EE 403 DIGITAL SIGNAL PROCESSING
TUTORIAL SHEET 4

1. A designer has available a number of 8 point FFT chips. Show explicitly how he should interconnect three such chips in order to compute a 24-point DFT.

2. The z-transform of the sequence $x(n) = u(n) - u(n-7)$ is sampled at five points on the unit circle as follows $x(k) = X(z)|_{z=e^{j2\pi k/5}}$ $k = 0, 1, 2, 3, 4$. Determine the inverse DFT $x'(n)$ of $X(k)$. Compare it with $x(n)$ and explain the results.

3. Compute the eight point DFT of the sequence

$$x(n) = \begin{matrix} 1 & 0 \leq n \leq 7 \\ 0 & otherwise \end{matrix}$$

by using decimation-in-frequency FFT algorithm.

4. Compute the 8 point circular convolution of the following sequences.
$x_1(n) = \left(\frac{1}{4}\right)^n$ $0 \leq n \leq 7$, $x_2(n) = \cos\frac{3\pi n}{8}$ $0 \leq n \leq 7$. Compute the DFT of the circular convolved sequence using the DFTs of $x_1(n)$ and $x_2(n)$.

5. Let $x_p(n)$ be a periodic sequence with fundamental period N. Consider the following DFTs $x_p(n) \longleftrightarrow X_1(k)$($N$-point DFT) and $x_p(n) \longleftrightarrow X_3(k)$($3N$ -point DFT)
(a) What is the relationship between $X_1(k)$ and $X_3(k)$?
(b) Verify the result in part (a) using the sequence $x_p(n) = \{......, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, ...\}$

6. *Computation of the even and odd harmonics using the DFT*
Let $x(n)$ be an N-point sequence with an N point DFT $X(k)$(N even).
(a) Consider the time aliased sequence

$$y(n) = \begin{matrix} \sum_{l=-\infty}^{\infty} x(n+lM) & 0 \leq n \leq M-1 \\ 0 & otherwise \end{matrix}$$

What is the relationship between the $M-$point DFT $Y(k)$ of $y(n)$ and the Fourier transform $X(\omega)$ of $x(n)$?
(b) Let

$$y(n) = \begin{matrix} x(n) + x(n+N/2) & 0 \leq n \leq N-1 \\ 0 & otherwise \end{matrix}$$

and $y(n) \longleftrightarrow Y(k)$($N/2$ point DFT).
Show that $X(k) = Y(k/2)$ $k = 2, 4, 6, ...N-2$
(c) Use the results in parts (a) and (b) to develop a procedure that computes the odd harmonics of $X(k)$ using an $N/2$ point DFT.