EE101: Bode plots

M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay
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* Bel turned out to be too large in practice → deciBel (i.e., one tenth of a Bel).
What is deciBel (dB)?

* dB is a unit that describes a quantity, on a log scale, with respect to a reference quantity.

\[ X \text{ (in dB)} = 10 \log_{10} \left( \frac{X}{X_{\text{ref}}} \right). \]
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\[ X \text{ (in dB)} = 10 \log_{10} (X/X_{\text{ref}}). \]

For example, if \( P_1 = 20 \text{ W} \) and \( P_{\text{ref}} = 1 \text{ W} \),
\[ P_1 = 10 \log (20 \text{ W}/1 \text{ W}) = 10 \log (20) = 13 \text{ dB}. \]
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For example, if \( V_1 = 1.2 \text{ V} \), \( V_{\text{ref}} = 1 \text{ mV} \), then

\[ V_1 = 10 \log \left( \frac{1.2 \text{ V}}{1 \text{ mV}} \right)^2 = 20 \log \left( 1.2/10^{-3} \right) = 61.6 \text{ dBm}. \]
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* The voltage gain of an amplifier is 
\[ A_V \text{ in dB} = 20 \log \left( \frac{V_o}{V_i} \right), \]
with \( V_i \) serving as the reference voltage.
Example

Given $V_i = 2.5 \text{ mV}$ and $A_V = 36.3 \text{ dB}$, compute $V_o$ in dBm and in mV. 

($V_i$ and $V_o$ are peak input and peak output voltages, respectively).
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**Method 1:**

\[
V_i = 20 \log \left( \frac{2.5 \text{ mV}}{1 \text{ mV}} \right) = 7.96 \text{ dBm}
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Since \( V_o \) (dBm) = 20 log \( \left( \frac{V_o \text{ (in mV)}}{1 \text{ mV}} \right) \),

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V_o = 10^x \times 1 \text{ mV}, \text{ where}
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x = \frac{1}{20} V_o \text{ (in dBm)}
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$\rightarrow V_o = 162.5 \text{ mV}$. 

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Example

\[ V_i \rightarrow \text{Amplifier} \rightarrow V_o \]

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**Method 2:**

\[
A_V = 36.3 \text{ dB}
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\rightarrow 20 \log A_V = 36.3 \rightarrow A_V = 65.
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$$A_V = 36.3 \text{ dB}$$

$$\rightarrow 20 \log A_V = 36.3 \rightarrow A_V = 65.$$  

$$V_o = A_V \times V_i = 65 \times 2.5 \text{ mV} = 162.5 \text{ mV}.$$
* When sound intensity is specified in dB, the reference pressure is $P_{\text{ref}} = 20 \mu Pa$ (our hearing threshold).

If the pressure corresponding to the sound being measured is $P$, we say that it is $20 \log \left( \frac{P}{P_{\text{ref}}} \right)$ dB.
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- normal conversation: 60 to 70 dB

windows break: 163 dB
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The transfer function of a circuit such as an amplifier or a filter is given by,
\[ H(s) = \frac{V_o(s)}{V_i(s)}, \]
where \( s = j\omega \).

\[ H(j\omega) = K_1 + s\tau = K_1 + j\omega\tau \]

* \( H(j\omega) \) is a complex number, and a complete description of \( H(j\omega) \) involves
  (a) a plot of \( |H(j\omega)| \) versus \( \omega \).
  (b) a plot of \( \angle H(j\omega) \) versus \( \omega \).

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A simple transfer function

\[ V_o = \frac{(1/sC)}{R + (1/sC)} V_s, \]

\[ \rightarrow H(s) = \frac{1}{1 + sRC} = \frac{1}{1 + (j\omega/\omega_0)}, \]

\[ \omega_0 = \frac{1}{RC}. \]
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\[ V_o = \frac{(1/s C)}{R + (1/s C)} V_s, \]

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* The circuit behaves like a low-pass filter.
  
  For \( \omega \ll \omega_0 \), \(|H(j\omega)| \rightarrow 1.\)
  
  For \( \omega \gg \omega_0 \), \(|H(j\omega)| \propto 1/\omega.\)
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* The magnitude and phase of \(H(j\omega)\) are given by,

\[ |H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right). \]
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* We are generally interested in a large variation in \( \omega \) (several orders), and its effect on \(|H|\) and \(\angle H\).
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\[ H(s) = \frac{1}{1+sRC} = \frac{1}{1+(j\omega/\omega_0)}, \]

\[ V_o = \frac{1}{sC}R + \frac{1}{sC}V_s, \]

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* We are generally interested in a large variation in \( \omega \) (several orders), and its effect on \(|H|\) and \(\angle H\).

* The magnitude \(|H|\) varies by orders of magnitude as well.
  
  The phase \(\angle H\) varies from 0 (for \( \omega \ll \omega_0 \)) to \(-\pi/2\) (for \( \omega \gg \omega_0 \)).
A simple transfer function: magnitude

$$V_o = \frac{(1/sC) V_s}{R + (1/sC)}$$

$$\rightarrow H(s) = \frac{1}{1+sRC} = \frac{1}{1+(j\omega/\omega_0)}.$$  

$$\omega_0 = \frac{1}{RC}.$$  

Since $$\omega$$ and $$|H(j\omega)|$$ vary by several orders of magnitude, a linear $$\omega$$- or $$|H|$$-axis is not appropriate $\rightarrow$ log $$|H|$$ is plotted against log $$\omega$$.

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\[ H(s) = \frac{1}{1 + \left(\frac{j\omega}{\omega_0}\right)} \]

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Note that the shape of the plot does not change.

\[ |H| \text{ (dB)} = 20 \log |H| \text{ is simply a scaled version of } \log |H|. \]
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\[ H(s) = \frac{1}{1+s\frac{1}{RC}} = \frac{1}{1+(j\omega/\omega_0)}, \]

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Since \( \omega = 2\pi f \), the shape of the plot does not change.
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A simple transfer function: phase

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\[ V_o = \frac{(1/sC)}{R + (1/sC)} V_s , \]

\[ \Rightarrow \]

\[ \angle H = -\tan^{-1}(\omega/\omega_0) \]

Since \( \angle H \) varies in a limited range (0° to −90° in this example), a linear axis is appropriate for \( \angle H \).

As in the magnitude plot, we use a log axis for \( \omega \), since we are interested in a wide range of \( \omega \).

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* As in the magnitude plot, we use a log axis for \( \omega \), since we are interested in a wide range of \( \omega \).
Consider \( H(s) = \frac{K (1 + s/z_1)(1 + s/z_2) \cdots (1 + s/z_M)}{(1 + s/p_1)(1 + s/p_2) \cdots (1 + s/p_N)} \).

\( -z_1, -z_2, \cdots \) are called the “zeros” of \( H(s) \).

\( -p_1, -p_2, \cdots \) are called the “poles” of \( H(s) \).

(In addition, there could be terms like \( s, s^2, \cdots \) in the numerator.)

We will assume, for simplicity, that the zeros (and poles) are real and distinct.

Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function \( \omega \).

(b) combining the various contributions to obtain \( |H| \) and \( \angle H \) versus \( \omega \).
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Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function \(\omega\).
Consider $H(s) = \frac{K (1 + s/z_1)(1 + s/z_2) \cdots (1 + s/z_M)}{(1 + s/p_1)(1 + s/p_2) \cdots (1 + s/p_N)}$.

$-z_1, -z_2, \cdots$ are called the “zeros” of $H(s)$.

$-p_1, -p_2, \cdots$ are called the “poles” of $H(s)$.

(In addition, there could be terms like $s, s^2, \cdots$ in the numerator.)

We will assume, for simplicity, that the zeros (and poles) are real and distinct.

Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function $\omega$.

(b) combining the various contributions to obtain $|H|$ and $\angle H$ versus $\omega$. 
Consider $H(s) = \frac{1}{1 + s/p} \rightarrow H(j\omega) = \frac{1}{1 + j(\omega/p)}$, $|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/p)^2}}$. 

Note that, at $\omega = p$, the actual value of $|H|$ is $1/\sqrt{2}$ (i.e., $-3$ dB).
Consider $H(s) = \frac{1}{1 + s/p} \rightarrow H(j\omega) = \frac{1}{1 + j(\omega/p)}$, $|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/p)^2}}$.

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1$, $20 \log |H| = 0$ dB.
Consider \( H(s) = \frac{1}{1 + s/p} \rightarrow H(j\omega) = \frac{1}{1 + j(\omega/p)} \), \(|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/p)^2}} \).

Asymptote 1: \( \omega \ll p \): \(|H| \rightarrow 1, \quad 20 \log |H| = 0 \text{ dB} \).

Asymptote 2: \( \omega \gg p \): \(|H| \rightarrow \frac{1}{\omega/p} = \frac{p}{\omega} \rightarrow |H| = 20 \log p - 20 \log \omega \text{ (dB)} \).
Consider $H(s) = \frac{1}{1 + s/p} \rightarrow H(j\omega) = \frac{1}{1 + j(\omega/p)}$, $|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/p)^2}}$.

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1, \ 20 \log |H| = 0 \text{ dB}$.

Asymptote 2: $\omega \gg p$: $|H| \rightarrow \frac{1}{\omega/p} = \frac{p}{\omega} \rightarrow |H| = 20 \log p - 20 \log \omega \ (\text{dB})$

Consider two values of $\omega$: $\omega_1$ and $10\omega_1$.

$|H|_1 = 20 \log p - 20 \log \omega_1 \ (\text{dB})$

$|H|_2 = 20 \log p - 20 \log (10\omega_1) \ (\text{dB})$
Contribution of a pole: magnitude

Consider \( H(s) = \frac{1}{1 + s/p} \rightarrow H(j\omega) = \frac{1}{1 + j(\omega/p)} \), \(|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/p)^2}}\).

Asymptote 1: \( \omega \ll p \): \(|H| \rightarrow 1, \quad 20 \log |H| = 0 \text{ dB} \).

Asymptote 2: \( \omega \gg p \): \(|H| \rightarrow \frac{1}{\omega/p} = \frac{p}{\omega} \rightarrow |H| = 20 \log p - 20 \log \omega \) (dB)

Consider two values of \( \omega \): \( \omega_1 \) and 10 \( \omega_1 \).

\(|H|_1 = 20 \log p - 20 \log \omega_1 \) (dB)

\(|H|_2 = 20 \log p - 20 \log (10 \omega_1) \) (dB)

\(|H|_1 - |H|_2 = -20 \log \frac{\omega_1}{10 \omega_1} = 20 \text{ dB} \).
Consider $H(s) = \frac{1}{1 + s/p} \rightarrow H(j\omega) = \frac{1}{1 + j(\omega/p)}$, $|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/p)^2}}$.

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1$, $20\log |H| = 0$ dB.
Asymptote 2: $\omega \gg p$: $|H| \rightarrow \frac{1}{\omega/p} = \frac{p}{\omega} \rightarrow |H| = 20 \log p - 20 \log \omega$ (dB)

Consider two values of $\omega$: $\omega_1$ and $10\omega_1$.

$|H|_1 = 20 \log p - 20 \log \omega_1$ (dB)
$|H|_2 = 20 \log p - 20 \log (10\omega_1)$ (dB)

$|H|_1 - |H|_2 = -20 \log \frac{\omega_1}{10\omega_1} = 20$ dB.

$\rightarrow |H|$ versus $\omega$ has a slope of $-20$ dB/decade.

Note that, at $\omega = p$, the actual value of $|H|$ is $1/\sqrt{2}$ (i.e., $-3$ dB).
Consider $H(s) = \frac{1}{1 + s/p} = \frac{1}{1 + j(\omega/p)} \rightarrow \angle H = -\tan^{-1}\left(\frac{\omega}{p}\right)$
Contribution of a pole: phase

Consider $H(s) = \frac{1}{1 + s/p} = \frac{1}{1 + j(\omega/p)} \rightarrow \angle H = -\tan^{-1}\left(\frac{\omega}{p}\right)$

Asymptote 1: $\omega \ll p$ (say, $\omega < p/10$): $\angle H = 0$. 

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Consider $H(s) = \frac{1}{1 + s/p} = \frac{1}{1 + j(\omega/p)} \rightarrow \angle H = -\tan^{-1}\left(\frac{\omega}{p}\right)$

Asymptote 1: $\omega \ll p$ (say, $\omega < p/10$): $\angle H = 0$.

Asymptote 2: $\omega \gg p$ (say, $\omega > 10p$): $\angle H = -\pi/2$. 
Consider $H(s) = \frac{1}{1 + s/p} = \frac{1}{1 + j(\omega/p)} \rightarrow \angle H = -\tan^{-1}\left(\frac{\omega}{p}\right)$

Asymptote 1: $\omega \ll p$ (say, $\omega < p/10$): $\angle H = 0$.

Asymptote 2: $\omega \gg p$ (say, $\omega > 10p$): $\angle H = -\pi/2$.

Asymptote 3: For $p/10 < \omega < 10p$, $\angle H$ is assumed to vary linearly with $\log \omega$
→ at $\omega = p$, $\angle H = -\pi/4$ (which is also the actual value of $\angle H$).
Consider \( H(s) = 1 + \frac{s}{z} \rightarrow H(j\omega) = 1 + j\left(\frac{\omega}{z}\right) \), \(|H(j\omega)| = \sqrt{1 + \left(\frac{\omega}{z}\right)^2}\).
Consider $H(s) = 1 + s/z \rightarrow H(j\omega) = 1 + j(\omega/z)$, $|H(j\omega)| = \sqrt{1 + (\omega/z)^2}$.

Asymptote 1:   $\omega \ll p$: $|H| \rightarrow 1$, $20 \log |H| = 0$ dB.
Consider $H(s) = 1 + s/z \rightarrow H(j\omega) = 1 + j(\omega/z)$, $|H(j\omega)| = \sqrt{1 + (\omega/z)^2}$.

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1, \ 20 \log |H| = 0$ dB.

Asymptote 2: $\omega \gg p$: $|H| \rightarrow \frac{\omega}{z} \rightarrow |H| = 20 \log \omega - 20 \log z$ (dB)
Consider $H(s) = 1 + s/z \rightarrow H(j\omega) = 1 + j(\omega/z)$, $|H(j\omega)| = \sqrt{1 + (\omega/z)^2}$.

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1$, $20 \log |H| = 0$ dB.

Asymptote 2: $\omega \gg p$: $|H| \rightarrow \frac{\omega}{z} \rightarrow |H| = 20 \log \omega - 20 \log z$ (dB)

Consider two values of $\omega$: $\omega_1$ and $10\omega_1$.

$|H|_1 = 20 \log \omega_1 - 20 \log z$ (dB)

$|H|_2 = 20 \log (10\omega_1) - 20 \log z$ (dB)
Contribution of a zero: magnitude

Consider $H(s) = 1 + s/z \rightarrow H(j\omega) = 1 + j(\omega/z)$, $|H(j\omega)| = \sqrt{1 + (\omega/z)^2}$.

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1$, $20 \log |H| = 0$ dB.

Asymptote 2: $\omega \gg p$: $|H| \rightarrow \frac{\omega}{z} \rightarrow |H| = 20 \log \omega - 20 \log z$ (dB)

Consider two values of $\omega$: $\omega_1$ and $10\omega_1$.

$|H|_1 = 20 \log \omega_1 - 20 \log z$ (dB)

$|H|_2 = 20 \log (10\omega_1) - 20 \log z$ (dB)

$|H|_1 - |H|_2 = 20 \log \frac{\omega_1}{10\omega_1} = -20$ dB.
Consider $H(s) = 1 + s/z \rightarrow H(j\omega) = 1 + j\left(\omega/z\right)$, $|H(j\omega)| = \sqrt{1 + (\omega/z)^2}$.

Asymptote 1: $\omega \ll p$: $|H| \rightarrow 1$, $20 \log |H| = 0$ dB.

Asymptote 2: $\omega \gg p$: $|H| \rightarrow \frac{\omega}{z} \rightarrow |H| = 20 \log \omega - 20 \log z$ (dB)

Consider two values of $\omega$: $\omega_1$ and $10 \omega_1$.

$|H|_1 = 20 \log \omega_1 - 20 \log z$ (dB)

$|H|_2 = 20 \log (10 \omega_1) - 20 \log z$ (dB)

$|H|_1 - |H|_2 = 20 \log \frac{\omega_1}{10 \omega_1} = -20$ dB.

$\rightarrow |H|$ versus $\omega$ has a slope of $+20$ dB/decade.

Note that, at $\omega = z$, the actual value of $|H|$ is $\sqrt{2}$ (i.e., 3 dB).
Consider \( H(s) = 1 + s/z = 1 + j(\omega/z) \rightarrow \angle H = \tan^{-1}\left(\frac{\omega}{z}\right) \)
Consider $H(s) = 1 + s/z = 1 + j(\omega/z) \rightarrow \angle H = \tan^{-1}\left(\frac{\omega}{z}\right)$

Asymptote 1: $\omega \ll z$ (say, $\omega < z/10$): $\angle H = 0$. 
Consider \( H(s) = 1 + s/z = 1 + j(\omega/z) \rightarrow \angle H = \tan^{-1}\left(\frac{\omega}{z}\right) \)

Asymptote 1: \( \omega \ll z \) (say, \( \omega < z/10 \)): \( \angle H = 0 \).

Asymptote 2: \( \omega \gg z \) (say, \( \omega > 10z \)): \( \angle H = \pi/2 \).
Consider \( H(s) = 1 + s/z = 1 + j(\omega/z) \rightarrow \angle H = \tan^{-1}\left(\frac{\omega}{z}\right) \)

Asymptote 1: \( \omega \ll z \) (say, \( \omega < z/10 \)): \( \angle H = 0 \).

Asymptote 2: \( \omega \gg z \) (say, \( \omega > 10z \)): \( \angle H = \pi/2 \).

Asymptote 3: For \( z/10 < \omega < 10z \), \( \angle H \) is assumed to vary linearly with \( \log \omega \rightarrow \) at \( \omega = z \), \( \angle H = \pi/4 \) (which is also the actual value of \( \angle H \)).
Contribution of $K$ (constant), $s$, and $s^2$

For $H(s) = K$, $20 \log|H| = 20 \log K$ (a constant), and $\angle H = 0$.
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For $H(s) = s$, i.e., $H(j\omega) = j\omega$, $|H| = \omega$. 
Contribution of \( K \) (constant), \( s \), and \( s^2 \)

For \( H(s) = K \), \( 20 \log |H| = 20 \log K \) (a constant), and \( \angle H = 0 \).

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For $H(s) = s$, i.e., $H(j\omega) = j\omega$, $|H| = \omega$.

$\rightarrow 20 \log |H| = 20 \log \omega$,
i.e., a straight line in the $|H|$ (dB)-log $\omega$ plane with a slope of 20 dB/decade, passing through $(1, 0)$. 
Contribution of $K$ (constant), $s$, and $s^2$

For $H(s) = K$, $20 \log |H| = 20 \log K$ (a constant), and $\angle H = 0$.

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$\angle H = \pi/2$ (irrespective of $\omega$).

For $H(s) = s^2$, i.e., $H(j\omega) = -\omega^2$, $|H| = \omega^2$. 
Contribution of $K$ (constant), $s$, and $s^2$

For $H(s) = K$, $20 \log |H| = 20 \log K$ (a constant), and $\angle H = 0$.

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$\rightarrow 20 \log |H| = 20 \log \omega$,
i.e., a straight line in the $|H|$ (dB)-log $\omega$ plane with a slope of 20 dB/decade,
passing through $(1, 0)$.
$\angle H = \pi/2$ (irrespective of $\omega$).

For $H(s) = s^2$, i.e., $H(j\omega) = -\omega^2$, $|H| = \omega^2$.

$\rightarrow 20 \log |H| = 40 \log \omega$,
Contribution of \( K \) (constant), \( s \), and \( s^2 \)

For \( H(s) = K \), \( 20 \log |H| = 20 \log K \) (a constant), and \( \angle H = 0 \).

For \( H(s) = s \), i.e., \( H(j\omega) = j\omega \), \( |H| = \omega \).
\[ \rightarrow 20 \log |H| = 20 \log \omega, \]
i.e., a straight line in the \(|H|(\text{dB})\)-log \( \omega \) plane with a slope of 20 dB/decade, passing through \((1, 0)\).
\[ \angle H = \pi/2 \] (irrespective of \( \omega \)).

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$\rightarrow 20 \log |H| = 40 \log \omega$,
i.e., a straight line in the $|H|$ (dB)-log $\omega$ plane with a slope of 40 dB/decade,
passing through $(1, 0)$.
$\angle H = \pi$ (irrespective of $\omega$).
Combining different terms

Consider $H(s) = H_1(s) \times H_2(s)$. 

Magnitude: $|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)|$.

Phase: $\angle H_1(j\omega) + \angle H_2(j\omega)$. 

In the Bode magnitude plot, the contributions due to $H_1(s)$ and $H_2(s)$ simply get added.

Phase: $\angle H(s) = \angle H_1(s) + \angle H_2(s)$. 

In the Bode phase plot, the contributions due to $H_1(s)$ and $H_2(s)$ also get added.

The same reasoning applies to more than two terms as well.
Combining different terms

Consider $H(s) = H_1(s) \times H_2(s)$.

**Magnitude:**

$|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)|$.

$20 \log |H| = 20 \log |H_1| + 20 \log |H_2|$.
Combining different terms

Consider $H(s) = H_1(s) \times H_2(s)$.

**Magnitude:**

$$|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)|.$$  

$$20 \log |H| = 20 \log |H_1| + 20 \log |H_2|.$$  

→ In the Bode magnitude plot, the contributions due to $H_1$ and $H_2$ simply get added.
Combining different terms

Consider $H(s) = H_1(s) \times H_2(s)$.

**Magnitude:**

$$|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)|.$$  
$$20 \log |H| = 20 \log |H_1| + 20 \log |H_2|.$$  
$\rightarrow$ In the Bode magnitude plot, the contributions due to $H_1$ and $H_2$ simply get added.

**Phase:**

$H_1(j\omega)$ and $H_2(j\omega)$ are complex numbers.

At a given $\omega$, let $H_1 = K_1 \angle \alpha = K_1 e^{j\alpha}$, and $H_2 = K_2 \angle \beta = K_2 e^{j\beta}$.

Then, $H_1 H_2 = K_1 K_2 e^{j(\alpha+\beta)} = K_1 K_2 \angle (\alpha + \beta)$.

i.e., $\angle H = \angle H_1 + \angle H_2$. 

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Combining different terms

Consider \( H(s) = H_1(s) \times H_2(s) \).

**Magnitude:**

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i.e., \( \angle H = \angle H_1 + \angle H_2 \).

In the Bode phase plot, the contributions due to \( H_1 \) and \( H_2 \) also get added.
Combining different terms

Consider \( H(s) = H_1(s) \times H_2(s) \).

**Magnitude:**

\[
|H(j\omega)| = |H_1(j\omega)| \times |H_2(j\omega)|.
\]

\[
20 \log |H| = 20 \log |H_1| + 20 \log |H_2|.
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In the Bode phase plot, the contributions due to \( H_1 \) and \( H_2 \) also get added.

The same reasoning applies to more than two terms as well.
Combining different terms: example

Consider \( H(s) = \frac{10s}{(1 + s/10^2)(1 + s/10^5)} \).
Combining different terms: example

Consider \( H(s) = \frac{10s}{(1 + s/10^2)(1 + s/10^5)} \).

Let \( H(s) = H_1(s) H_2(s) H_3(s) H_4(s) \), where

\( H_1(s) = 10 \),
\( H_2(s) = s \),
\( H_3(s) = \frac{1}{1 + s/p_1} \), \( p_1 = 10^2 \text{ rad/s} \),
\( H_4(s) = \frac{1}{1 + s/p_2} \), \( p_2 = 10^5 \text{ rad/s} \).
Combining different terms: example

Consider \( H(s) = \frac{10s}{(1 + s/10^2)(1 + s/10^5)} \).

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\( H_1(s) = 10 \),
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\( H_4(s) = \frac{1}{1 + s/p_2} \), \( p_2 = 10^5 \text{ rad/s} \).

We can now plot the magnitude and phase of \( H_1, H_2, H_3, H_4 \) individually versus \( \omega \) and then simply add them to obtain \( |H| \) and \( \angle H \).
Magnitude plot ($|H|$ in dB)

$H_1(s) = 10$

$H(s) = 10 s (1 + s/10^2) (1 + s/10^5)$
Magnitude plot ($|H|$ in dB)

\[
H_1(s) = 10
\]

\[
H_2(s) = s
\]

\[
H_3(s) = 1 + \frac{s}{10^2}
\]

\[
H_4(s) = 1 + \frac{s}{10^5}
\]
Magnitude plot ($|H|$ in dB)

$p_{\text{approx.}} = 10 \times s \left( 1 + \frac{s}{10^2} \right) \left( 1 + \frac{s}{10^5} \right)$
Magnitude plot ($|H|$ in dB)

$H_1(s) = 10$

$H_2(s) = s$

$H_3(s) = \frac{1}{1 + s/10^2}$

$H_4(s) = \frac{1}{1 + s/10^5}$

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Magnitude plot ($|H|$ in dB)

$H_1(s) = 10$

$H_2(s) = s$

$H_3(s) = \frac{1}{1 + s/10^2}$

$H_4(s) = \frac{1}{1 + s/10^5}$

$H(s) = \frac{10s}{(1 + s/10^2)(1 + s/10^5)}$
Phase plot

\[ H_1(s) = 10 \]

\[ H_2(s) = s \]

\[ H_3(s) = \frac{1}{1 + \frac{s}{10^2}} \]

\[ H_4(s) = \frac{1}{1 + \frac{s}{10^5}} \]

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Phase plot

\[ H_1(s) = 10 \]

\[ H_2(s) = s \]
Phase plot

\begin{align*}
H_1(s) &= 10 \\
H_2(s) &= s \\
H_3(s) &= \frac{1}{1 + s/10^2}
\end{align*}

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Phase plot

$H_1(s) = 10$

$H_2(s) = s$

$H_3(s) = \frac{1}{1 + \frac{s}{10^2}}$

$H_4(s) = \frac{1}{1 + \frac{s}{10^5}}$

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Phase plot

\[ H_1(s) = 10 \]

\[ H_2(s) = s \]

\[ H_3(s) = \frac{1}{1 + s/10^2} \]

\[ H_4(s) = \frac{1}{1 + s/10^5} \]

\[ H(s) = \frac{10s}{(1 + s/10^2)(1 + s/10^5)} \]
How good are the approximations?

* As we have seen, the contribution of a pole to the magnitude and phase plots is well represented by the asymptotes when $\omega \ll p$ or $\omega \gg p$ (similarly for a zero).
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* If two poles \( p_1 \) and \( p_2 \) are close to each other (say, separated by less than a decade in \( \omega \)), the error becomes larger (next slide).
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* If two poles \( p_1 \) and \( p_2 \) are close to each other (say, separated by less than a decade in \( \omega \)), the error becomes larger (next slide).

* When the poles and zeros are not sufficiently separated, the Bode approximation should be used only for a rough estimate, followed by a numerical calculation. However, even in such cases, it does give a good idea of the asymptotic magnitude and phase plots, which is valuable in amplifier design.
How good are the approximations?

Consider \( H(s) = \frac{10s}{(1 + s/p_1)(1 + s/p_2)} \).
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\[
\begin{array}{c|c|c|c|c|c|c|c}
\text{Frequency (rad/s)} & 0 & -45 & -90 & 45 & 90 \\
\hline
p_1 = 10^2 & & & & & \\
p_2 = 10^4 & & & & & \\
\hline
p_1 = 10^2 & & & & & \\
p_2 = 5 \times 10^2 & & & & & \\
\end{array}
\]
How good are the approximations?

Consider \( H(s) = \frac{10s}{(1+s/p_1)(1+s/p_2)} \).