Department of Electrical Engineering
Digital Signal Processing (EE 403)

Tutorial 3

1. Determine the causal signal $x[n]$ if its z-transform is given by

   (a) $X(z) = \frac{1+2z^{-1}+z^{-2}}{1+4z^{-1}+4z^{-2}}$

   (b) $X(z)$ is specified by pole-zero pattern in Figure 1.

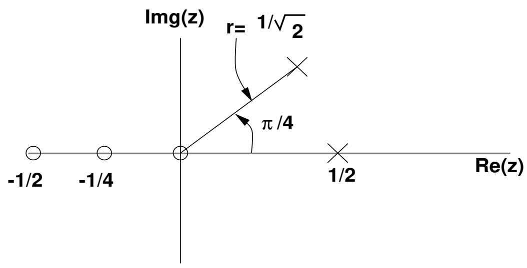

Figure 1: Pole-zero plot.

2. Do the following.

   (a) Draw the pole-zero pattern for the signal

$$x_1[n] = (r^n \sin \omega_0 n) \cdot u[n] \qquad 0 < r < 1$$

   (b) Compute the z-transform $X_2(z)$, which corresponds to the pole-zero pattern in part (a)

   (c) Compare $X_1(z)$ and $X_2(z)$. Are they identical? If not, indicate a method to derive $X_1(z)$ from the pole-zero pattern

3. Let $x[n]$ be a causal sequence with z-transform $X(z)$, whose pole-zero plot is shown in Figure 2. Sketch the pole-zero plots and the ROC of the following

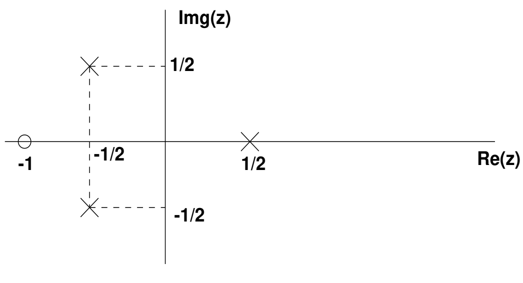

Figure 2: Pole-zero plot

sequences:

   (a) $x_1[n] = x[-n+2]$

   (b) $x_2[n] = e^{j(\pi/3)n}x[n]$

4. Determine a state-space model and the corresponding realization for the following FIR system.

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

5. Consider the system described by the state-space equations

$$v[n+1] = \begin{bmatrix} -0.3 & 0.4 \\ 0.4 & -0.3 \end{bmatrix} \cdot v[n] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot x[n]$$

$$y[n] = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot v[n] + x[n]$$

   (a) Determine the characteristic polynomial and eigenvalues of the system.

   (b) Determine the state transition matrix $\Phi[n]$ for $n \geq 0$.

   (c) Determine the impulse response of the system.

   (d) Compute the step response of the system if $v[0] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   (e) Sketch a state-space realization for the system.

6. Let $x[n]$ be a causal sequence.

   (a) What conclusion can you draw about the value of its z-transform $X(z)$ at $z = \infty$?

   (b) Use the result in part (a) to check which of the following transforms cannot be associated with a causal sequence.

      i. $X(z) = \frac{\left(1-\frac{1}{2}z^{-1}\right)^2}{\left(1-\frac{1}{3}z^{-1}\right)}$

      ii. $X(z) = \frac{\left(z-\frac{1}{3}\right)^2}{\left(z-\frac{1}{2}\right)^3}$