EE101: Resonance in $RLC$ circuits

M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay
Resonance in series $RLC$ circuits

$I = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} \equiv I_m \angle \theta$, where

$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$.
Resonance in series $RLC$ circuits

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* As $\omega$ is varied, both $I_m$ and $\theta$ change.
Resonance in series $RLC$ circuits

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- As $\omega$ is varied, both $I_m$ and $\theta$ change.
- When $\omega L = 1/\omega C$, $I_m$ reaches its maximum value, $I_m^{\text{max}} = V_m/R$, and $\theta$ becomes 0, i.e., the current $I$ is in phase with the applied voltage.
Resonance in series $RLC$ circuits

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* When $\omega L = 1/\omega C$, $I_m$ reaches its maximum value, $I_m^{\text{max}} = V_m/R$, and $\theta$ becomes 0, i.e., the current $I$ is in phase with the applied voltage.
* The above condition is called “resonance,” and the corresponding frequency is called the “resonance frequency” ($\omega_0$).

\[ \omega_0 = 1/\sqrt{LC} \]
Resonance in series $RLC$ circuits

\[ I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \], \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].

As $\omega$ deviates from $\omega_0$, $I_m$ decreases.

As $\omega \to 0$, the term $1/\omega C$ dominates, and $\theta \to \pi/2$.

As $\omega \to \infty$, the term $\omega L$ dominates, and $\theta \to -\pi/2$. 
Resonance in series $RLC$ circuits

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Resonance in series $RLC$ circuits

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\[
\begin{align*}
R &= 10 \Omega \\
L &= 1 \text{ mH} \\
C &= 1 \mu \text{F}
\end{align*}
\]
Resonance in series $RLC$ circuits

\[ I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]. \]

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* As $\omega \to 0$, the term $1/\omega C$ dominates, and $\theta \to \pi/2$.
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$R = 10 \Omega$
$L = 1 \text{ mH}$
$C = 1 \mu\text{F}$
Resonance in series $RLC$ circuits

\[ l_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]. \]

* As $\omega$ deviates from $\omega_0$, $l_m$ decreases.
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(SEQUEL file: ee101_reso_rlc_1.sqproj)
The maximum power that can be absorbed by the resistor is \( P_{\text{max}} = \frac{1}{2} \left( \frac{I_{\text{max}}}{\sqrt{2}} \right)^2 R = \frac{1}{2} \frac{V_m^2}{R} \).

Define \( \omega_1 \) and \( \omega_2 \) (see figure) as frequencies at which \( I_{\text{max}} = \frac{I_{\text{max}}}{\sqrt{2}} \), i.e., the power absorbed by \( R \) is \( P_{\text{max}}/2 \).

The bandwidth of a resonant circuit is defined as \( B = \omega_2 - \omega_1 \), and the quality factor as \( Q = \frac{\omega_0}{B} \). Quality is a measure of the sharpness of the \( I_{\text{m}} \) versus frequency relationship.
Resonance in series $RLC$ circuits

* The maximum power that can be absorbed by the resistor is

$$P_{\text{max}} = \frac{1}{2} \left( I_{\text{m}}^{\text{max}} \right)^2 \ R = \frac{1}{2} \ V_m^2 / R.$$
The maximum power that can be absorbed by the resistor is
\[ P_{\text{max}} = \frac{1}{2} (I_m^{\text{max}})^2 \]
\[ R = \frac{1}{2} V_m^2 / R. \]

Define \( \omega_1 \) and \( \omega_2 \) (see figure) as frequencies at which \( I_m = I_m^{\text{max}} / \sqrt{2} \), i.e., the power absorbed by \( R \) is \( P_{\text{max}} / 2 \).
Resonance in series *RLC* circuits

* The maximum power that can be absorbed by the resistor is

\[ P_{\text{max}} = \frac{1}{2} \left( I_m^{\text{max}} \right)^2 R = \frac{1}{2} V_m^2 / R. \]

* Define \( \omega_1 \) and \( \omega_2 \) (see figure) as frequencies at which \( I_m = I_m^{\text{max}} / \sqrt{2} \), i.e., the power absorbed by \( R \) is \( P_{\text{max}} / 2 \).

* The *bandwidth* of a resonant circuit is defined as \( B = \omega_2 - \omega_1 \), and the *quality factor* as \( Q = \omega_0 / B \). Quality is a measure of the sharpness of the \( I_m \) versus frequency relationship.
Resonance in series $RLC$ circuits

\[ I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \]

For $\omega = \omega_0$, $I_m = I_m^{\text{max}} = V_m / R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{\text{max}} / \sqrt{2}$.
Resonance in series $RLC$ circuits

\[ I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \]

For $\omega = \omega_0$, \( I_m = I_m^{\text{max}} = V_m / R. \)

For $\omega = \omega_1$ or $\omega = \omega_2$, \( I_m = I_m^{\text{max}} / \sqrt{2}. \)

\[ \Rightarrow \frac{1}{\sqrt{2}} \left( \frac{V_m}{R} \right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \]

for $\omega = \omega_{1,2}$. 

Bandwidth $B = \omega_2 - \omega_1 = \frac{R}{L}$.

Quality $Q = \frac{\omega_0}{B} = \frac{\omega_0 L}{R}$. 

* Show that, at resonance (i.e., $\omega = \omega_0$), $|V_L| = |V_C| = Q V_m$. 

* Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$. 

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Resonance in series $RLC$ circuits

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$$  

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For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{\text{max}} / \sqrt{2}$.

$$\Rightarrow \frac{1}{\sqrt{2}} \left( \frac{V_m}{R} \right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2}.$$  

$$2R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C).$$
Resonance in series $RLC$ circuits

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For $\omega = \omega_0$, $I_m = I_{m_{\text{max}}} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_{m_{\text{max}}}/\sqrt{2}$.

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$$2R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C).$$

Solving for $\omega$ (and discarding negative solutions), we get

$$\omega_{1,2} = \pm \frac{R}{2L} + \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}}.$$
Resonance in series $RLC$ circuits

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For $\omega = \omega_0$, $I_m = I_m^{\text{max}} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{\text{max}}/\sqrt{2}$.

$$\Rightarrow \frac{1}{\sqrt{2}} \left( \frac{V_m}{R} \right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2}.$$ 

$$2\,R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C).$$

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* Bandwidth $B = \omega_2 - \omega_1 = R/L$. 

\[ \text{Graph:} \]
Resonance in series $RLC$ circuits

\[ I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \]

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For $\omega = \omega_1$ or $\omega = \omega_2$, \( I_m = I_m^{\text{max}}/\sqrt{2} \).

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* Bandwidth $B = \omega_2 - \omega_1 = R/L$.
* Quality $Q = \omega_0/B = \omega_0 L/R$.
Resonance in series \( RLC \) circuits

\[ I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \]

For \( \omega = \omega_0 \), \( I_m = I_m^{\text{max}} = \frac{V_m}{R} \).

For \( \omega = \omega_1 \) or \( \omega = \omega_2 \), \( I_m = \frac{I_m^{\text{max}}}{\sqrt{2}} \).

\[
\Rightarrow \frac{1}{\sqrt{2}} \left( \frac{V_m}{R} \right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for} \quad \omega = \omega_{1,2}.
\]

\[ 2R^2 = R^2 + (\omega L - 1/\omega C)^2 \rightarrow R = \pm(\omega L - 1/\omega C). \]

Solving for \( \omega \) (and discarding negative solutions), we get

\[ \omega_{1,2} = \pm \frac{R}{2L} + \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}}. \]

* Bandwidth \( B = \omega_2 - \omega_1 = R/L \).
* Quality \( Q = \omega_0/B = \omega_0 L/R \).
* Show that, at resonance (i.e., \( \omega = \omega_0 \)), \(|V_L| = |V_C| = Q V_m \).
Resonance in series $RLC$ circuits

\[ I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}. \]

For $\omega = \omega_0$, $I_m = I_m^{\text{max}} = V_m/R$.

For $\omega = \omega_1$ or $\omega = \omega_2$, $I_m = I_m^{\text{max}}/\sqrt{2}$.

\[ \Rightarrow \frac{1}{\sqrt{2}} \left( \frac{V_m}{R} \right) = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{for } \omega = \omega_{1,2}. \]

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Solving for $\omega$ (and discarding negative solutions), we get

\[ \omega_{1,2} = \pm \frac{R}{2L} \pm \sqrt{\left( \frac{R}{2L} \right)^2 + \frac{1}{LC}}. \]

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* Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.
As $R$ is increased,
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* The quality factor $Q = \omega_0 L/R$ decreases, i.e., $I_m$ versus $\omega$ curve becomes broader.
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- The quality factor $Q = \omega_0 L / R$ decreases, i.e., $I_m$ versus $\omega$ curve becomes broader.

- The maximum current (at $\omega = \omega_0$) decreases (since $I_{m,max} = V_m / R$).
As $R$ is increased,

* The quality factor $Q = \omega_0 L / R$ decreases, i.e., $I_m$ versus $\omega$ curve becomes broader.

* The maximum current (at $\omega = \omega_0$) decreases (since $I_{m\text{max}} = V_m / R$).

* The resonance frequency ($\omega_0 = 1 / \sqrt{LC}$) is not affected.
Resonance in series $RLC$ circuits

\[
I = \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} = I_m \angle \theta, \text{ where}
\]

\[
l_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1} \left[ \frac{\omega L - 1/\omega C}{R} \right].
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Resonance in series $RLC$ circuits

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* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.
Resonance in series $RLC$ circuits

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* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.

* For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.
Resonance in series $RLC$ circuits

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* For $\omega < \omega_0$, $\omega L < 1/\omega C$, the net impedance is capacitive, and the current leads the applied voltage.

* For $\omega = \omega_0$, $\omega L = 1/\omega C$, the net impedance is purely resistive, and the current is in phase with the applied voltage.

* For $\omega > \omega_0$, $\omega L > 1/\omega C$, the net impedance is inductive, and the current lags the applied voltage.
Resonance in series \( RLC \) circuits

\[
\begin{align*}
I &= \frac{V_m \angle 0}{R + j\omega L + 1/j\omega C} = \frac{V_m}{R + j(\omega L - 1/\omega C)} = I_m \angle \theta, \text{ where} \\
I_m &= \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].
\end{align*}
\]

* For \( \omega < \omega_0, \omega L < 1/\omega C \), the net impedance is capacitive, and the current leads the applied voltage.

* For \( \omega = \omega_0, \omega L = 1/\omega C \), the net impedance is purely resistive, and the current is in phase with the applied voltage.

* For \( \omega > \omega_0, \omega L > 1/\omega C \), the net impedance is inductive, and the current lags the applied voltage.

* Let us look at an example (next slide).
Resonance in series $RLC$ circuits

\[ f = 4.3 \text{ kHz} \]

\[ f = 5 \text{ kHz} \approx f_0 \]

\[ f = 5.9 \text{ kHz} \]

$R = 10 \Omega$

$L = 1 \text{ mH}$

$C = 1 \mu\text{F}$

![Diagram of RLC circuit]

$V_s (V)$ (left axis)

$i (A)$ (right axis)
Resonance in series $RLC$ circuits: phasor diagrams

$R = 10 \, \Omega$
$L = 1 \, \text{mH}$
$C = 1 \, \mu\text{F}$

$f = f_0 \approx 5 \, \text{kHz}$
$f = 4.3 \, \text{kHz}$
$f = 5.9 \, \text{kHz}$
Resonance in parallel $RLC$ circuits

$I_m \angle 0 = Y V$, where $Y = G + j\omega C + 1/j\omega L$ ($G = 1/R$).

$V = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta$, where

$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}$, $\theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]$. 
Resonance in parallel $RLC$ circuits

$I_m \angle 0 = YV$, where $Y = G + j\omega C + 1/j\omega L$ ($G = 1/R$).

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$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}$, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]$.

* As $\omega$ is varied, both $V_m$ and $\theta$ change.
Resonance in parallel $RLC$ circuits

$I_m \angle 0 = Y V$, where $Y = G + j\omega C + 1/j\omega L \ (G = 1/R)$.

$V = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta$, where

$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].$

* As $\omega$ is varied, both $V_m$ and $\theta$ change.

* When $\omega C = 1/\omega L$, $V_m$ reaches its maximum value, $V_m^{\max} = I_m/G = I_mR$, and $\theta$ becomes 0, i.e., the voltage $V$ is in phase with the source current.
Resonance in parallel $RLC$ circuits

$I_m \angle 0 = Y V$, where $Y = G + j\omega C + 1/j\omega L \quad (G = 1/R)$.

$V = \frac{I_m \angle 0}{G + j\omega C + 1/j\omega L} = \frac{I_m}{G + j(\omega C - 1/\omega L)} \equiv V_m \angle \theta$, where

$V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}$, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]$.

* As $\omega$ is varied, both $V_m$ and $\theta$ change.
* When $\omega C = 1/\omega L$, $V_m$ reaches its maximum value, $V_m^{\text{max}} = I_m/G = I_m R$, and $\theta$ becomes 0, i.e., the voltage $V$ is in phase with the source current.
* The above condition is called “resonance,” and the corresponding frequency is called the “resonance frequency” ($\omega_0$).

$\omega_0 = 1/\sqrt{LC}$
Resonance in parallel $RLC$ circuits

Series $RLC$ circuit: $I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$, $\theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$.

Parallel $RLC$ circuit: $V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}$, $\theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]$.

The two situations are identical if we make the following substitutions: $I \leftrightarrow V$, $R \leftrightarrow 1/R$, $L \leftrightarrow C$.

Thus, our results for series $RLC$ circuits can be easily extended to parallel $RLC$ circuits.

* Show that $\omega_1$, $\omega_2 = \pm \sqrt{\frac{1}{2RC} + \frac{s}{2} - \frac{1}{LC}}$.

* Show that, at resonance (i.e., $\omega = \omega_0$), $|I_L| = |I_C| = Q I_m$.

* Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.

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Resonance in parallel *RLC* circuits

Series *RLC* circuit: \( I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \), \( \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right] \).

Parallel *RLC* circuit: \( V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}} \), \( \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right] \).

* The two situations are identical if we make the following substitutions:
  - \( I \leftrightarrow V \),
  - \( R \leftrightarrow 1/R \),
  - \( L \leftrightarrow C \).
Resonance in parallel $RLC$ circuits

Series $RLC$ circuit: $I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$, $\theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$.

Parallel $RLC$ circuit: $V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}$, $\theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]$.

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  $I \leftrightarrow V$,
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* Thus, our results for series $RLC$ circuits can be easily extended to parallel $RLC$ circuits.
Resonance in parallel $RLC$ circuits

Series $RLC$ circuit: $I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$.

Parallel $RLC$ circuit: $V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right]$.

* The two situations are identical if we make the following substitutions:
  $I \leftrightarrow V,$
  $R \leftrightarrow 1/R,$
  $L \leftrightarrow C.$

* Thus, our results for series $RLC$ circuits can be easily extended to parallel $RLC$ circuits.

* Show that $\omega_{1,2} = \pm \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$

  $\Rightarrow$ Bandwidth $B = 1/RC$.

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Resonance in parallel \( RLC \) circuits

Series \( RLC \) circuit:  
\[
I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right].
\]

Parallel \( RLC \) circuit:  
\[
V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}, \quad \theta = -\tan^{-1}\left[\frac{\omega C - 1/\omega L}{G}\right].
\]

* The two situations are identical if we make the following substitutions:  
  \( I \leftrightarrow V \),  
  \( R \leftrightarrow 1/R \),  
  \( L \leftrightarrow C \).

* Thus, our results for series \( RLC \) circuits can be easily extended to parallel \( RLC \) circuits.

* Show that \( \omega_{1,2} = \pm \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \)  
  \( \Rightarrow \) Bandwidth \( B = 1/RC \).

* Show that, at resonance (i.e., \( \omega = \omega_0 \)),  
  \( |I_L| = |I_C| = Q I_m \).
Resonance in parallel $RLC$ circuits

Series $RLC$ circuit: $I_m = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$, $\theta = -\tan^{-1} \left[ \frac{\omega L - 1/\omega C}{R} \right]$.

Parallel $RLC$ circuit: $V_m = \frac{I_m}{\sqrt{G^2 + (\omega C - 1/\omega L)^2}}$, $\theta = -\tan^{-1} \left[ \frac{\omega C - 1/\omega L}{G} \right]$.

* The two situations are identical if we make the following substitutions:
  $I \leftrightarrow V$,
  $R \leftrightarrow 1/R$,
  $L \leftrightarrow C$.

* Thus, our results for series $RLC$ circuits can be easily extended to parallel $RLC$ circuits.

* Show that $\omega_{1,2} = \pm \frac{1}{2RC} + \sqrt{\left( \frac{1}{2RC} \right)^2 + \frac{1}{LC}}$.
  $\Rightarrow$ Bandwidth $B = 1/RC$.

* Show that, at resonance (i.e., $\omega = \omega_0$), $|I_L| = |I_C| = Q I_m$.

* Show that $\omega_0 = \sqrt{\omega_1 \omega_2}$.
Calculate $\omega_0$, $f_0$, $B$, $Q$.

Calculate $I_R$, $I_L$, $I_C$ at $\omega = \omega_0$, $\omega_1$, $\omega_2$.

Verify graphically that $I_R + I_L + I_C = I_s$ in each case.

Plot the power absorbed by $R$ as a function of frequency for $f_0/10 < f < 10 f_0$. 

$I_m = 50$ mA

$R = 2$ k$\Omega$

$L = 40$ mH

$C = 0.25$ $\mu$F