving block cipher with block length n.
- A uniform value $\texttt{ctr}$ of length n is first chosen.
- $c_0 = \texttt{ctr}$. For $i = 1, \ldots, l$, $c_i \coloneqq F_k(\texttt{ctr} + i) \oplus m_i$.
- For $i = 1, 2, \ldots, l$, $m_i \coloneqq F_k(\texttt{ctr} + i) \oplus c_{i-1}$.
- This mode has a ciphertext which is larger than the plaintext by n bits.
- Both encryption and decryption can be parallelized.

4 Data Encryption Standard (DES)

- DES was proposed by IBM in 1974 in response to a call for proposals from the US National Bureau of Standards (now NIST)
- Adopted as a US federal standard from 1979 to 2005
- In 2000, AES selected as successor to DES.
- DES considered insecure now but still interesting for historical reasons.

4.1 Construction

- Based on the *Feistel transform*
- Let $f : \{0,1\}^n \to \{0,1\}^n$ be any function. The Feistel transform of f is the function $FSTL_f : \{0,1\}^{2n} \to \{0,1\}^{2n}$ is defined by

$$FSTL_f(L, R) = (R, f(R) \oplus L)$$

- Even if f is not a bijection, $FSTL_f$ is a bijection.
- The inverse is given by

$$FSTL_f^{-1}(X, Y) = (Y \oplus f(X), X)$$

- DES has a key length of 56 bits and a block length of $n = 64$ bits. It consists of 16 *rounds* of a Feistel transform.
- First the 56-bit key K is expanded to a sequence of 16 subkeys $K_1, K_2, \ldots, K_{16}$.
- See pages 41–44 of Bellare-Rogaway notes for full description.

5 References and Additional Reading

- Section 3.5, 3.6 from Katz/Lindell
- Chapter 3 of *Introduction to Modern Cryptography* by Mihir Bellare, Phillip Rogaway, 2005. `http://web.cs.ucdavis.edu/~rogaway/classes/227/spring05/book/main.pdf`