

EE101: BJT circuits (Part 1)



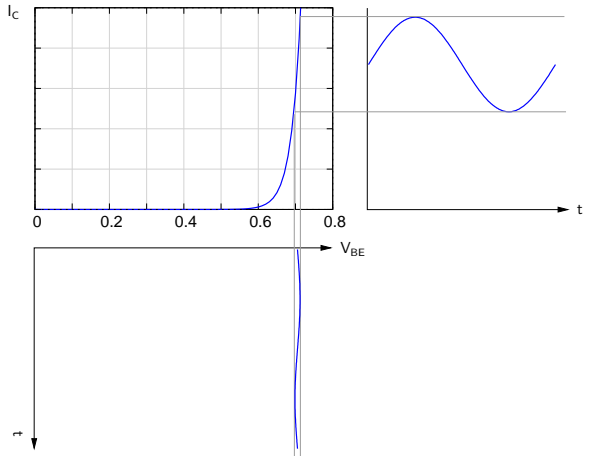
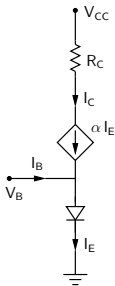
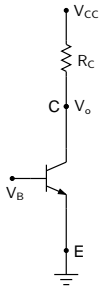
M. B. Patil

mbpatil@ee.iitb.ac.in

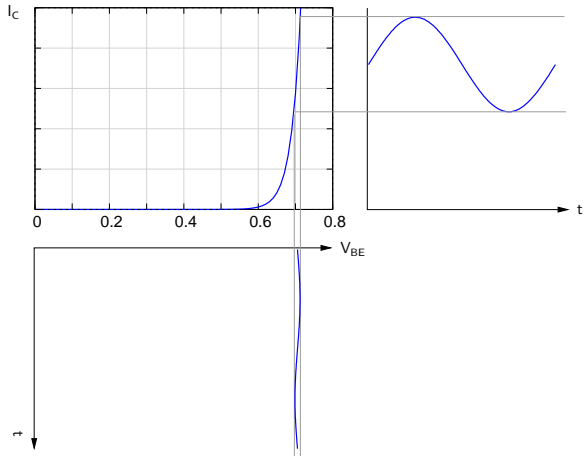
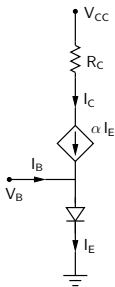
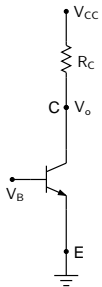
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Department of Electrical Engineering
Indian Institute of Technology Bombay

BJT amplifier: basic operation

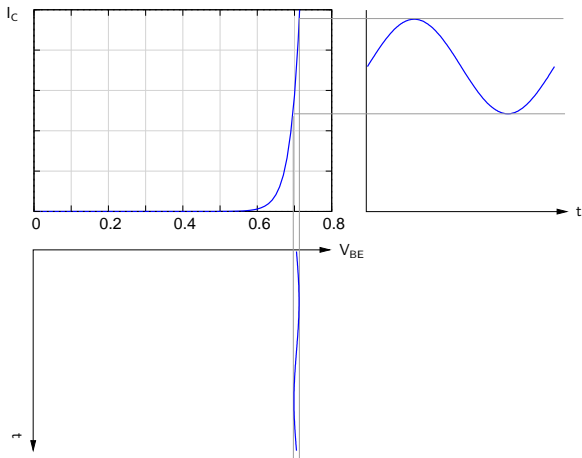
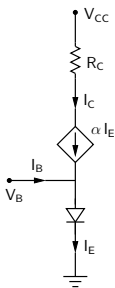
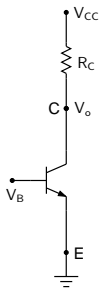


BJT amplifier: basic operation



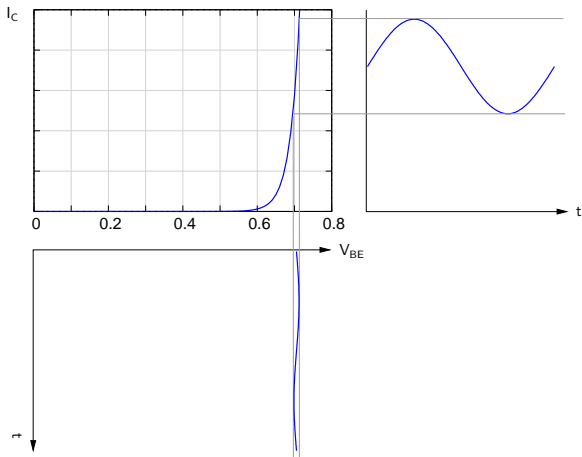
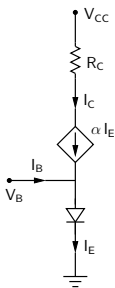
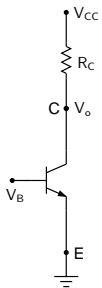
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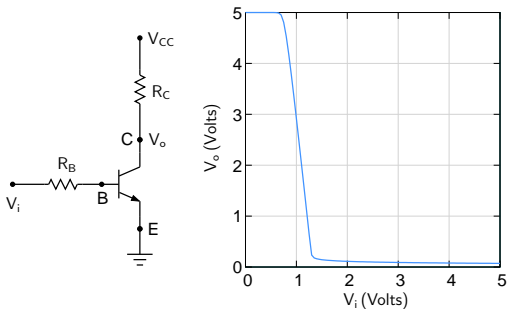
- * In the active mode, I_C changes exponentially with V_{BE} .
- * $V_o = V_{CC} - I_C R_C \Rightarrow$ the amplitude of V_o is simply $I_C R_C$ which can be made much larger than the input voltage amplitude.

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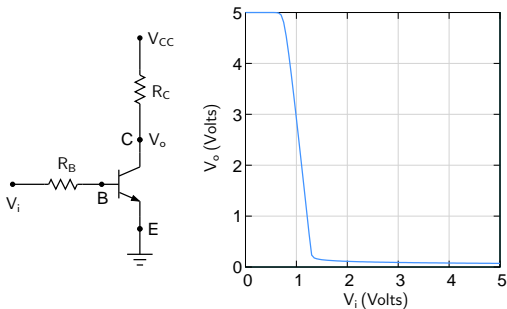
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- * $V_o = V_{CC} - I_C R_C \Rightarrow$ the amplitude of V_o is simply $I_C R_C$ which can be made much larger than the input voltage amplitude.
- * Note that both the input (V_{BE}) and output (V_o) voltages have DC ("bias") components.

BJT amplifier biasing: why?



Consider a more realistic BJT amplifier circuit, with R_B added to limit the base current (and thus protect the transistor).

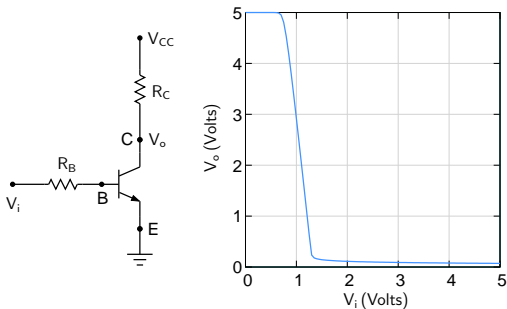
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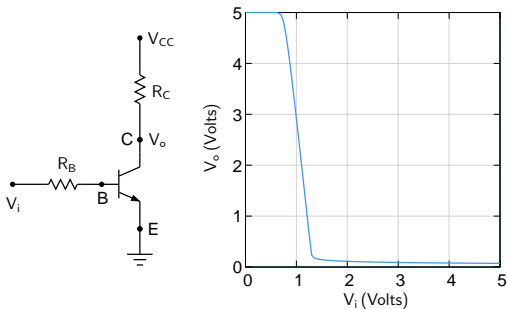
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- * Since V_o is nearly constant for $V_i < 0.7$ V (due to cut-off) and $V_i > 1.3$ V (due to saturation), the circuit will not work as an amplifier in this range.

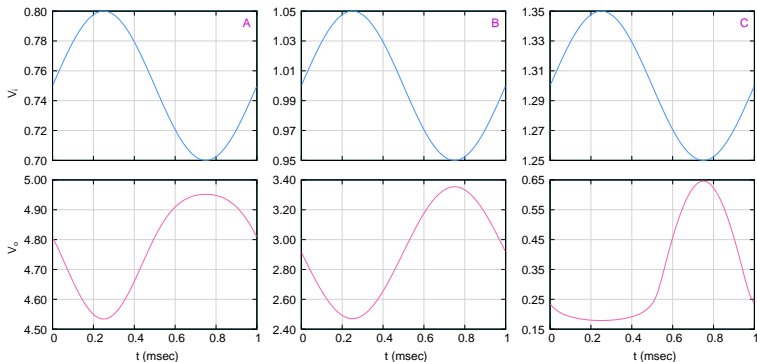
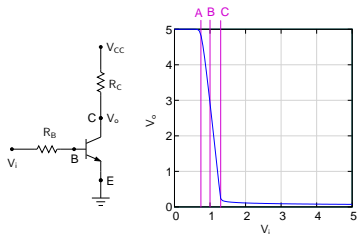
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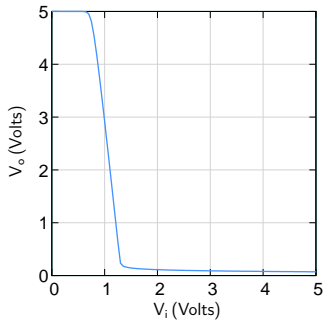
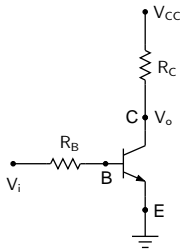
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- * Since V_o is nearly constant for $V_i < 0.7$ V (due to cut-off) and $V_i > 1.3$ V (due to saturation), the circuit will not work as an amplifier in this range.
- * Further, to get a large swing in V_o without distortion, the DC bias of V_i should be at the centre of the amplifying region, i.e., $V_i \approx 1$ V.

BJT amplifier biasing: why?



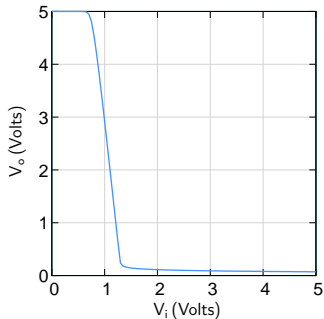
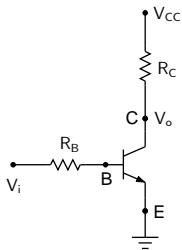
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BJT amplifier



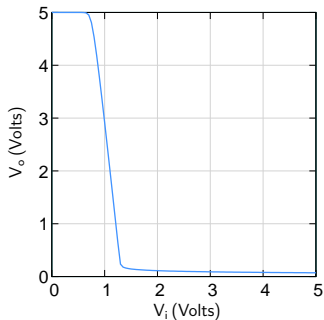
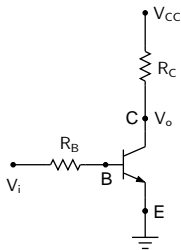
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BJT amplifier



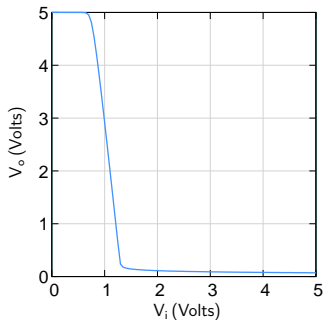
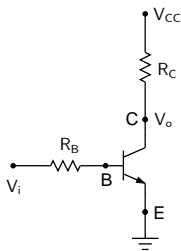
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 - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of I_C .

BJT amplifier



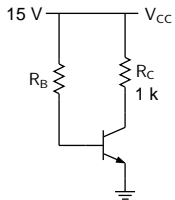
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BJT amplifier



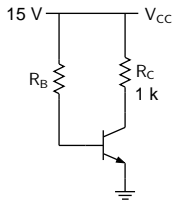
- * The key challenges in realizing this amplifier in practice are
 - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of I_C .
 - mixing the input DC bias with the signal voltage.
- * The first issue is addressed by using a suitable biasing scheme, and the second by using “coupling” capacitors.

BJT amplifier: a simple biasing scheme



“Biasing” an amplifier \Rightarrow selection of component values for a certain DC value of I_C (i.e., when no signal is applied).

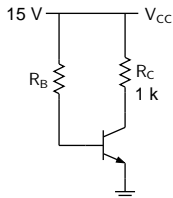
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Equivalently, we may bias an amplifier for a certain DC value of V_{CE} , since I_C and V_{CE} are related: $V_{CE} + I_C R_C = V_{CC}$.

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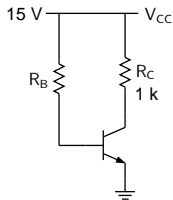


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As an example, for $R_C = 1\text{ k}$, $\beta = 100$, let us calculate R_B which will give $I_C = 3.3\text{ mA}$, assuming the BJT to be operating in the active mode.

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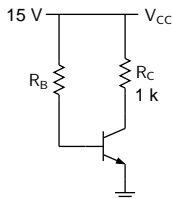
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$$I_B = \frac{I_C}{\beta} = \frac{3.3\text{ mA}}{100} = 33\text{ }\mu\text{A} = \frac{V_{CC} - V_{BE}}{R_B}$$

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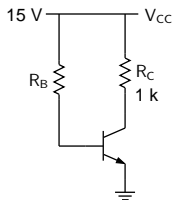
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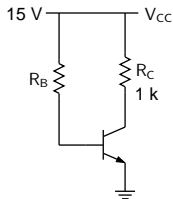
$$\rightarrow R_B = \frac{14.3\text{ V}}{33\text{ }\mu\text{A}} = 430\text{ k}\Omega.$$

BJT amplifier: a simple biasing scheme (continued)



With $R_B = 430 \text{ k}$, we expect $I_C = 3.3 \text{ mA}$, assuming $\beta = 100$.

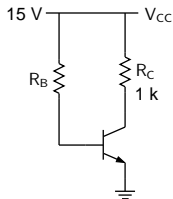
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However, in practice, there is a substantial variation in the β value (even for the same transistor type). The manufacturer may specify the nominal value of β as 100, but the actual value may be 150, for example.

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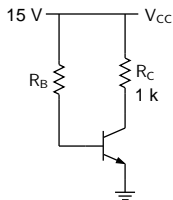
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With $\beta = 150$, the actual I_C is,

$$I_C = \beta \times \frac{V_{CC} - V_{BE}}{R_B} = 150 \times \frac{(15 - 0.7) \text{ V}}{430 \text{ k}} = 5 \text{ mA},$$

which is significantly different than the intended value, 3.3 mA.

BJT amplifier: a simple biasing scheme (continued)



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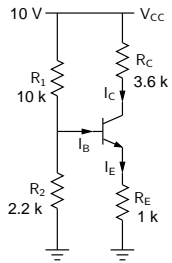
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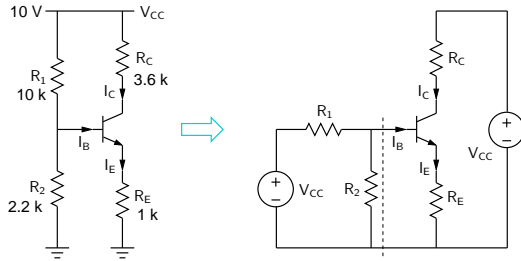
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⇒ need a biasing scheme which is not so sensitive to β .

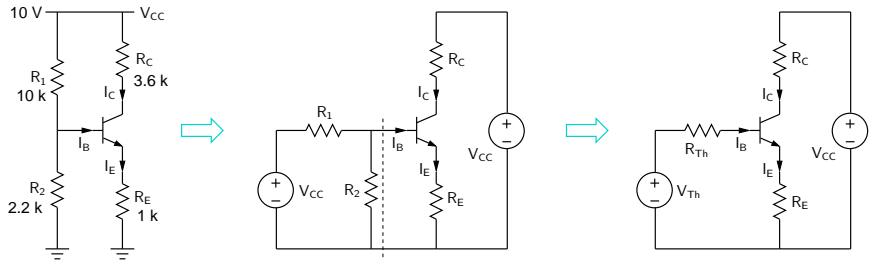
BJT amplifier: improved biasing scheme



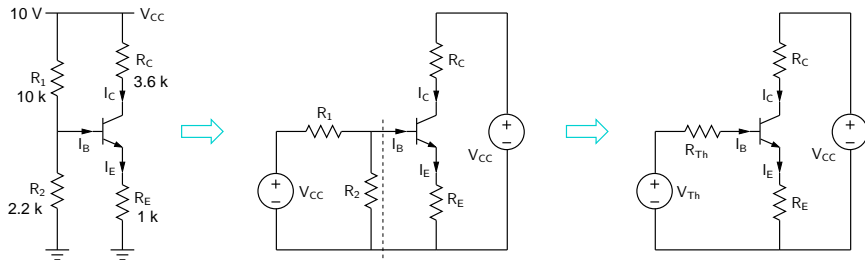
BJT amplifier: improved biasing scheme



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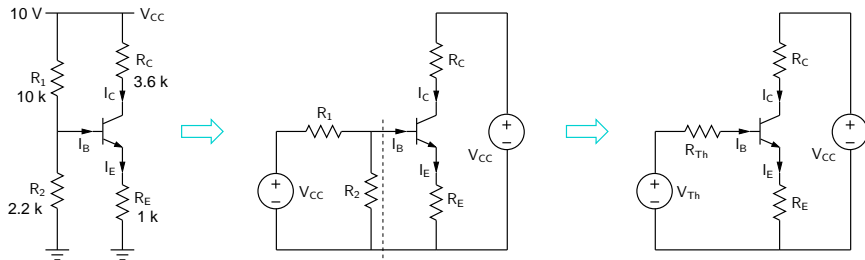


BJT amplifier: improved biasing scheme



$$V_{Th} = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{2.2\text{ k}}{10\text{ k} + 2.2\text{ k}} \times 10\text{ V} = 1.8\text{ V}, \quad R_{Th} = R_1 \parallel R_2 = 1.8\text{ k}$$

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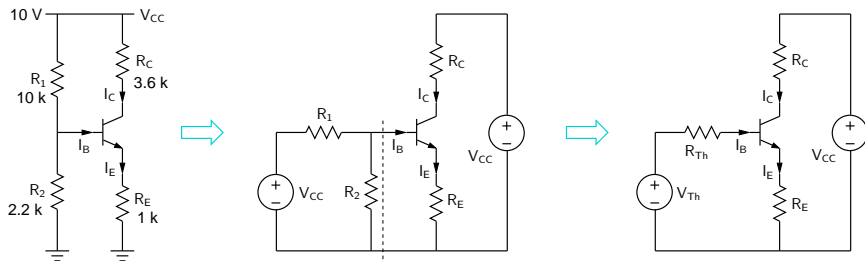


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Assuming the BJT to be in the active mode,

$$\text{KCL: } V_{Th} = R_{Th} I_B + V_{BE} + R_E I_E = R_{Th} I_B + V_{BE} + (\beta + 1) I_B R_E$$

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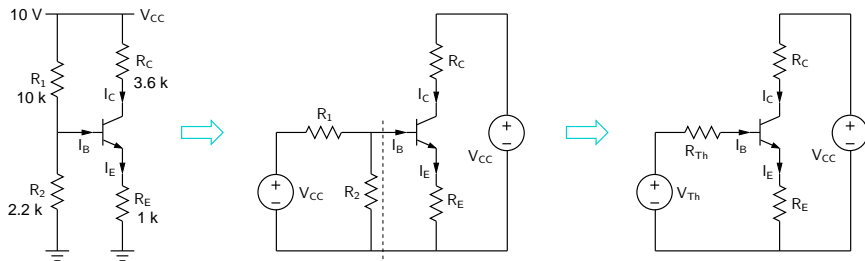
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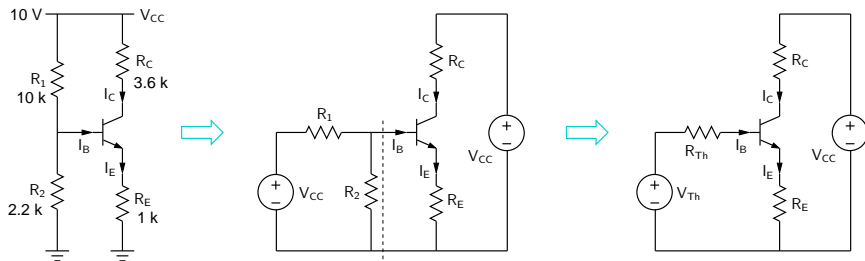
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For $\beta = 100$, $I_C = 1.07\text{ mA}$.

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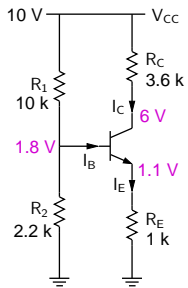
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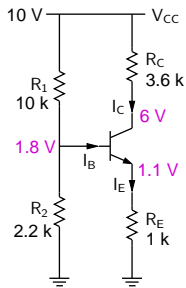
For $\beta = 200$, $I_C = 1.085\text{ mA}$.

BJT amplifier: improved biasing scheme (continued)



With $I_C = 1.1$ mA, the various DC (“bias”) voltages are,

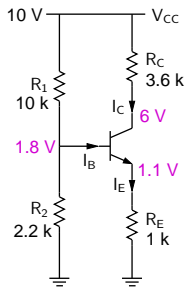
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$$V_E = I_E R_E \approx 1.1\text{ mA} \times 1\text{ k} = 1.1\text{ V},$$

BJT amplifier: improved biasing scheme (continued)

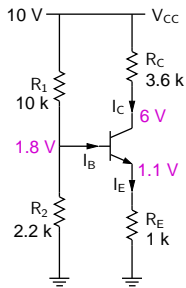


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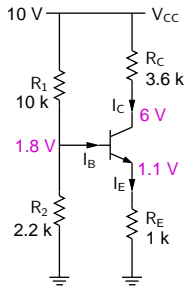
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$$V_C = V_{CC} - I_C R_C = 10 \text{ V} - 1.1 \text{ mA} \times 3.6 \text{ k} \approx 6 \text{ V},$$

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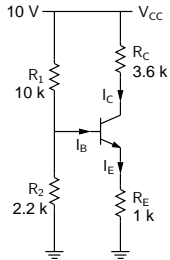
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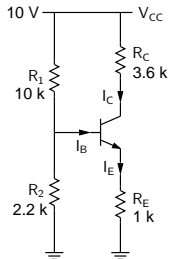
$$V_{CE} = V_C - V_E = 6 - 1.1 = 4.9 \text{ V}.$$

BJT amplifier: improved biasing scheme (continued)



A quick estimate of the bias values can be obtained by ignoring I_B (which is fair if β is large). In that case,

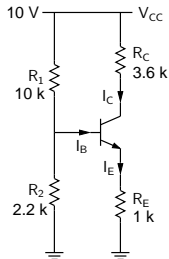
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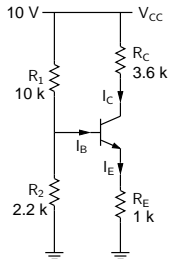


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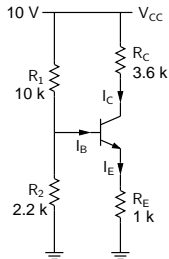
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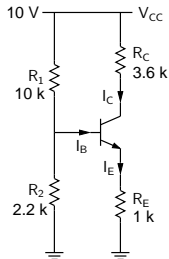
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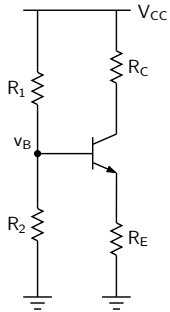
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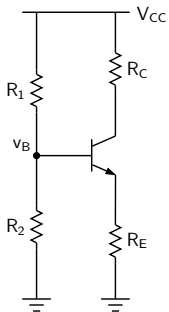
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Adding signal to bias

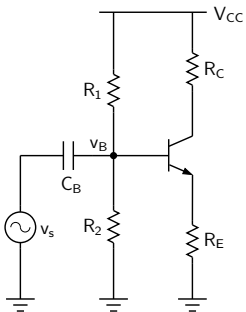


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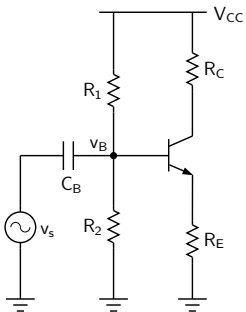
- * As we have seen earlier, the input signal $v_s(t) = \hat{V} \sin \omega t$ (for example) needs to be mixed with the desired bias value V_B so that the net voltage at the base is $v_B(t) = V_B + \hat{V} \sin \omega t$.

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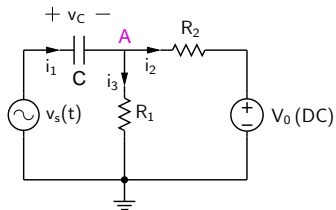
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- * Let us take a simple circuit to illustrate how a coupling capacitor works.

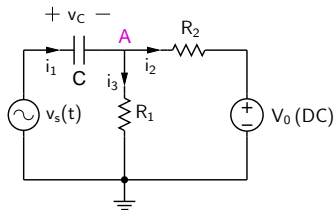
Coupling capacitor: example



Let v_A be the instantaneous node voltage at A. KCL gives,

$$-i_1 + i_3 + i_2 = 0 \rightarrow -C \frac{dv_C}{dt} + \frac{v_A}{R_1} + \frac{v_A - V_0}{R_2} = 0.$$

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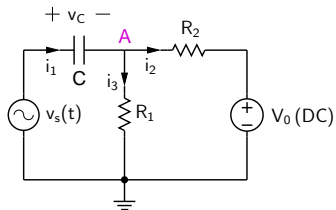


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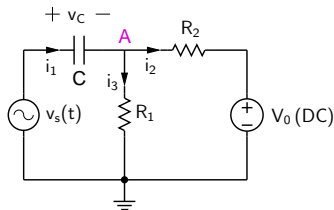
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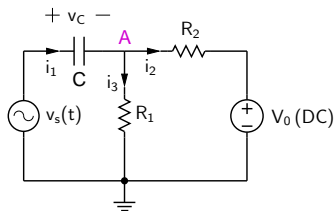
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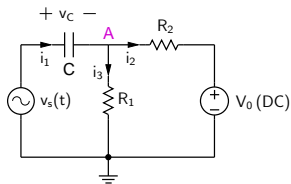
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$$-C \frac{d(-V_A)}{dt} + \frac{V_A}{R_1} + \frac{V_A - V_0}{R_2} = 0, \text{ and } -C \frac{d(v_s - v_a)}{dt} + \frac{v_a}{R_1} + \frac{v_a}{R_2} = 0.$$

Coupling capacitor: example (continued)

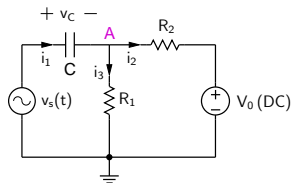


Since $\frac{dV_A}{dt} = 0$ (V_A being constant), we get

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Coupling capacitor: example (continued)

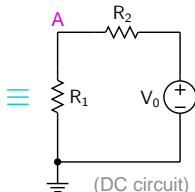
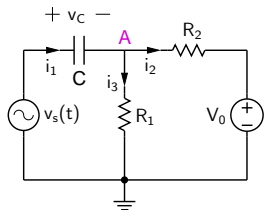


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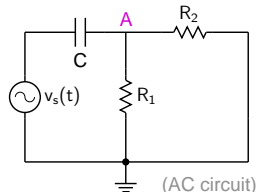
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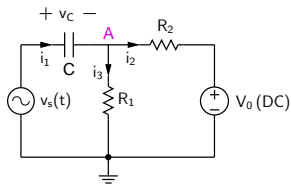
In other words, the original circuit can be thought of as



AND



Coupling capacitor: example (continued)

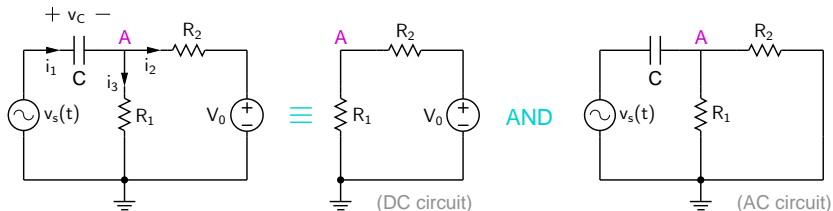


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We can get V_A from the first circuit, $v_a(t)$ from the second, and then combine them to get the actual $v_A(t)$: $v_A(t) = V_A + v_a(t)$

Capacitors in an amplifier circuit

- * Split the original circuit into two circuits:
 - DC circuit: replace each capacitor with an open circuit, each AC voltage source with a short circuit.
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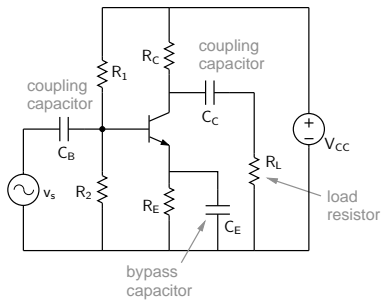
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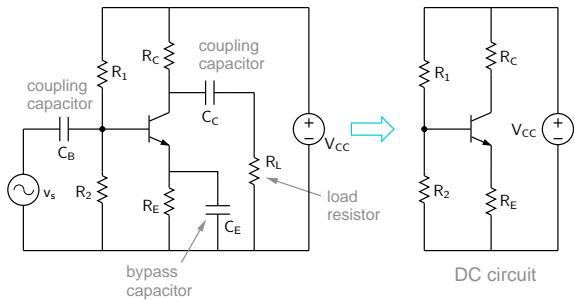
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- * The procedure described above also applies to a nonlinear circuit such as a BJT amplifier IF the “small-signal approximation” is satisfied. We will soon see what that means.

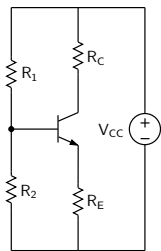
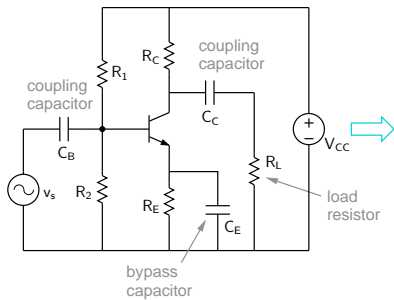
Common-emitter amplifier



Common-emitter amplifier

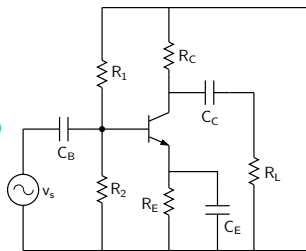


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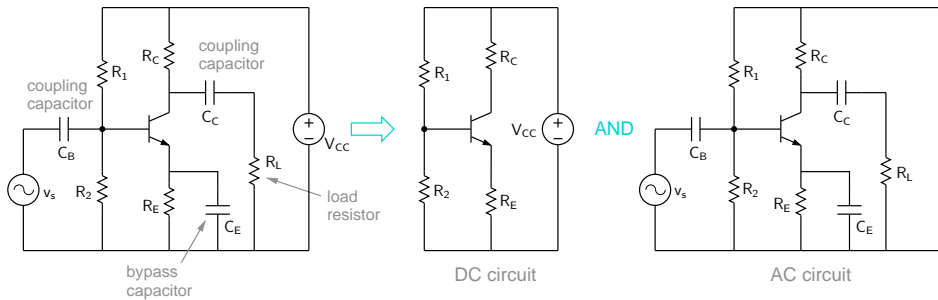
DC circuit

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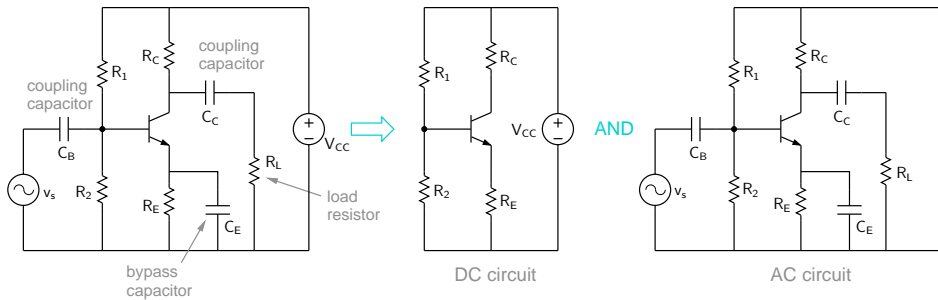
AC circuit

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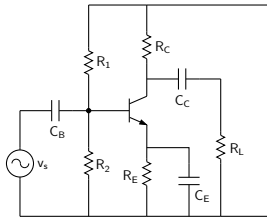
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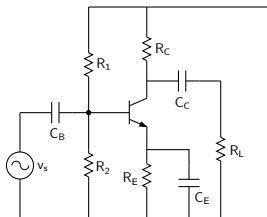


- * The coupling capacitors ensure that the signal source and the load resistor do not affect the DC bias of the amplifier. (We will see the purpose of C_E a little later.)
- * This enables us to bias the amplifier without worrying about what load it is going to drive.

Common-emitter amplifier: AC circuit



Common-emitter amplifier: AC circuit



- * The coupling and bypass capacitors are “large” (typically, a few μF), and at frequencies of interest, their impedance is small.

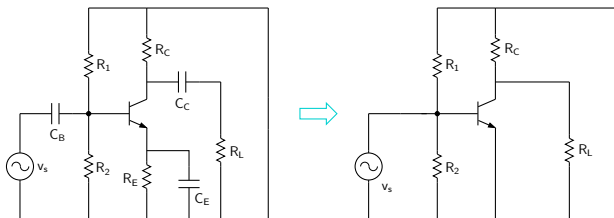
For example, for $C = 10 \mu F$, $f = 1 \text{ kHz}$,

$$Z_C = \frac{1}{2\pi \times 10^3 \times 10 \times 10^{-6}} = 16 \Omega,$$

which is much smaller than typical values of R_1 , R_2 , R_C , R_E (a few $\text{k}\Omega$).

$\Rightarrow C_B$, C_C , C_E can be replaced by short circuits at the frequencies of interest.

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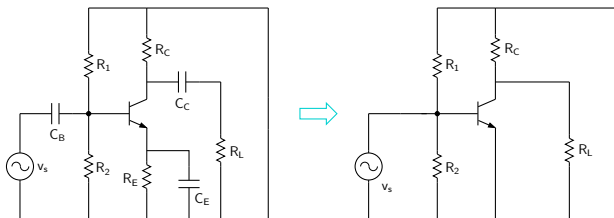
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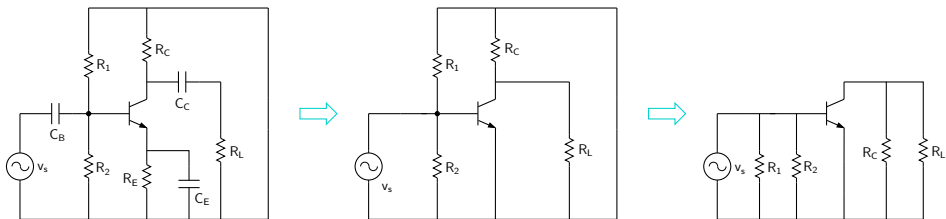
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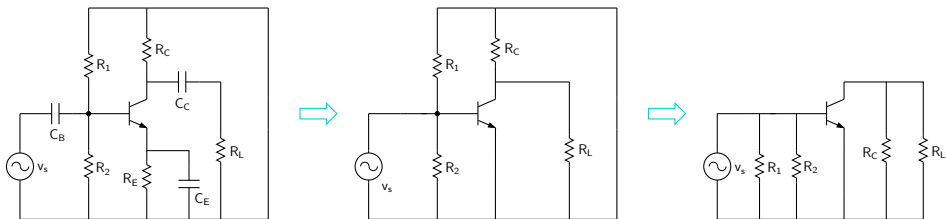
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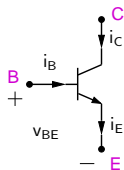
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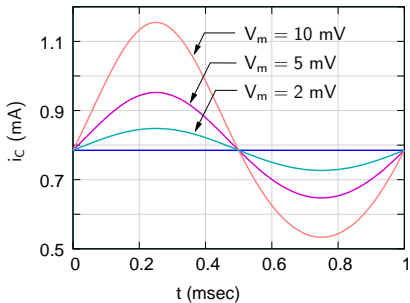
- * The circuit can be re-drawn in a more friendly format.
- * We now need to figure out the AC description of a BJT.

BJT amplifier: basic operation

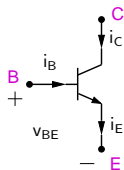


$$v_{BE}(t) = V_0 + V_m \sin \omega t$$

$$V_0 = 0.65 \text{ V}, f = 1 \text{ kHz}$$

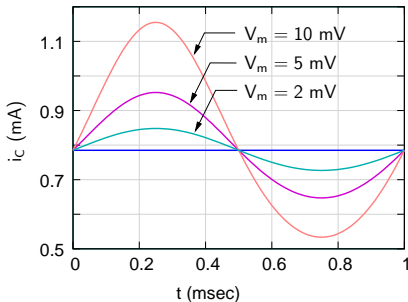


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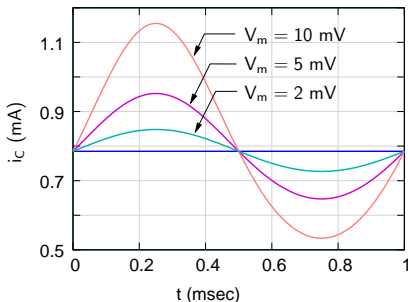
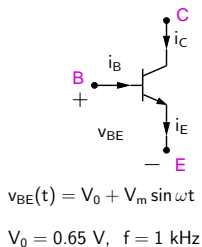
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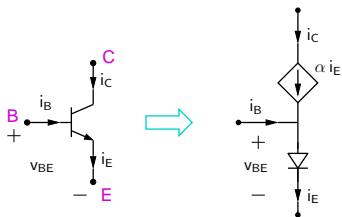
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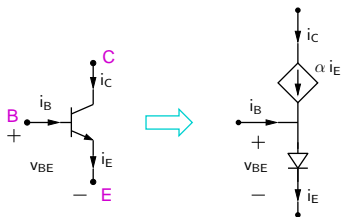
- * As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid \rightarrow distortion.
- * If $v_{be}(t)$, i.e., the time-varying part of v_{BE} , is kept small, i_C varies linearly with v_{BE} . How small? Let us look at this in more detail.

BJT: small-signal model



Let $v_{BE}(t) = V_{BE} + v_{be}(t)$ (bias+signal), and $i_C(t) = I_C + i_c(t)$.

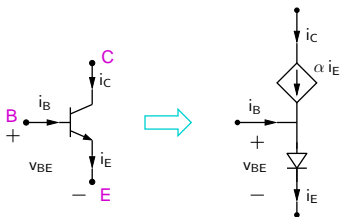
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Assuming active mode, $i_C(t) = \alpha i_E(t) = \alpha I_{ES} \left[\exp\left(\frac{v_{BE}(t)}{V_T}\right) - 1 \right]$.

BJT: small-signal model



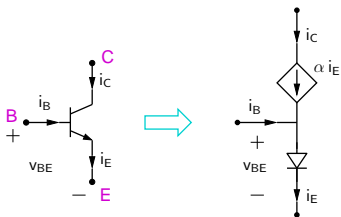
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Since the B-E junction is forward-biased, $\exp\left(\frac{v_{BE}(t)}{V_T}\right) \gg 1$, and we get

$$\begin{aligned} i_C(t) &= \alpha I_{ES} \exp\left(\frac{v_{BE}(t)}{V_T}\right) = \alpha I_{ES} \exp\left(\frac{V_{BE} + v_{be}(t)}{V_T}\right) \\ &= \alpha I_{ES} \exp\left(\frac{V_{BE}}{V_T}\right) \times \exp\left(\frac{v_{be}(t)}{V_T}\right) \end{aligned}$$

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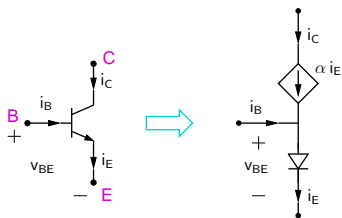
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If $v_{be}(t) = 0$, $i_C(t) = I_C$ (the bias value of i_C), i.e.,

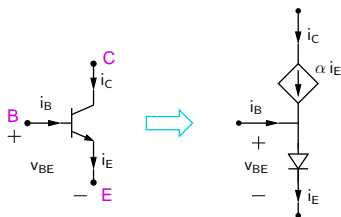
$$I_C = \alpha I_{ES} \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow i_C(t) = I_C \exp\left(\frac{v_{be}(t)}{V_T}\right).$$

BJT: small-signal model



$$i_C(t) = I_C \exp\left(\frac{v_{be}(t)}{V_T}\right) = I_C [1 + x + x^2 + \dots], \quad x = v_{be}(t)/V_T.$$

BJT: small-signal model

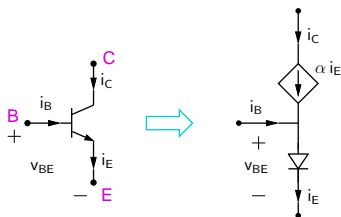


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$$i_C(t) = I_C \left[1 + \frac{v_{be}(t)}{V_T}\right].$$

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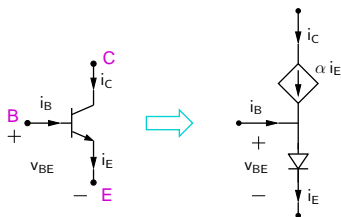
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BJT: small-signal model



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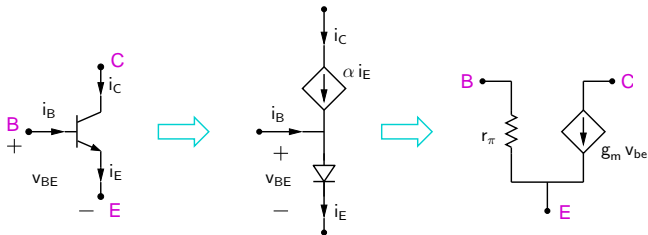
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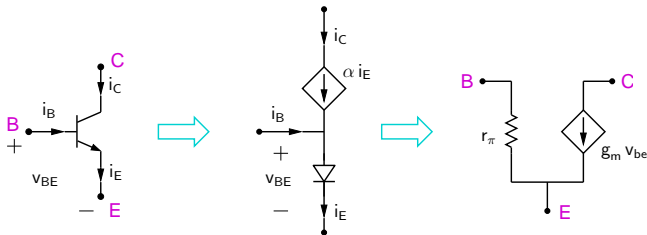
$$i_C(t) = I_C + i_c(t) = I_C \left[1 + \frac{v_{be}(t)}{V_T}\right] \Rightarrow \boxed{i_c(t) = \frac{I_C}{V_T} v_{be}(t)}$$

BJT: small-signal model



The relationship, $i_c(t) = \frac{I_C}{V_T} v_{be}(t)$ can be represented by a VCCS, $i_c(t) = g_m v_{be}(t)$, where $g_m = I_C/V_T$.

BJT: small-signal model



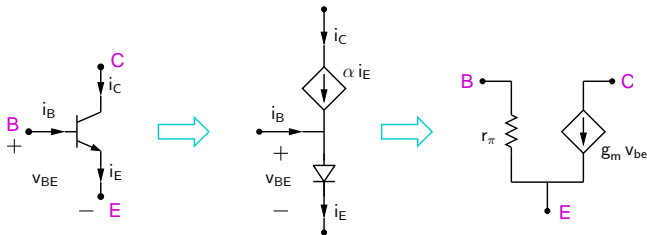
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For the base current, we have,

$$i_B(t) = I_B + i_b(t) = \frac{1}{\beta} [I_C + i_c(t)]$$

$$\rightarrow i_b(t) = \frac{1}{\beta} i_c(t) = \frac{1}{\beta} g_m v_{be}(t) \rightarrow v_{be}(t) = (\beta/g_m) i_b(t).$$

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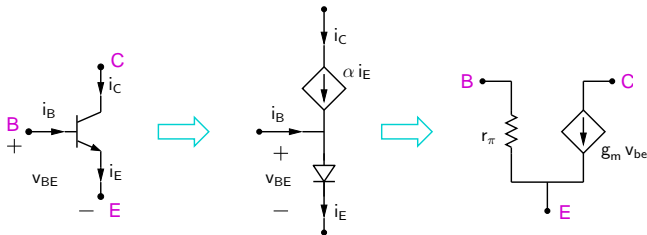
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The above relationship is represented by a resistance, $r_\pi = \beta/g_m$, connected between B and E.

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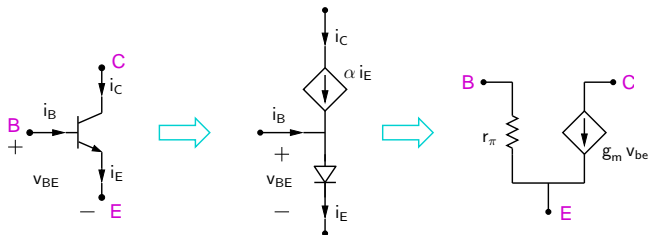
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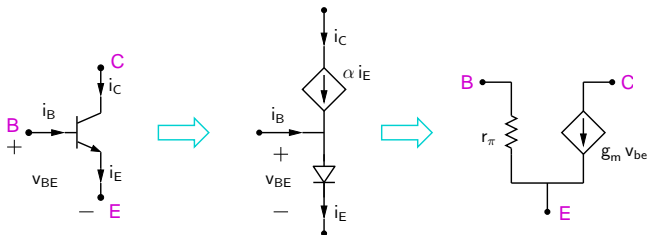
The resulting model is called the π -model for small-signal description of a BJT.

BJT: small-signal model



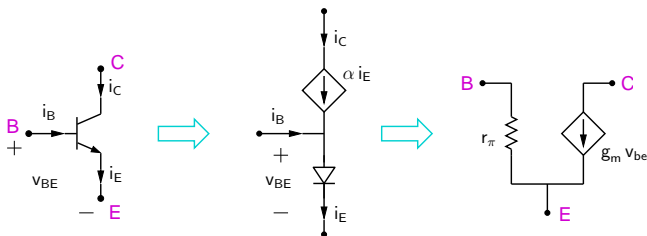
- * The transconductance g_m depends on the biasing of the BJT, since $g_m = I_C/V_T$. For $I_C = 1 \text{ mA}$, $V_T \approx 25 \text{ mV}$ (room temperature), $g_m = 1 \text{ mA}/25 \text{ mV} = 40 \text{ m}\Omega$.

BJT: small-signal model



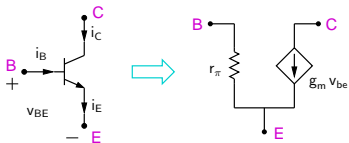
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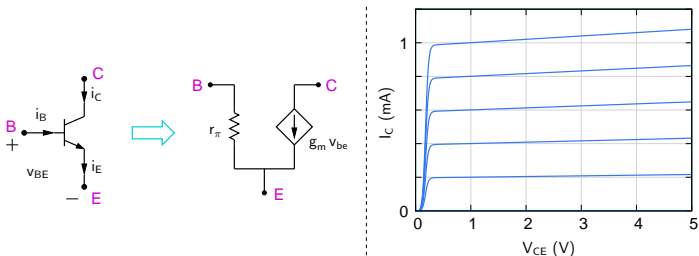
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- * Note that the small-signal model is valid only for small v_{be} (compared to V_T), as we have seen earlier.

BJT: small-signal model



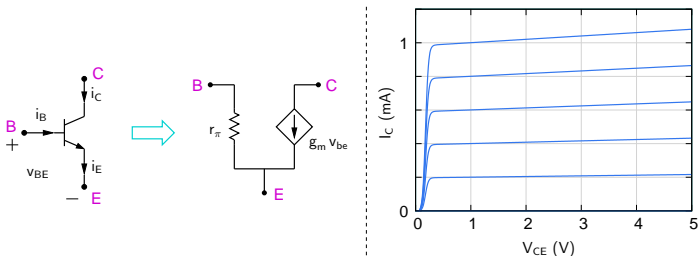
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BJT: small-signal model



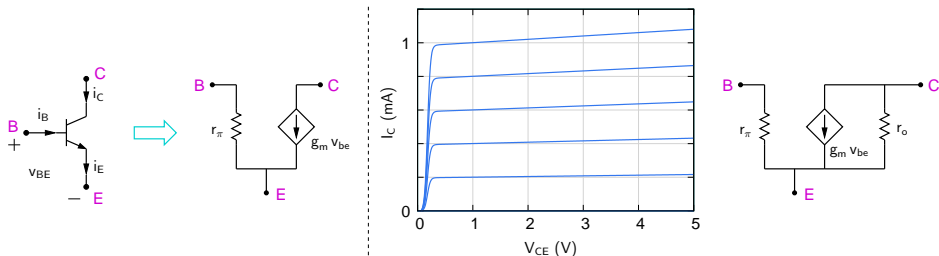
- * In the above model, note that i_C is independent of v_{ce} .
- * In practice, i_C does depend on v_{ce} because of the Early effect, and $\frac{di_C}{dV_{CE}} \approx \text{constant} = 1/r_o$, where r_o is called the output resistance.

BJT: small-signal model



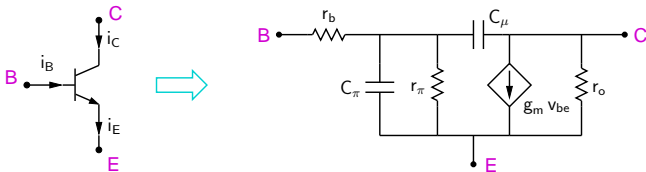
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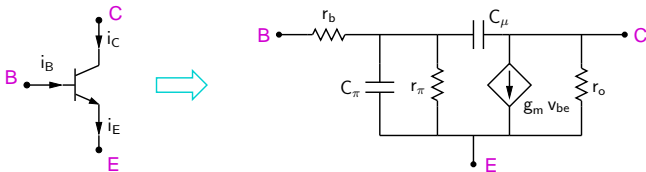
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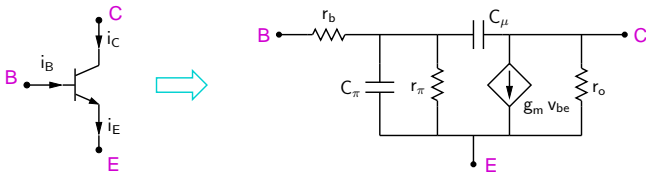
- * A few other components are required to make the small-signal model complete:
 - r_b : base spreading resistance
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- * Note that the small-signal models we have described are valid in the active region only.