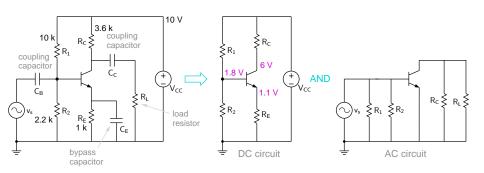
EE101: BJT circuits (Part 2)



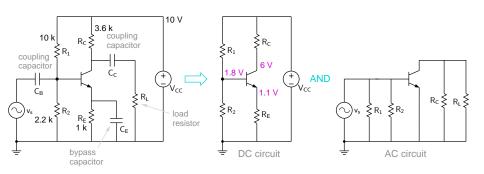
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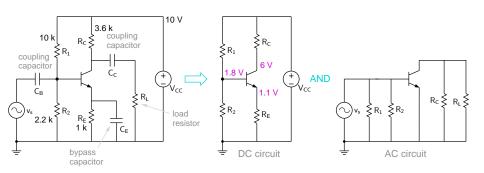
Department of Electrical Engineering Indian Institute of Technology Bombay



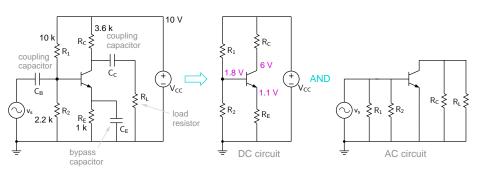
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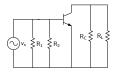
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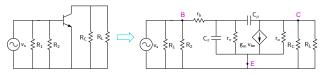


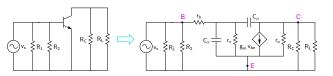
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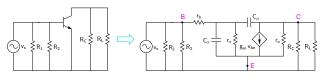
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- * We will then get the complete solution by simply adding the DC and AC results, e.g., $i_C(t) = I_C + i_C(t)$.
- * We will assume that C_B , C_C , C_E are large enough so that, at the signal frequency (say, 1 kHz), they can be replaced by short circuits.



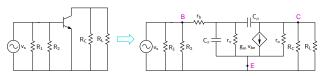




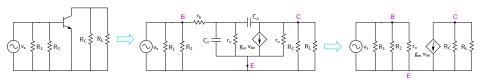
- * The parasitic capacitances C_{π} and C_{μ} are in the pF range. At a signal frequency of 1 kHz, their impedance is $1/\omega C \sim 1/(2\pi \times 10^3 \times 10^{-12})$, i.e., $\sim 100 \, \text{M}\Omega$. ightarrow C_{π} and C_{μ} can be replaced by open circuits.



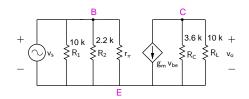
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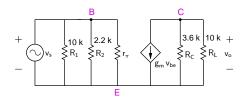
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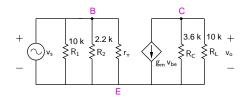
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(superscript L is used because the gain includes the effect of R_L .)

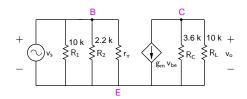


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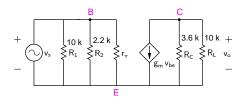
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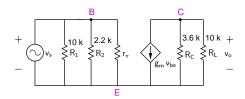
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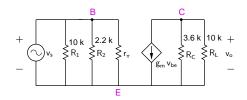
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For $v_s(t) = (2 \text{ mV}) \sin \omega t$, the AC output voltage is,

$$v_o = A_V^L v_s = -(112.5)(2 \,\mathrm{mV}) \sin \omega t = -(125 \,\mathrm{mV}) \sin \omega t$$



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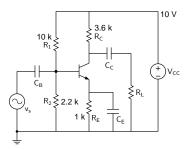
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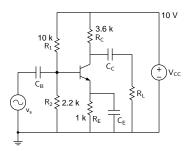
$$i_c = g_m v_{be} = g_m v_s = -42.5 \,\mathrm{m} \Im \times (2 \,\mathrm{m} V) \,\sin \omega t = -85 \,\sin \omega t \,\mu A.$$



For $v_s(t)=(2\,\mathrm{m}\,V)\sin\omega t$, we can now obtain expressions for the instantaneous currents and voltages:

$$v_C(t) = V_C + v_c(t) = V_C + v_o(t) = 6 \ V - (125 \ \mathrm{m} V) \sin \omega t \ .$$

$$i_C(t) = I_C + i_c(t) = 1.1\,\mathrm{m}A - 0.085\,\sin\omega t\,\,\mathrm{m}A\,.$$

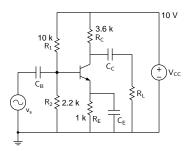


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Note that the above procedure (DC + AC analysis) can be used *only if* the small-signal approximation (i.e., $|v_{be}| \ll V_T$) is valid. In the above example, the amplitude of v_{be} is 2 mV, which is much smaller than V_T .



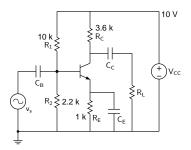
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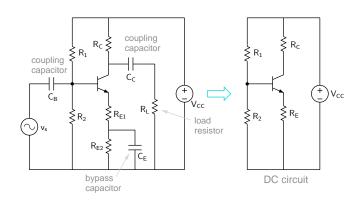
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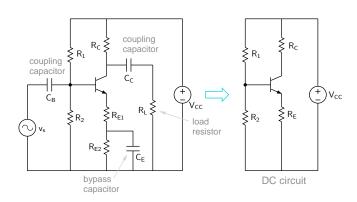
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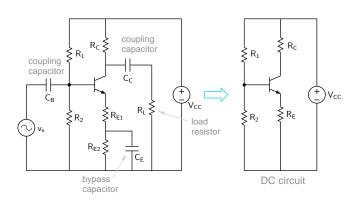
For $v_s(t) = (20 \,\mathrm{mV}) \sin \omega t$, for example, the small-signal approximation will not hold, and a numerical simulation will be required to obtain the currents and voltages of interest.

In practice, such a situation is anyway not prevalent (because it gives rise to distortion in the output voltage) except in special types of amplifiers.

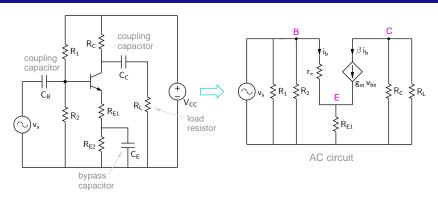




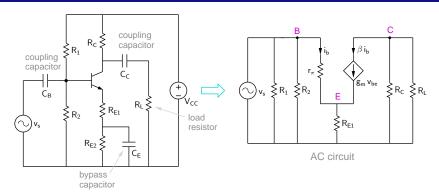
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- * Bypassing a part of R_E (as opposed to all of it) does have an impact on the voltage gain (see next slide).

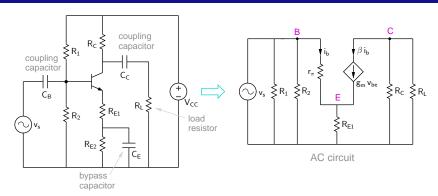


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$$v_s = v_{be} = i_b r_{\pi} + (\beta + 1) i_b R_{E1} \rightarrow i_b = \frac{v_s}{r_{\pi} + (\beta + 1) R_{E1}}.$$

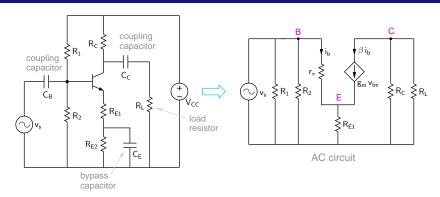


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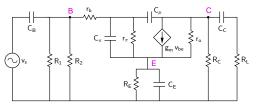
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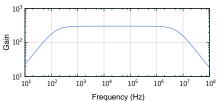
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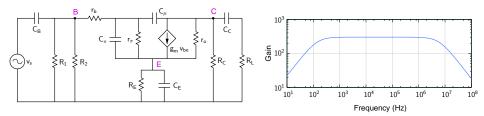
$$v_o = -\beta i_b \times (R_C \parallel R_L) \rightarrow \frac{v_o}{v_s} = -\frac{\beta (R_C \parallel R_L)}{r_\pi + (\beta + 1) R_{E1}}$$

Note: R_{E1} gets multiplied by $(\beta + 1)$.

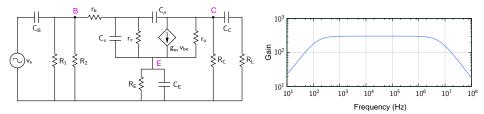




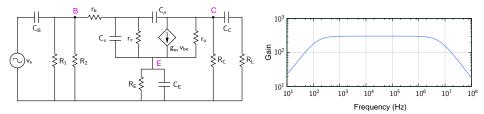




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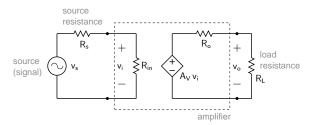


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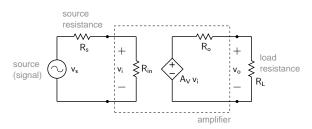
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- * In the intermediate range (called "mid-band"), the large capacitances behave like short circuits, and the small capacitances behave like open circuits. In this range, the gain is independent of frequency.

General representation of an amplifier



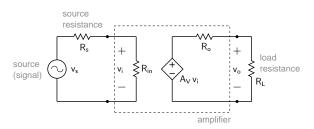
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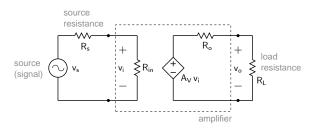
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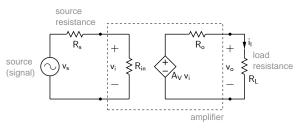
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- * The DC bias of the circuit can affect parameter values in the AC equivalent circuit $(A_V, R_{\rm in}, R_{\rm o})$. For example, for the common-emitter amplifier, $A_V \propto g_m = I_C/V_T$, I_C being the DC (bias) value of the collector current.

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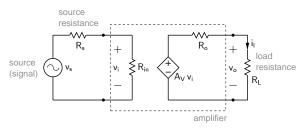
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- * Suppose we are given an amplifier as a "black box" and asked to find A_V , $R_{\rm in}$, and R_o . What experiments would give us this information?

Voltage gain A_V



If $R_L \to \infty$, $i_l \to 0$, and $v_o \to A_V v_i$.

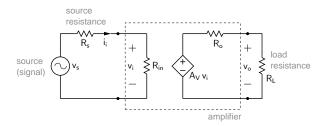
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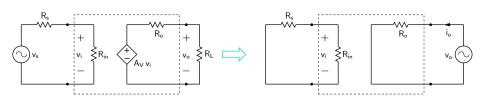
We can remove R_L (i.e., replace it with an open circuit), measure v_i and v_o , then use $A_V = v_o/v_i$.

Input resistance R_{in}



Measurement of v_i and i_i yields $R_{in} = v_i/i_i$.

Output resistance R_o



Method 1:

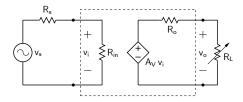
If $v_s \rightarrow 0$, $A_V v_i \rightarrow 0$.

Now, connect a test source v_o , and measure i_o .

Clearly, $R_o = v_o/i_o$.

This method works fine on paper, but it is difficult to use experimentally.

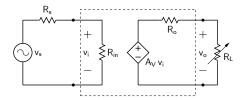
Output resistance R_o



Method 2:

$$v_o = \frac{R_L}{R_L + R_o} A_V v_i.$$

Output resistance Ro

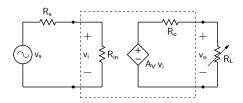


Method 2:

$$v_o = \frac{R_L}{R_L + R_o} A_V v_i$$

If
$$R_L \to \infty$$
, $v_{o1} = A_V v_i$.

Output resistance R_o



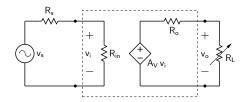
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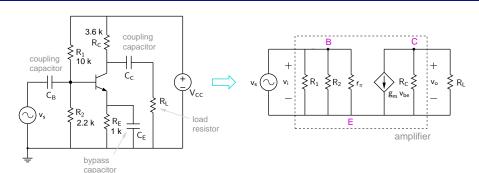
Procedure:

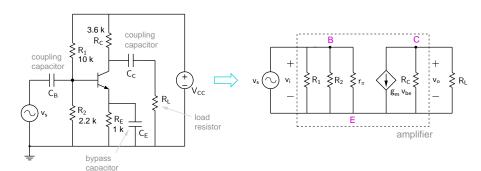
Measure v_{o1} with $R_L \to \infty$ (i.e., R_L removed).

Vary R_L to obtain $v_o = v_{o1}/2$.

The corresponding R_L is the same as R_o .

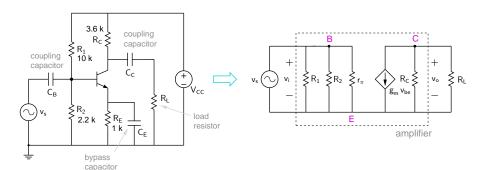






$$A_V = \frac{v_o}{v_i}$$
, with $R_L \to \infty$.

$$A_V = \frac{-g_m v_{be} R_C}{v_i} = -g_m R_C = -42.5 \,\mathrm{m} \odot \times 3.6 \,\mathrm{k}{=}153.$$

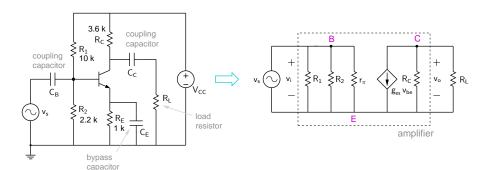


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The input resistance of the amplifier is, by inspection, $R_{\rm in} = (R_1 \parallel R_2) \parallel r_{\pi}$.

$$r_{\pi} = \beta/g_m = 100/42.5 \,\mathrm{m} \mho = 2.35 \,\mathrm{k} \to R_{\mathrm{in}} = 1.24 \,\mathrm{k}.$$



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The output resistance is R_C (by "Method 1" seen previously).

