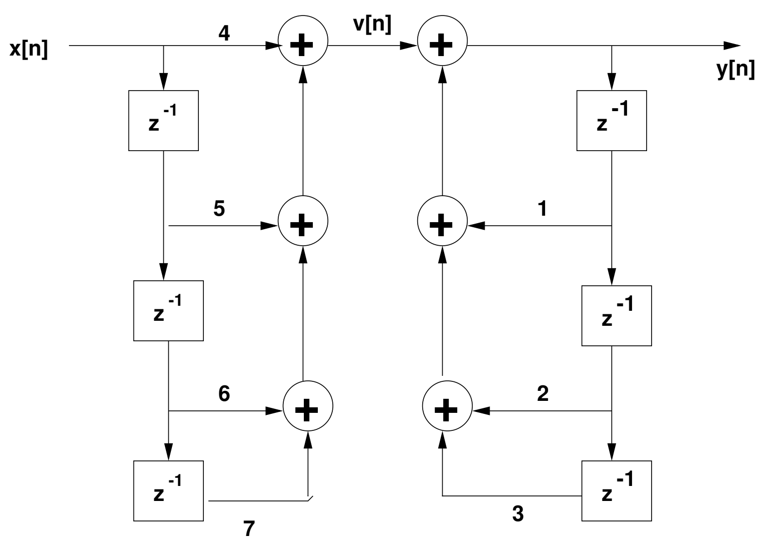
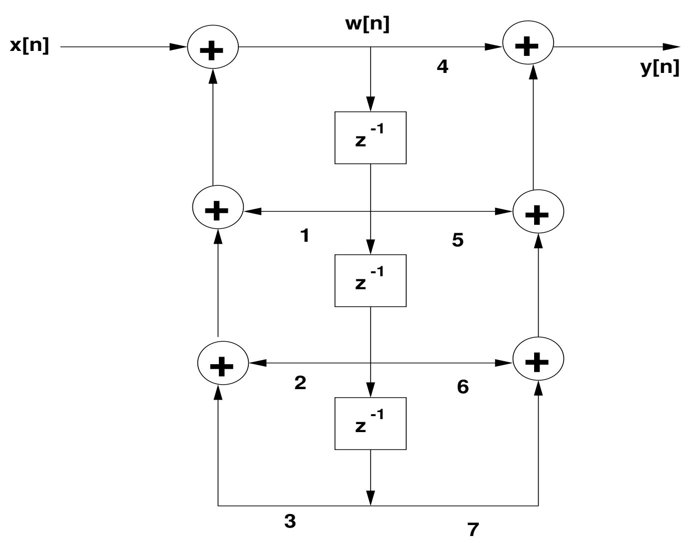
EE 403 DIGITAL SIGNAL PROCESSING
TUTORIAL SHEET NO 2

1. (a) Write the difference equations for each system .(Hint: Write it as a pair of difference equations using an intermediate variable.) Looking at these two pairs would you say these systems are same or different?
(b) Write these systems as z-transforms. Are the z-transforms of transfer functions same or different?
(c) An interchange of this kind is possible in LTI systems. (as demonstrated in this question.) Is it possible for non LTI systems?

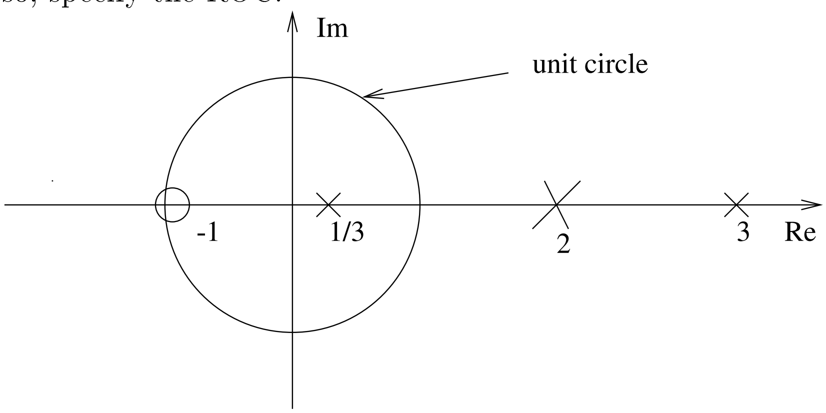




2. Fibonacci numbers f_k satisfy the difference equation $f_{k+2} = f_{k+1} + f_k$,$k \geq 0$ with $f_0 = 0$ and $f_1 = 1$.
(a) Find general expression for f_k.
(b) Show that the ratio $\frac{f_k}{f_{k+1}} = \frac{2}{\sqrt{5}+1}$as $k \to \infty$.(Known as golden mean).

3. For an LTI system $h(n) = \frac{N-n}{N}$ if $0 \leq n \leq 5$ and zero elsewhere. If the input-output relation is written in matrix vector multiplication form $\underline{y} = H\underline{x}$, obtain the matrix H when $N = 10$.If $h(n)$ is periodic with period N i.e. $h(n+N) = h(n)$, obtain the corresponding matrix H.

4. Consider the z-transform $X(z)$ whose pole-zero plot is as shown in the figure.
(a) Determine the ROC of $X(z)$ given that Fourier transform exists.In this case, determine whether the sequence is right sided, left sided or two-sided.
(b) How many possible two-sided sequences have the pole-zero plot shown in figure?
(c) Is it possible to associate sequence that is causal and stable with the pole-zero plot? If so, specify the ROC.



5. Show that any discrete-time signal $x(n)$ can be expressed as

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k)$$

where $u(n)$ is the unit step function.

6. Show that the output of an LTI system can be expressed in terms of its unit step response $s(n)$ as follows.

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, s(n-k)$$

7. You have borrowed Rs.1,00,000 from a bank and plan to pay it up by equal monthly payments of Rs.X starting from the next month. The bank charges an interest of 12% per annuam compounded monthly on the unpaid balance.
Obtain a difference equation to represent the above problem.If the loan is to be paid up in 30 years, what should be the monthly payment?
Consider a special case when the RBI decides to reduce the lending rate to only 6% per annuam after 10 years. If the person has been paying the same amount as obtained in the previous part of the problem when would the loan be fully repaid? Is this an LTI system?