EE101: Digital circuits (Part 1)

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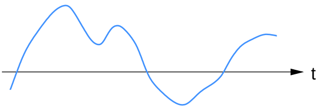
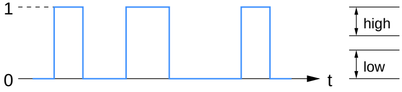
Digital circuits

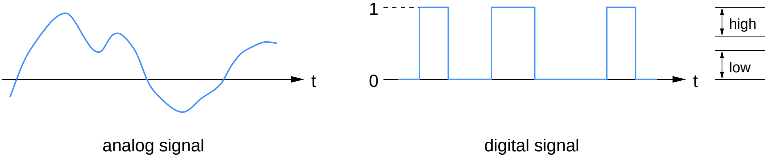


analog signal

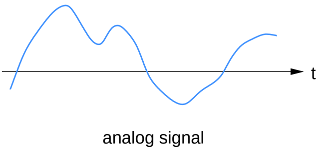


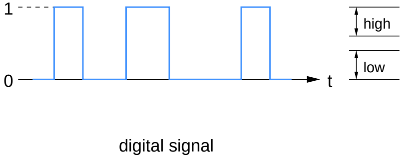
digital signal

Digital circuits



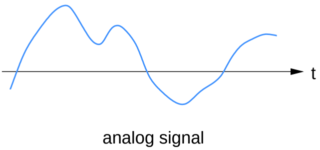
* For an analog signal $x(t)$, the actual value (a real number) at a given time is important.

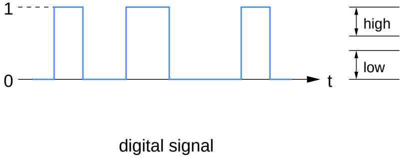




* For an analog signal $x(t)$, the actual value (a real number) at a given time is important.
* A digital signal, on the other hand, is "binary" in nature, i.e., it takes on only two values: low (0) or high (1).

Digital circuits





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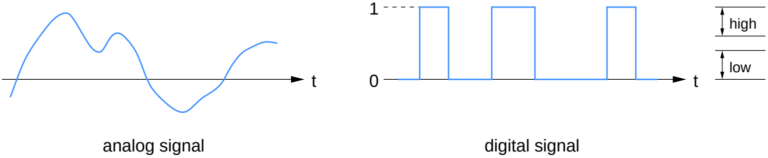
Digital circuits



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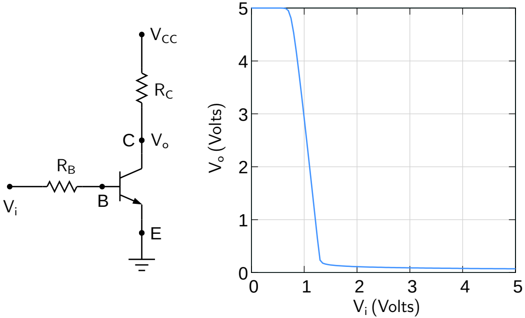
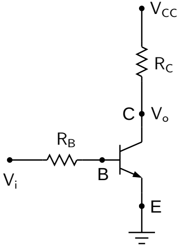
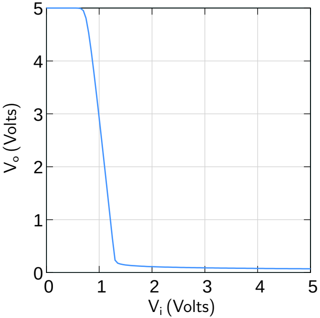
* Although we have shown 0 and 1 as constant levels, in reality, that is not required. Any value in the low (high) band will be interpreted as 0 (1) by digital circuits.
* The definition of low and high bands depends on the technology used, such as TTL (Transitor-Transitor Logic), CMOS (Complementary MOS), ECL (Emitter-Coupled Logic), etc.


V_{CC}
R_C
C
V_o
R_B
B
V_i
E
V_o (Volts)
V_i (Volts)
0
1
2
3
4
5

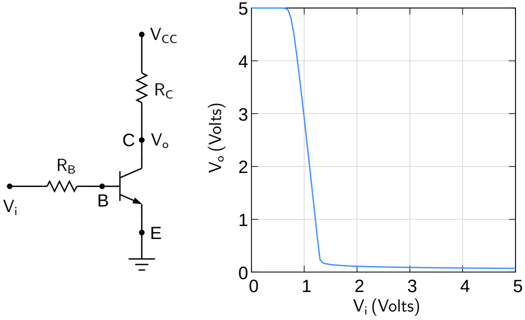
A simple digital circuit





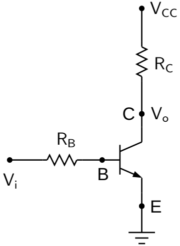
* If V_i is low ("0"), V_o is high ("1").
  If V_i is high ("1"), V_o is low ("0").

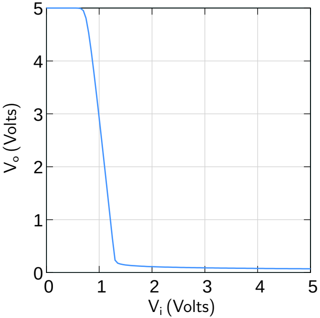
A simple digital circuit



* If V_i is low ("0"), V_o is high ("1").
  If V_i is high ("1"), V_o is low ("0").
* The circuit is called an "inverter" because it inverts the logic level of the input.
  If the input is 0, it makes the output 1, and vice versa.

A simple digital circuit





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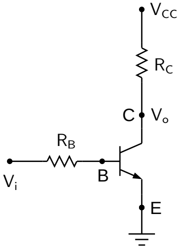

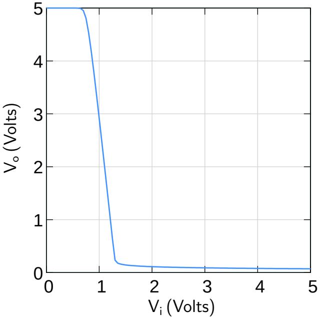

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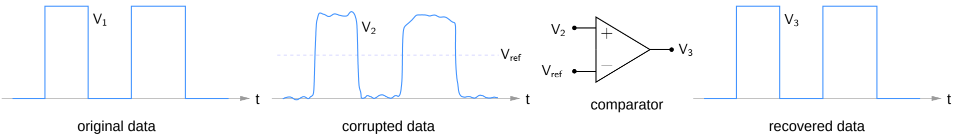
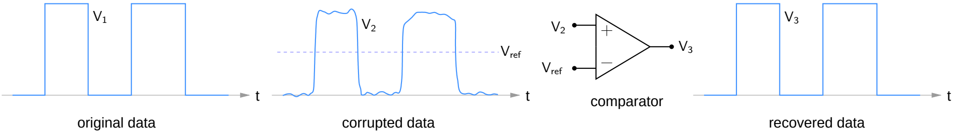
* Most of the VLSI circuits today employ the MOS technology because of the high packing density and low power consumption it offers.

Digital circuits



Digital circuits



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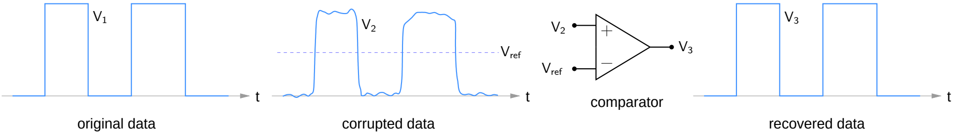
Digital circuits



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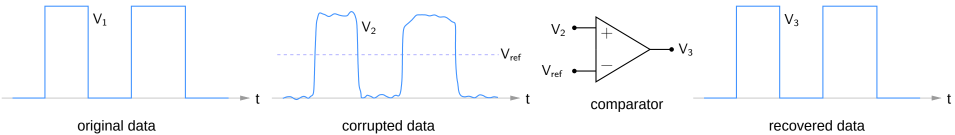
* There are several other benefits of using digital representation:

Digital circuits



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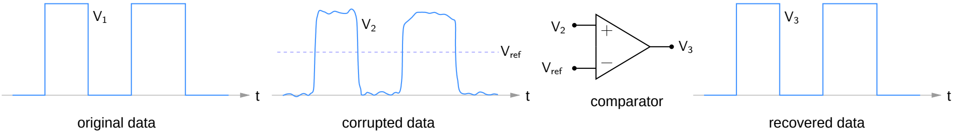
Digital circuits



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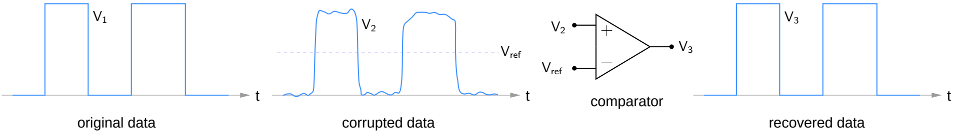
Digital circuits



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* There are several other benefits of using digital representation:
  - can use computers to process the data.
  - can store in a variety of storage media.
  - can *program* the functionality. For example, the behaviour of a digital filter can be changed simply by changing its coefficients.

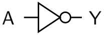
Logical operations

Operation	NOT	AND	OR
Gate			
Truth table			
Notation			

Logical operations

Operation	NOT	AND	OR

Gate



Truth table

A	Y
0	1
1	0

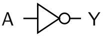
Notation $Y = \overline{A}$

Logical operations

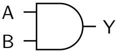
Operation	NOT	AND	OR

Gate

NOT:



AND:



Truth table

NOT:

A	Y
0	1
1	0

AND:

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Notation

NOT: $Y = \overline{A}$

AND:

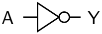
$$\begin{aligned} Y &= A \cdot B \\ &= A\,B \end{aligned}$$

Logical operations

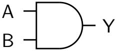
Operation: NOT, AND, OR

Gate

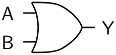
NOT:



AND:



OR:



Truth table

NOT:

A	Y
0	1
1	0

AND:

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Notation

NOT: $Y = \overline{A}$

AND: $Y = A \cdot B$
$= A\,B$

OR: $Y = A + B$

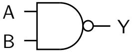
Logical operations

Operation	NAND	NOR	XOR
Gate			
Truth table			
Notation			

Logical operations

Operation	NAND	NOR	XOR

Gate



Truth table

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Notation

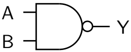
$$\mathrm{Y} = \overline{\mathrm{A} \cdot \mathrm{B}}$$
$$= \overline{\mathrm{A}\,\mathrm{B}}$$

Logical operations

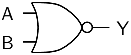
Operation	NAND	NOR	XOR

Gate

NAND:



NOR:



Truth table

NAND:

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR:

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Notation

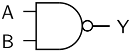
NAND: $Y = \overline{A \cdot B} = \overline{A\,B}$

NOR: $Y = \overline{A + B}$

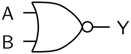
Operation	NAND	NOR	XOR

Gate

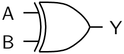
NAND:



NOR:



XOR:



Truth table

NAND:

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR:

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XOR:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Notation

NAND: $Y = \overline{A \cdot B} = \overline{A\,B}$

NOR: $Y = \overline{A + B}$

XOR: $Y = A \oplus B = A\,\overline{B} + \overline{A}\,B$

* The AND operation is *commutative*.
  $\rightarrow A \cdot B = B \cdot A$.

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  $\rightarrow (A \cdot B) \cdot C = A \cdot (B \cdot C)$.

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  $\rightarrow A + B = B + A$.

Logical operations

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  $\rightarrow (A \cdot B) \cdot C = A \cdot (B \cdot C)$.
* The OR operation is *commutative*.
  $\rightarrow A + B = B + A$.
* The OR operation is *associative*.
  $\rightarrow (A + B) + C = A + (B + C)$.

* Theorem: $\overline{\overline{A}} = A$.

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  The theorem can be proved by constructing a truth table:

A	$\overline{A}$	$\overline{\overline{A}}$
0	1	0
1	0	1

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  Therefore, for all possible values that A can take (i.e., 0 and 1), $\overline{\overline{A}}$ is the same as A.

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* Similarly, the following theorems can be proved:

  $A + 0 = A \qquad A \cdot 1 = A$

  $A + 1 = 1 \qquad A \cdot 0 = 0$

  $A + A = A \qquad A \cdot A = A$

  $A + \overline{A} = 1 \qquad A \cdot \overline{A} = 0$

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  The theorem can be proved by constructing a truth table:

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1	0	1

  Therefore, for all possible values that A can take (i.e., 0 and 1), $\overline{\overline{A}}$ is the same as A.

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$$A + A = A \qquad A \cdot A = A$$

$$A + \overline{A} = 1 \qquad A \cdot \overline{A} = 0$$

  Note the duality: $(+ \leftrightarrow \cdot)$ and $(1 \leftrightarrow 0)$.

De Morgan's theorems

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0								
0	1								
1	0								
1	1								

De Morgan's theorems

A	B	$A + B$	$\overline{A + B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$	$A \cdot B$	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	0							
0	1	1							
1	0	1							
1	1	1							

De Morgan's theorems

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1						
0	1	1	0						
1	0	1	0						
1	1	1	0						

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0	0	0	1	1					
0	1	1	0	1					
1	0	1	0	0					
1	1	1	0	0					

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0	0	0	1	1	1				
0	1	1	0	1	0				
1	0	1	0	0	1				
1	1	1	0	0	0				

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0	0	0	1	1	1	1			
0	1	1	0	1	0	0			
1	0	1	0	0	1	0			
1	1	1	0	0	0	0			

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0	0	0	1	1	1	1	0		
0	1	1	0	1	0	0	0		
1	0	1	0	0	1	0	0		
1	1	1	0	0	0	0	1		

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0	0	0	1	1	1	1	0	1	
0	1	1	0	1	0	0	0	1	
1	0	1	0	0	1	0	0	1	
1	1	1	0	0	0	0	1	0	

De Morgan's theorems

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0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

De Morgan's theorems

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0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

* Comparing the truth tables for $\overline{A+B}$ and $\overline{A}\,\overline{B}$, we conclude that $\overline{A+B} = \overline{A}\,\overline{B}$.

De Morgan's theorems

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

* Comparing the truth tables for $\overline{A+B}$ and $\overline{A}\,\overline{B}$, we conclude that $\overline{A+B}=\overline{A}\,\overline{B}$.
* Similiarly, $\overline{A\cdot B}=\overline{A}+\overline{B}$.

De Morgan's theorems

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

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A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

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0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

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0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

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Distributive laws

1. $A \cdot (B + C) = A\,B + A\,C$.

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A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

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A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0				
0	0	1	1				
0	1	0	1				
0	1	1	1				
1	0	0	0				
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1	1	0	1				
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0	0	0	0	0			
0	0	1	1	0			
0	1	0	1	0			
0	1	1	1	0			
1	0	0	0	0			
1	0	1	1	1			
1	1	0	1	1			
1	1	1	1	1			

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A	B	C	$B+C$	$A \cdot (B+C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0	0	0		
0	0	1	1	0	0		
0	1	0	1	0	0		
0	1	1	1	0	0		
1	0	0	0	0	0		
1	0	1	1	1	0		
1	1	0	1	1	1		
1	1	1	1	1	1		

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0	0	0	0	0	0	0	
0	0	1	1	0	0	0	
0	1	0	1	0	0	0	
0	1	1	1	0	0	0	
1	0	0	0	0	0	0	
1	0	1	1	1	0	1	
1	1	0	1	1	1	0	
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0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
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1	1	1	1	1	1	1	1

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0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

$\uparrow$ $\uparrow$

2. $A + B \cdot C = (A + B) \cdot (A + C)$.

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A	B	C	$B\,C$	$A + B\,C$	$A + B$	$A + C$	$(A + B)\,(A + C)$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
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A	B	C	$B\,C$	$A + B\,C$	$A + B$	$A + C$	$(A + B)\,(A + C)$
0	0	0	0				
0	0	1	0				
0	1	0	0				
0	1	1	1				
1	0	0	0				
1	0	1	0				
1	1	0	0				
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0	0	0	0	0			
0	0	1	0	0			
0	1	0	0	0			
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0	0	0	0	0	0		
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0	1	0	0	0	1		
0	1	1	1	1	1		
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0	0	0	0	0	0	0	
0	0	1	0	0	0	1	
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0	1	1	1	1	1	1	
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  Proof: $$\begin{aligned} A \cdot (A + B) &= A \cdot A + A \cdot B \\ &= A + A\,B \\ &= A \end{aligned}$$

Duality

$A + AB = A \quad \longleftrightarrow \quad A \cdot (A + B) = A.$

Note the duality between OR and AND.

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Dual of $A + A\,B$ (LHS): $A\,B \rightarrow A + B$

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Useful theorems

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  Dual theorem: $A \cdot (\overline{A} + B) = A\,B$.

Useful theorems

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  $= 1 \cdot (A + B)$
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* $A\,B + A\,\overline{B} = A.$

  Proof: $$\begin{aligned} A\,B + A\,\overline{B} &= A \cdot (B + \overline{B}) \quad \text{(by distributive law)} \\ &= A \cdot 1 \\ &= A \end{aligned}$$

  Dual theorem: $(A + B) \cdot (A + \overline{B}) = A.$

A game of words

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Let T $\equiv$ Tendulkar scores a century.

S $\equiv$ Sehwag scores a century.

W $\equiv$ Warne fails.

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Let $T \equiv$ Tendulkar scores a century.
$S \equiv$ Sehwag scores a century.
$W \equiv$ Warne fails.
$I \equiv$ India wins.

$$I = T + \overline{T}\,W + \overline{T}\,S$$

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$$
\begin{aligned}
I &= T + \overline{T}\,W + \overline{T}\,S \\
&= T + T + \overline{T}\,W + \overline{T}\,S \\
&= (T + \overline{T}\,W) + (T + \overline{T}\,S)
\end{aligned}
$$

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$$\begin{aligned} I &= T + \overline{T}\,W + \overline{T}\,S \\ &= T + T + \overline{T}\,W + \overline{T}\,S \\ &= (T + \overline{T}\,W) + (T + \overline{T}\,S) \\ &= (T + \overline{T}) \cdot (T + W) + (T + \overline{T}) \cdot (T + S) \end{aligned}$$

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$$
\begin{aligned}
I &= T + \overline{T}\,W + \overline{T}\,S \\
&= T + T + \overline{T}\,W + \overline{T}\,S \\
&= (T + \overline{T}\,W) + (T + \overline{T}\,S) \\
&= (T + \overline{T}) \cdot (T + W) + (T + \overline{T}) \cdot (T + S) \\
&= T + W + T + S \\
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$$
\begin{aligned}
I &= T + \overline{T}\,W + \overline{T}\,S \\
&= T + T + \overline{T}\,W + \overline{T}\,S \\
&= (T + \overline{T}\,W) + (T + \overline{T}\,S) \\
&= (T + \overline{T}) \cdot (T + W) + (T + \overline{T}) \cdot (T + S) \\
&= T + W + T + S \\
&= T + W + S
\end{aligned}
$$

i.e., India will win if one or more of the following hold:

(a) Tendulkar strikes, (b) Warne fails, (c) Sehwag strikes.

Consider a function X of three variables A, B, C:

$$X = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$
$$\equiv X_1 \quad + X_2 \quad + X_3 \quad + X_4$$

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This form is called the "sum of products" form ("sum" corresponding to OR and "product" corresponding to AND).

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Logical functions in standard forms

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(1) Enumerate all possible combinations of A, B, C.
Since each of A, B, C can take two values (0 or 1), we have 2^3 possibilities.

Consider a function X of three variables A, B, C:

$$X = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$
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$$X = X_1 + X_2 + X_3 + X_4 = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

A	B	C	X_1	X_2	X_3	X_4	X
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
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0	0	1					
0	1	0	1				
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0	0	1	0				
0	1	0	1				
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1	0	0	0				
1	0	1	0				
1	1	0	0				
1	1	1	0				

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1	0	0	0	0	1		
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1	1	0	0	0			
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0	1	1	0	1	0		
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0	1	1	0	1	0	0	
1	0	0	0	0	1	0	
1	0	1	0	0	0	0	
1	1	0	0	0	0	1	
1	1	1	0	0	0	0	

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0	1	1	0	1	0	0	1
1	0	0	0	0	1	0	1
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$$Y = Y_1 Y_2 Y_3 Y_4 = (A + B + C)(A + B + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$$

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0	0	0	0	1	1	1	0
0	0	1	1	0	1	1	0
0	1	0	1	1	1	1	
0	1	1	1	1	1	1	
1	0	0	1	1	1	1	
1	0	1	1	1	0	1	0
1	1	0	1	1	1	1	
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0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
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0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	1	0	1	0
1	1	0	1	1	1	1	1
1	1	1	1	1	1	0	0

Note that Y is identical to X (seen two slides back). This is an example of how the same function can be written in two seemingly different forms (in this case, the sum-of-products form and the product-of-sums form).

Standard sum-of-products form

Consider a function X of three variables A, B, C:

$X = A\,B\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$

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In the truth table for X, the numbers of 1s is the same as the number of minterms, as we have seen in an example.

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X can be rewritten as,

$$\begin{aligned} X &= A\,B\,\overline{C} + \overline{A}\,B\,(C + \overline{C}) \\ &= A\,B\,\overline{C} + \overline{A}\,B. \end{aligned}$$

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$$X = A\,B\,\overline{C} + \overline{A}\,B\,(C + \overline{C})$$
$$= A\,B\,\overline{C} + \overline{A}\,B.$$

This is also a sum-of-products form, but not the standard one.

Standard product-of-sums form

Consider a function X of three variables A, B, C:

$X = (A + B + C)\,(\overline{A} + B + C)\,(\overline{A} + \overline{B} + C)$

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&= (A + B + C)\,(\overline{A} + C + B\,\overline{B}) \\
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This is also a product-of-sums form, but not the standard one.

The "don't care" condition

I want to design a box (with inputs A, B, C, and output S) which will help in scheduling my appointments.

$A \equiv$ I am in town, and the time slot being suggested for the appointment is free.

$B \equiv$ My favourite player is scheduled to play a match (which I can watch on TV).

$C \equiv$ The appointment is crucial for my business.

$S \equiv$ Schedule the appointment.

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The following truth table summarizes the expected functioning of the box.

A	B	C	S
0	X	X	0
1	0	X	1
1	1	0	0
1	1	1	1

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$S \equiv$ Schedule the appointment.

The following truth table summarizes the expected functioning of the box.

A	B	C	S
0	X	X	0
1	0	X	1
1	1	0	0
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Note that we have a new entity called X in the truth table.

The "don't care" condition

I want to design a box (with inputs A, B, C, and output S) which will help in scheduling my appointments.

$A \equiv$ I am in town, and the time slot being suggested for the appointment is free.

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Don't care conditions can often be used to get a more efficient implementation of a logical function.