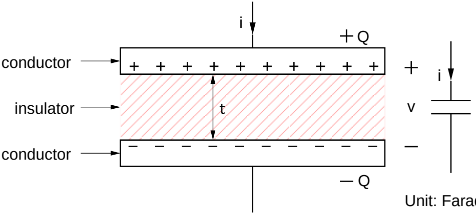
EE101: RC and RL Circuits (with DC sources)

M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay

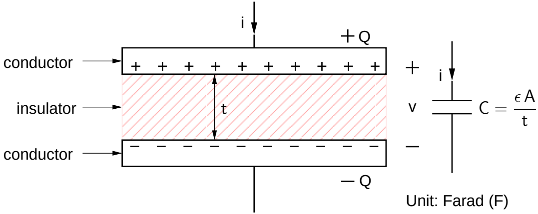
Capacitors

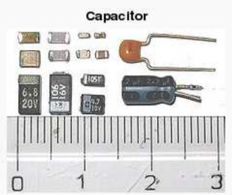


$C = \dfrac{\epsilon A}{t}$

Unit: Farad (F)

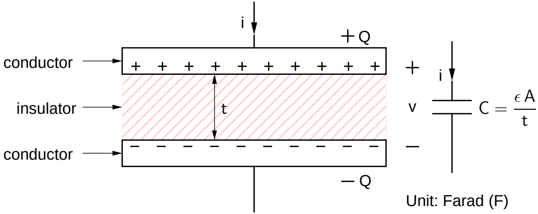
Capacitors



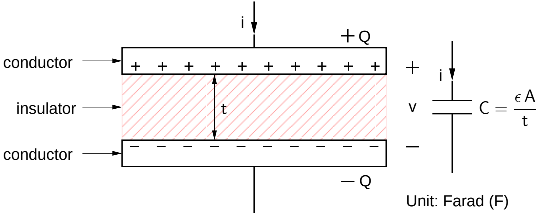


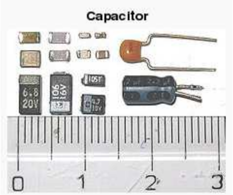
* In practice, capacitors are available in a wide range of shapes and values, and they differ significantly in the way they are fabricated. (`http://en.wikipedia.org/wiki/Capacitor`)

Capacitors



* In practice, capacitors are available in a wide range of shapes and values, and they differ significantly in the way they are fabricated.
(`http://en.wikipedia.org/wiki/Capacitor`)
* To make C larger, we need (a) high ϵ, (b) large area, (c) small thickness.

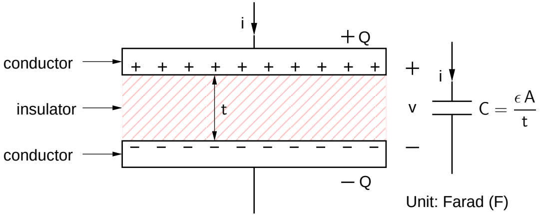




* In practice, capacitors are available in a wide range of shapes and values, and they differ significantly in the way they are fabricated. (`http://en.wikipedia.org/wiki/Capacitor`)
* To make C larger, we need (a) high ϵ, (b) large area, (c) small thickness.
* For a constant capacitance,

$$Q(t) = C\,v(t)\,, \quad \frac{dQ}{dt} = C\,\frac{dv}{dt}\,, \text{ i.e, } i(t) = C\,\frac{dv}{dt}\,.$$

Capacitors



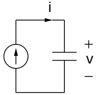
* In practice, capacitors are available in a wide range of shapes and values, and they differ significantly in the way they are fabricated. (`http://en.wikipedia.org/wiki/Capacitor`)
* To make C larger, we need (a) high ϵ, (b) large area, (c) small thickness.
* For a constant capacitance,

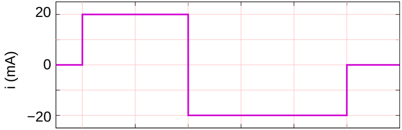
$$Q(t) = C\, v(t)\,, \quad \frac{dQ}{dt} = C\,\frac{dv}{dt}\,, \text{ i.e, } i(t) = C\,\frac{dv}{dt}\,.$$

* If $v =$ constant, $i = 0$, i.e., a capacitor behaves like an open circuit in DC conditions as one would expect from two conducting plates separated by an insulator.

Example

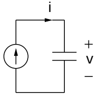
Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.

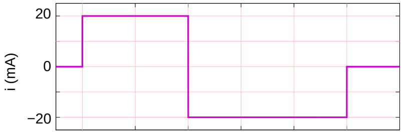




Example

Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.



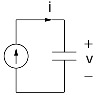


$$i(t) = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int i(t)\,dt$$

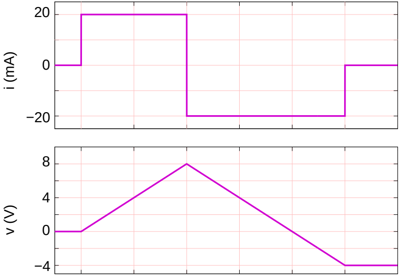
Example

Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.



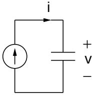
$$i(t) = C\,\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int i(t)\,dt$$



Example

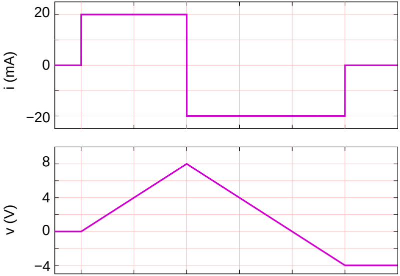
Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.



$$i(t) = C\,\frac{dv}{dt}$$

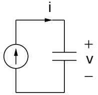
$$v(t) = \frac{1}{C}\int i(t)\,dt$$

$$p(t) = v(t) \times i(t)$$



Example

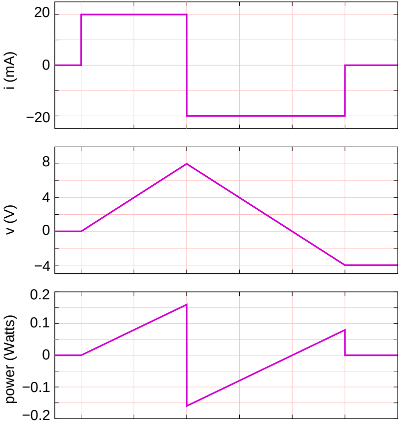
Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.



$$i(t) = C\,\frac{dv}{dt}$$

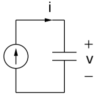
$$v(t) = \frac{1}{C}\int i(t)\,dt$$

$$p(t) = v(t) \times i(t)$$



Example

Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.

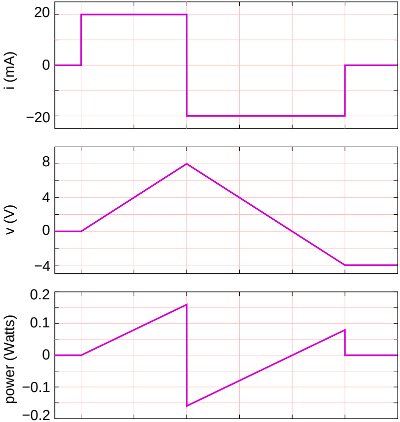


$$i(t) = C\,\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int i(t)\,dt$$

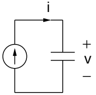
$$p(t) = v(t) \times i(t)$$

$$W(t) = \int p(t)\,dt$$



Example

Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.

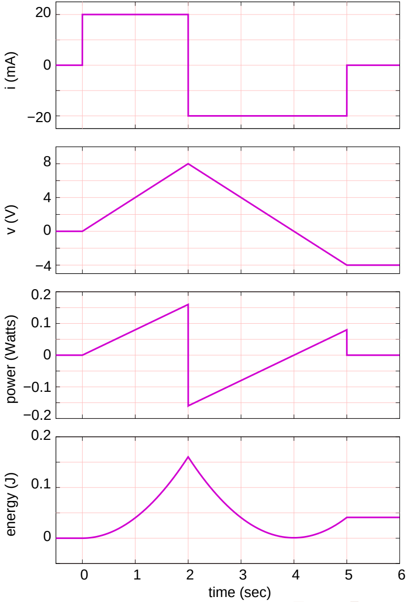


$$i(t) = C\,\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int i(t)\,dt$$

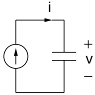
$$p(t) = v(t) \times i(t)$$

$$W(t) = \int p(t)\,dt$$



Example

Plot v, p, and W versus time for the given source current. Assume $v(0) = 0$ V, $C = 5$ mF.



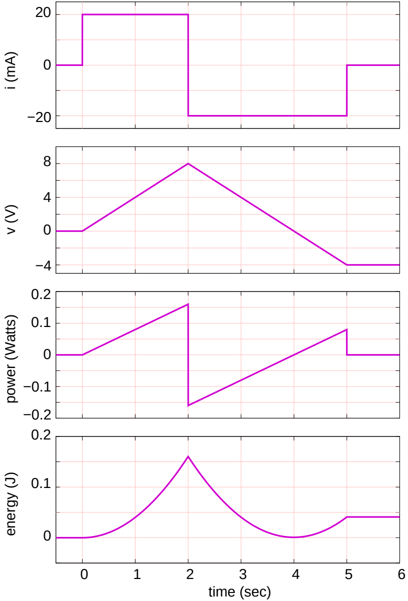
$$i(t) = C\,\frac{dv}{dt}$$

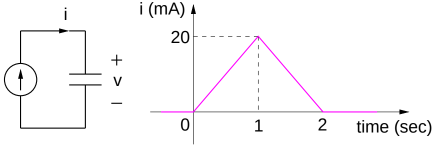
$$v(t) = \frac{1}{C}\int i(t)\,dt$$

$$p(t) = v(t) \times i(t)$$

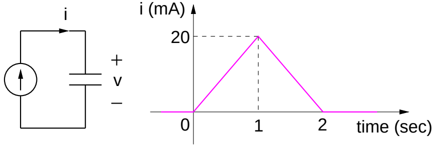
$$W(t) = \int p(t)\,dt$$

$$\begin{aligned} W(t) &= \int p(t)\,dt \\ &= C\int v\,\frac{dv}{dt}\,dt \\ &= C\int v\,dv \\ &= \frac{1}{2}\,C\,v^2 \end{aligned}$$



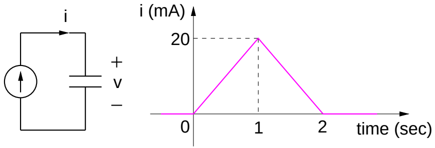

i
+
v
−
i (mA)
20
0
1
2
time (sec)

Home work



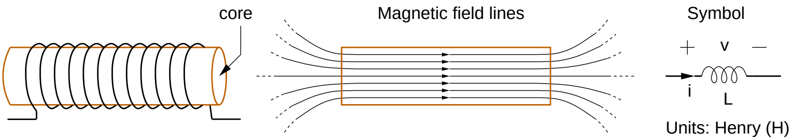
* For the given source current, plot $v(t)$, $p(t)$, and $W(t)$, assuming $v(0) = 0\ V$, $C = 5\ \text{m}F$.

Home work

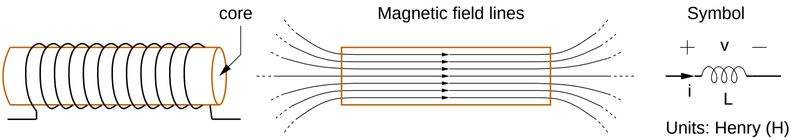


* For the given source current, plot $v(t)$, $p(t)$, and $W(t)$, assuming $v(0) = 0\ V$, $C = 5\ \text{m}F$.
* Verify your results with circuit simulation.

Inductors

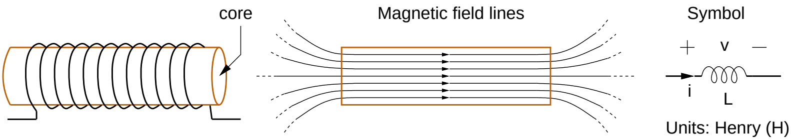


Inductors



* An inductor is basically a conducting coil wound around a "core."

Inductors

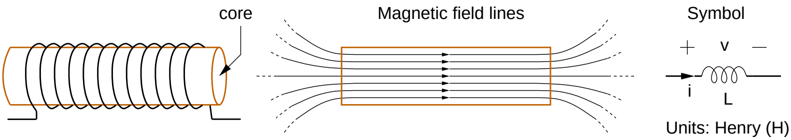


* An inductor is basically a conducting coil wound around a "core."
* $V = N\,\dfrac{d\phi}{dt} = N\,\dfrac{d}{dt}\,(\mathbf{B}\cdot\mathbf{A}) = N\,\dfrac{d}{dt}\left[\left(\dfrac{\mu\,N\,i}{l}\right)\,A\right].$

  Compare with $v = L\,\dfrac{di}{dt}$.

  $\Rightarrow L = \mu\,N^2\,\dfrac{A}{l} = \mu_r\mu_0\,N^2\,\dfrac{A}{l}$.

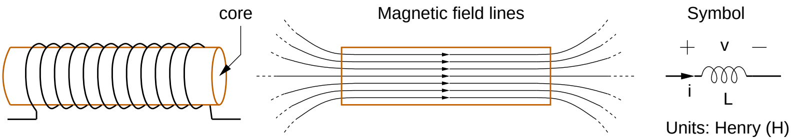
Inductors



* An inductor is basically a conducting coil wound around a "core."
* $V = N\,\dfrac{d\phi}{dt} = N\,\dfrac{d}{dt}\,(\mathbf{B}\cdot\mathbf{A}) = N\,\dfrac{d}{dt}\left[\left(\dfrac{\mu\,N\,i}{l}\right)\,A\right].$

  Compare with $v = L\,\dfrac{di}{dt}$.

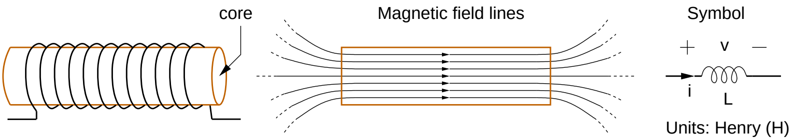
  $\Rightarrow L = \mu\,N^2\,\dfrac{A}{l} = \mu_r\mu_0\,N^2\,\dfrac{A}{l}$.
* To make L larger, we need (a) high μ_r, (b) large area, (c) large number of turns.



- An inductor is basically a conducting coil wound around a "core."
- $V = N\,\dfrac{d\phi}{dt} = N\,\dfrac{d}{dt}\,(\mathbf{B}\cdot\mathbf{A}) = N\,\dfrac{d}{dt}\left[\left(\dfrac{\mu\,N\,i}{l}\right)\,A\right].$

  Compare with $v = L\,\dfrac{di}{dt}$.

  $\Rightarrow L = \mu\,N^2\,\dfrac{A}{l} = \mu_r\mu_0\,N^2\,\dfrac{A}{l}$.
- To make L larger, we need (a) high μ_r, (b) large area, (c) large number of turns.
- For 99.8 % pure iron, $\mu_r \simeq 5,000$.
  For "supermalloy" (Ni: 79 %, Mo: 5 %, Fe): $\mu_r \simeq 10^6$.

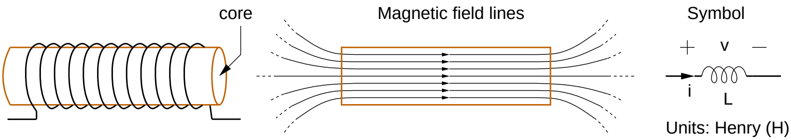


* An inductor is basically a conducting coil wound around a "core."
* $V = N\,\dfrac{d\phi}{dt} = N\,\dfrac{d}{dt}\,(\mathbf{B}\cdot\mathbf{A}) = N\,\dfrac{d}{dt}\left[\left(\dfrac{\mu\,N\,i}{l}\right)\,A\right].$

  Compare with $v = L\,\dfrac{di}{dt}$.

  $\Rightarrow L = \mu\,N^2\,\dfrac{A}{l} = \mu_r\mu_0\,N^2\,\dfrac{A}{l}$.
* To make L larger, we need (a) high μ_r, (b) large area, (c) large number of turns.
* For 99.8 % pure iron, $\mu_r \simeq 5,000$.
  For "supermalloy" (Ni: 79 %, Mo: 5 %, Fe): $\mu_r \simeq 10^6$.
* If i = constant, $v = 0$, i.e., an inductor behaves like a short circuit in DC conditions as one would expect from a highly conducting coil.

Inductors

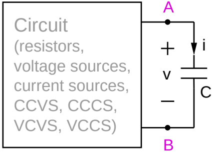


* An inductor is basically a conducting coil wound around a "core."
* $V = N\,\dfrac{d\phi}{dt} = N\,\dfrac{d}{dt}\,(\mathbf{B}\cdot\mathbf{A}) = N\,\dfrac{d}{dt}\left[\left(\dfrac{\mu\,N\,i}{l}\right)\,A\right]$.

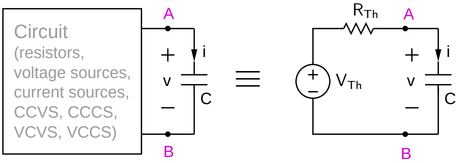
  Compare with $v = L\,\dfrac{di}{dt}$.

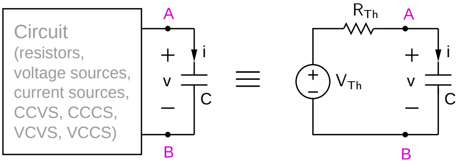
  $\Rightarrow L = \mu\,N^2\,\dfrac{A}{l} = \mu_r\mu_0\,N^2\,\dfrac{A}{l}$.
* To make L larger, we need (a) high μ_r, (b) large area, (c) large number of turns.
* For 99.8 % pure iron, $\mu_r \simeq 5,000$.
  For "supermalloy" (Ni: 79 %, Mo: 5 %, Fe): $\mu_r \simeq 10^6$.
* If $i =$ constant, $v = 0$, i.e., an inductor behaves like a short circuit in DC conditions as one would expect from a highly conducting coil.
* Note: $B = \mu\,H$ is an approximation. In practice, B may be a nonlinear function of H, depending on the core material.

RC circuits with DC sources



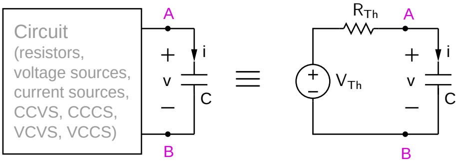
RC circuits with DC sources



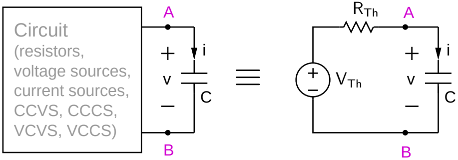


* If all sources are DC (constant), $V_{Th} = \text{constant}$.

RC circuits with DC sources



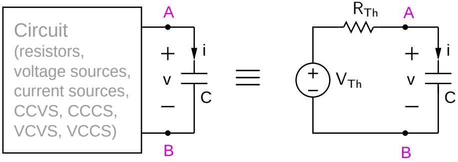
* If all sources are DC (constant), $V_{Th} =$ constant.
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th}\, C\, \dfrac{dv}{dt} + v$.



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C \dfrac{dv}{dt} + v$.
* Homogeneous solution:

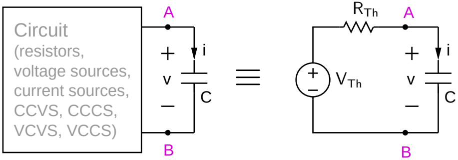
$$\frac{dv}{dt} + \frac{1}{\tau}\, v = 0\,, \text{ where } \tau = R_{Th}\, C \text{ is the "time constant."}$$

$$\rightarrow v^{(h)} = K\, \exp(-t/\tau)\,.$$



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C \, \dfrac{dv}{dt} + v$.
* Homogeneous solution:
  $\dfrac{dv}{dt} + \dfrac{1}{\tau}\, v = 0$, where $\tau = R_{Th}\, C$ is the "time constant."
  $\rightarrow v^{(h)} = K \, \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i = 0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).

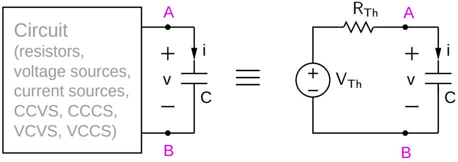
RC circuits with DC sources



* If all sources are DC (constant), $V_{Th} =$ constant .
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C\, \dfrac{dv}{dt} + v$.
* Homogeneous solution:

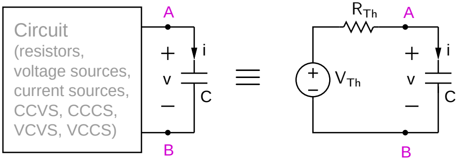
  $\dfrac{dv}{dt} + \dfrac{1}{\tau}\, v = 0$, where $\tau = R_{Th}\, C$ is the "time constant."

  $\rightarrow v^{(h)} = K\, \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i = 0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).
* $v = v^{(h)} + v^{(p)} = K\, \exp(-t/\tau) + V_{Th}$.

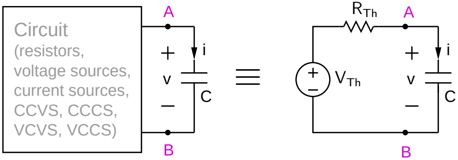


* If all sources are DC (constant), $V_{Th}=$ constant.
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C \dfrac{dv}{dt} + v$.
* Homogeneous solution:
  $\dfrac{dv}{dt} + \dfrac{1}{\tau} v = 0$, where $\tau = R_{Th}\, C$ is the "time constant."
  $\rightarrow v^{(h)} = K \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i=0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).
* $v = v^{(h)} + v^{(p)} = K \exp(-t/\tau) + V_{Th}$.
* In general, $v(t) = A \exp(-t/\tau) + B$, where A and B can be obtained from known conditions on v.

RC circuits with DC sources (continued)



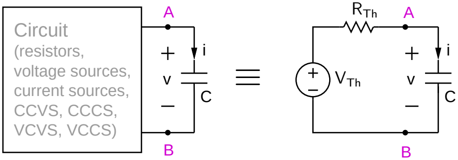
* If all sources are DC (constant), we have
  $v(t) = A \exp(-t/\tau) + B$, $\tau = RC$.



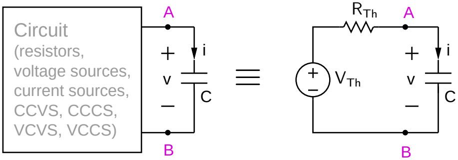
* If all sources are DC (constant), we have
  $v(t) = A\,\exp(-t/\tau) + B\,,\ \tau = RC\,.$

* $i(t) = C\,\dfrac{dv}{dt} = C \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,exp(-t/\tau)\,.$

RC circuits with DC sources (continued)

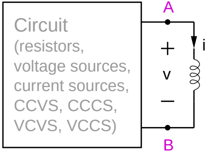


* If all sources are DC (constant), we have
  $v(t) = A\exp(-t/\tau) + B$, $\tau = RC$.
* $i(t) = C\,\dfrac{dv}{dt} = C \times A\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,exp(-t/\tau)$.
* As $t \to \infty$, $i \to 0$, i.e., the capacitor behaves like an open circuit since all derivatives vanish.

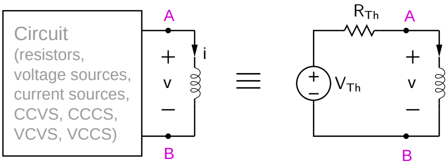


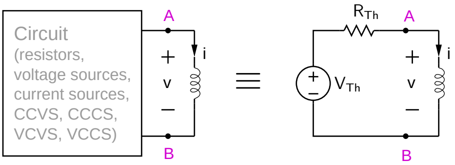
* If all sources are DC (constant), we have
  $v(t) = A\,\exp(-t/\tau) + B\,,\ \tau = RC\,.$
* $i(t) = C\,\dfrac{dv}{dt} = C \times A\,\exp(-t/\tau)\,\left(-\dfrac{1}{\tau}\right) \equiv A'\,exp(-t/\tau)\,.$
* As $t \rightarrow \infty$, $i \rightarrow 0$, i.e., the capacitor behaves like an open circuit since all derivatives vanish.
* Since the circuit in the black box is linear, *any* variable (current or voltage) in the circuit can be expressed as
  $x(t) = K_1\,\exp(-t/\tau) + K_2\,,$
  where K_1 and K_2 can be obtained from suitable conditions on $x(t)$.

RL circuits with DC sources

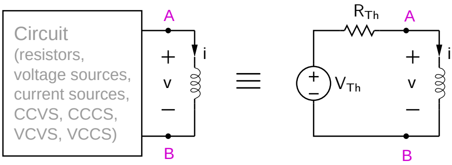


RL circuits with DC sources

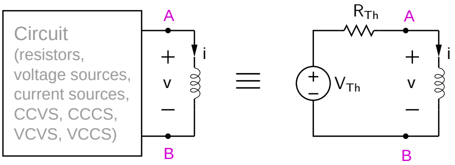




* If all sources are DC (constant), V_{Th} = constant .



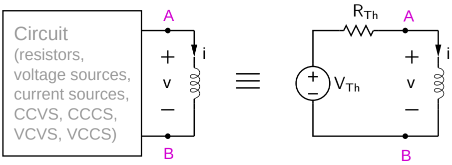
* If all sources are DC (constant), $V_{Th} =$ constant .
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.
* Homogeneous solution:

$$\frac{di}{dt} + \frac{1}{\tau}\, i = 0\,, \text{ where } \tau = L/R_{Th}$$

$$\rightarrow i^{(h)} = K\,\exp(-t/\tau)\,.$$

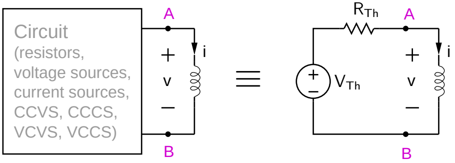


* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.
* Homogeneous solution:

$$\frac{di}{dt} + \frac{1}{\tau}\, i = 0\,, \text{ where } \tau = L/R_{Th}$$

$$\rightarrow i^{(h)} = K\, \exp(-t/\tau)\,.$$

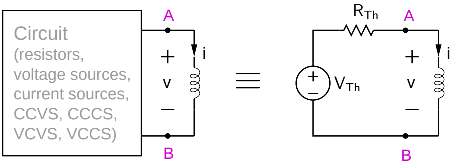
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\, \dfrac{di}{dt}$.
* Homogeneous solution:

  $\dfrac{di}{dt} + \dfrac{1}{\tau}\, i = 0$, where $\tau = L/R_{Th}$

  $\rightarrow i^{(h)} = K\, \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).
* $i = i^{(h)} + i^{(p)} = K\, \exp(-t/\tau) + V_{Th}/R_{Th}$.

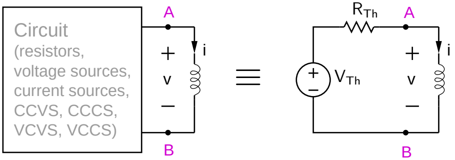


* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\, \dfrac{di}{dt}$.
* Homogeneous solution:

  $\dfrac{di}{dt} + \dfrac{1}{\tau}\, i = 0$, where $\tau = L/R_{Th}$

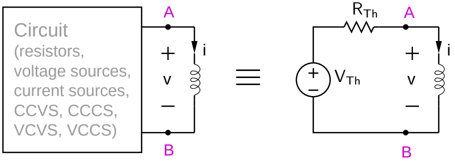
  $\rightarrow i^{(h)} = K \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differntial equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).
* $i = i^{(h)} + i^{(p)} = K \exp(-t/\tau) + V_{Th}/R_{Th}$.
* In general, $i(t) = A \exp(-t/\tau) + B$, where A and B can be obtained from known conditions on i.

RL circuits with DC sources (continued)



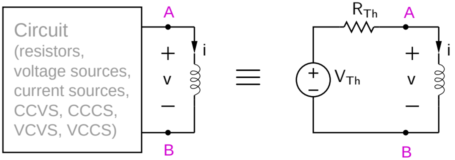
* If all sources are DC (constant), we have $i(t) = A\,\exp(-t/\tau) + B\,,\ \tau = L/R\,.$

RL circuits with DC sources (continued)

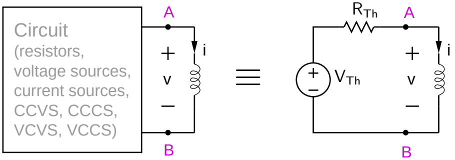


* If all sources are DC (constant), we have
  $i(t) = A\ \exp(-t/\tau) + B$, $\tau = L/R$.

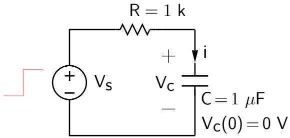
* $v(t) = L\,\dfrac{di}{dt} = L \times A\ \exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\ exp(-t/\tau)$.

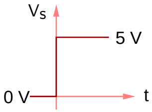


* If all sources are DC (constant), we have
  $i(t) = A\,\exp(-t/\tau) + B\,,\ \tau = L/R\,.$
* $v(t) = L\,\dfrac{di}{dt} = L \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,exp(-t/\tau)\,.$
* As $t \rightarrow \infty$, $v \rightarrow 0$, i.e., the inductor behaves like a short circuit since all derivatives vanish.

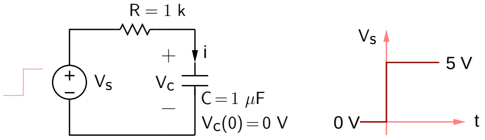


* If all sources are DC (constant), we have
  $i(t) = A\,\exp(-t/\tau) + B\,,\ \tau = L/R\,.$
* $v(t) = L\,\dfrac{di}{dt} = L \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,exp(-t/\tau)\,.$
* As $t \rightarrow \infty$, $v \rightarrow 0$, i.e., the inductor behaves like a short circuit since all derivatives vanish.
* Since the circuit in the black box is linear, *any* variable (current or voltage) in the circuit can be expressed as
  $x(t) = K_1\,\exp(-t/\tau) + K_2\,,$
  where K_1 and K_2 can be obtained from suitable conditions on $x(t)$.

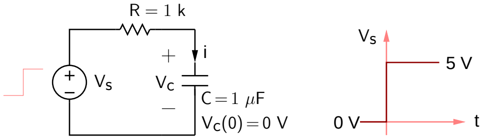

R = 1 k
i
+
Vs
Vc
C = 1 μF
−
Vc(0) = 0 V


Vs
5 V
0 V
t

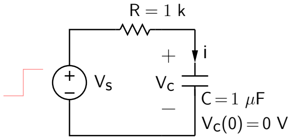
RC circuits: Can V_c change "suddenly?"

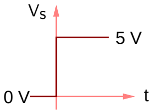


* V_s changes from 0 V (at $t = 0^-$), to 5 V (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?



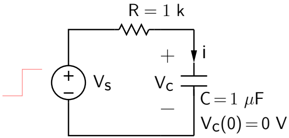
* V_s changes from 0 V (at $t = 0^-$), to 5 V (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by 1 V in 1 μs at a constant rate of 1 $V/1\ \mu\text{s} = 10^6\ V/\text{s}$?

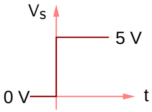




* V_s changes from 0 V (at $t = 0^-$), to 5 V (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by 1 V in 1 μs at a constant rate of 1 $V/1\ \mu\text{s} = 10^6\ V/\text{s}$?
* $i = C\,\dfrac{dV_c}{dt} = 1\ \mu F \times 10^6\ \dfrac{V}{\text{s}} = 1\ A$.

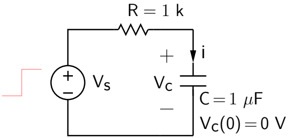
RC circuits: Can V_c change "suddenly?"

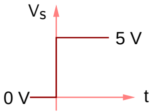




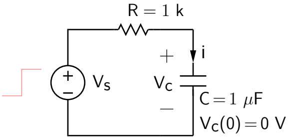
* V_s changes from 0 V (at $t = 0^-$), to 5 V (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by 1 V in 1 μs at a constant rate of 1 $V/1\ \mu\text{s} = 10^6\ V/\text{s}$?
* $i = C\,\dfrac{dV_c}{dt} = 1\ \mu F \times 10^6\ \dfrac{V}{\text{s}} = 1\ A\,.$
* With $i = 1\ A$, the voltage drop across R would be 1000 V! Not allowed by KVL.

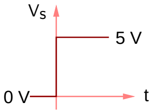
RC circuits: Can V_c change "suddenly?"





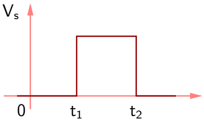
* V_s changes from 0 V (at $t=0^-$), to 5 V (at $t=0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by 1 V in 1 μs at a constant rate of 1 $V/1\ \mu\text{s} = 10^6\ V/\text{s}$?
* $i = C\,\dfrac{dV_c}{dt} = 1\ \mu F \times 10^6\ \dfrac{V}{\text{s}} = 1\ A$.
* With $i=1\ A$, the voltage drop across R would be 1000 V! Not allowed by KVL.
* We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.





* V_s changes from $0\ V$ (at $t = 0^-$), to $5\ V$ (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by $1\ V$ in $1\ \mu s$ at a constant rate of $1\ V/1\ \mu s = 10^6\ V/s$?
* $i = C\,\dfrac{dV_c}{dt} = 1\ \mu F \times 10^6\ \dfrac{V}{s} = 1\ A\,.$
* With $i = 1\ A$, the voltage drop across R would be $1000\ V$! Not allowed by KVL.
* We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.
* Similarly, an inductor does not allow abrupt changes in i_L.

* Identify intervals in which the source voltages/currents are constant. For example,

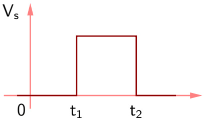


(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

* Identify intervals in which the source voltages/currents are constant. For example,



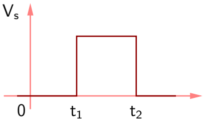
(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
$x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
$x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
$x(t) = A_3 \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Identify intervals in which the source voltages/currents are constant. For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

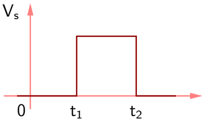
(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
  $x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
  $x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
  $x(t) = A_3 \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Work out suitable conditions on $x(t)$ at specific time points using

Analysis of RC/RL circuits with a piece-wise constant source

* Identify intervals in which the source voltages/currents are constant. For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

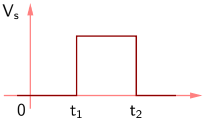
* For *any* current or voltage $x(t)$, write general expressions such as,

$x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
$x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
$x(t) = A_3 \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Work out suitable conditions on $x(t)$ at specific time points using
  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.

$$\Rightarrow i_C = C\,\frac{dV_c}{dt} = 0\,, \text{ and } V_L = L\,\frac{di_L}{dt} = 0\,.$$

* Identify intervals in which the source voltages/currents are constant. For example,



(1) $t < t_1$

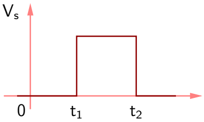
(2) $t_1 < t < t_2$

(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
$x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
$x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
$x(t) = A_3 \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Work out suitable conditions on $x(t)$ at specific time points using
  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.
  $\Rightarrow i_C = C\,\dfrac{dV_c}{dt} = 0$, and $V_L = L\,\dfrac{di_L}{dt} = 0$.
  (b) When a source voltage (or current) changes, say, at $t = t_0$, $V_c(t)$ or $i_L(t)$ cannot change abruptly, i.e.,
  $V_c(t_0^+) = V_c(t_0^-)$, and $i_L(t_0^+) = i_L(t_0^-)$.

* Identify intervals in which the source voltages/currents are constant. For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

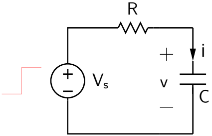
(3) $t > t_2$

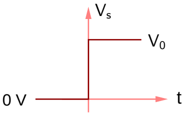
* For *any* current or voltage $x(t)$, write general expressions such as,
$x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
$x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
$x(t) = A_3 \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Work out suitable conditions on $x(t)$ at specific time points using
  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.
  $$\Rightarrow i_C = C\,\frac{dV_c}{dt} = 0\,, \text{ and } V_L = L\,\frac{di_L}{dt} = 0\,.$$
  (b) When a source voltage (or current) changes, say, at $t = t_0$, $V_c(t)$ or $i_L(t)$ cannot change abruptly, i.e.,
  $$V_c(t_0^+) = V_c(t_0^-)\,, \text{ and } i_L(t_0^+) = i_L(t_0^-)\,.$$

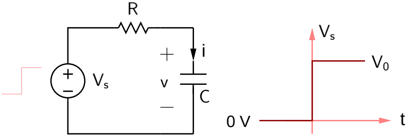
* Compute A_1, B_1, $\cdots$ using the conditions on $x(t)$.

RC circuits: charging and discharging transients

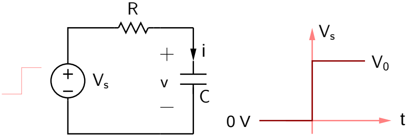




RC circuits: charging and discharging transients



Let $v(t) = A \exp(-t/\tau) + B, \quad t > 0$ (A)



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

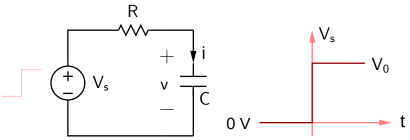
$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty\,, i \to 0 \;\to v(\infty) = V_s(\infty) = V_0$

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty$, $i \rightarrow 0 \;\rightarrow v(\infty) = V_s(\infty) = V_0$

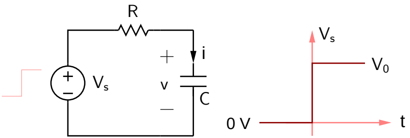
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B$,

$t \rightarrow \infty$: $V_0 = B$.

i.e., $A = V_0$, $B = -V_0$

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty, i \rightarrow 0 \rightarrow v(\infty) = V_S(\infty) = V_0$

Imposing (1) and (2) on Eq. (A), we get

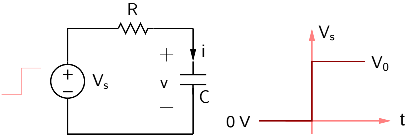
$t = 0^+$: $0 = A + B$,

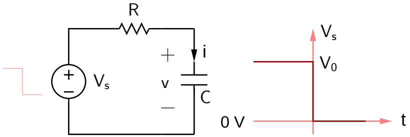
$t \rightarrow \infty$: $V_0 = B$.

i.e., $A = V_0, B = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$

RC circuits: charging and discharging transients





Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0 \qquad \text{(A)}$

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty\,, i \rightarrow 0 \ \rightarrow v(\infty) = V_s(\infty) = V_0$

Imposing (1) and (2) on Eq. (A), we get

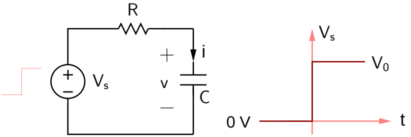
$t = 0^+$: $0 = A + B\,,$

$t \rightarrow \infty$: $V_0 = B\,.$

i.e., $A = V_0\,, B = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty$, $i \rightarrow 0 \rightarrow v(\infty) = V_s(\infty) = V_0$

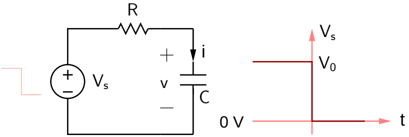
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B$,

$t \rightarrow \infty$: $V_0 = B$.

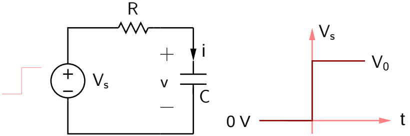
i.e., $A = V_0$, $B = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty\,, i \rightarrow 0\ \rightarrow v(\infty) = V_s(\infty) = V_0$

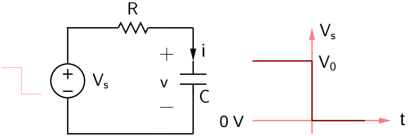
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B\,,$

$t \rightarrow \infty$: $V_0 = B\,.$

i.e., $A = V_0\,, B = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = V_0$

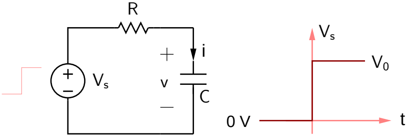
$v(0^+) \simeq v(0^-) = V_0$

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty\,, i \rightarrow 0\ \rightarrow v(\infty) = V_s(\infty) = 0$ V

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0 \qquad$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty\,, i \rightarrow 0 \; \rightarrow v(\infty) = V_S(\infty) = V_0$

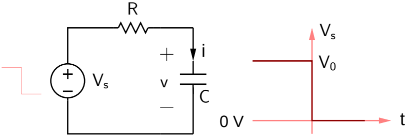
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B\,,$

$t \rightarrow \infty$: $V_0 = B\,.$

i.e., $A = V_0\,, B = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0 \qquad$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = V_0$

$v(0^+) \simeq v(0^-) = V_0$

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty\,, i \rightarrow 0 \; \rightarrow v(\infty) = V_S(\infty) = 0$ V

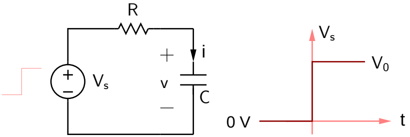
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $V_0 = A + B\,,$

$t \rightarrow \infty$: $0 = B\,.$

i.e., $A = V_0\,, B = 0$

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty$, $i \to 0$ $\to v(\infty) = V_S(\infty) = V_0$

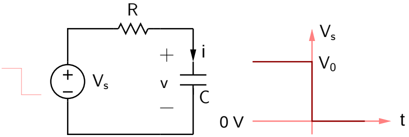
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B$,

$t \to \infty$: $V_0 = B$.

i.e., $A = V_0$, $B = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = V_0$

$v(0^+) \simeq v(0^-) = V_0$

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty$, $i \to 0$ $\to v(\infty) = V_S(\infty) = 0$ V

Imposing (1) and (2) on Eq. (A), we get

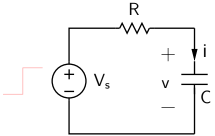
$t = 0^+$: $V_0 = A + B$,

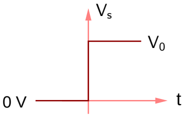
$t \to \infty$: $0 = B$.

i.e., $A = V_0$, $B = 0$

$$\boxed{v(t) = V_0\exp(-t/\tau)}$$

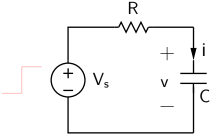
RC circuits: charging and discharging transients

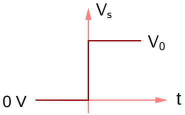




Compute i(t), $t > 0$.

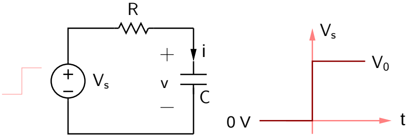
RC circuits: charging and discharging transients





Compute $i(t)$, $t > 0$.

(A) $$i(t) = C\frac{d}{dt}V_0\left[1 - \exp(-t/\tau)\right]$$
$$= \frac{CV_0}{\tau}\exp(-t/\tau) = \frac{V_0}{R}\exp(-t/\tau)$$



Compute $i(t)$, $t > 0$.

(A) $i(t) = C \dfrac{d}{dt} V_0 \left[1 - \exp(-t/\tau)\right]$

$$= \frac{CV_0}{\tau} \exp(-t/\tau) = \frac{V_0}{R} \exp(-t/\tau)$$

(B) Let $i(t) = A' \exp(-t/\tau) + B'$, $\quad t > 0$.

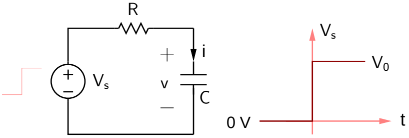
$t = 0^+$: $v = 0$, $V_s = V_0 \Rightarrow i(0^+) = V_0/R$.

$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = \frac{V_0}{R}, \; B' = 0 \Rightarrow i(t) = \frac{V_0}{R} \exp(-t/\tau)$$

RC circuits: charging and discharging transients



Compute $i(t)$, $t > 0$.

(A) $i(t) = C\dfrac{d}{dt}V_0\left[1 - \exp(-t/\tau)\right]$

$$= \frac{CV_0}{\tau}\exp(-t/\tau) = \frac{V_0}{R}\exp(-t/\tau)$$

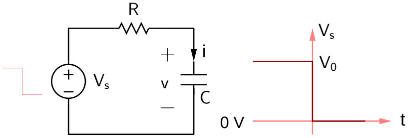
(B) Let $i(t) = A'\exp(-t/\tau) + B'\,,\quad t > 0\,.$

$t = 0^+$: $v = 0\,,\ V_s = V_0 \ \Rightarrow i(0^+) = V_0/R\,.$

$t \rightarrow \infty$: $i(t) = 0\,.$

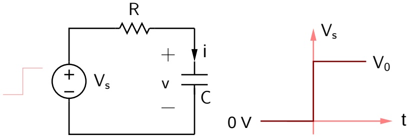
Using these conditions, we obtain

$$A' = \frac{V_0}{R}\,,\ B' = 0 \ \Rightarrow i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



Compute $i(t)$, $t > 0$.

RC circuits: charging and discharging transients



Compute $i(t)$, $t > 0$.

(A) $i(t) = C \dfrac{d}{dt} V_0 \left[1 - \exp(-t/\tau)\right]$

$$= \frac{CV_0}{\tau} \exp(-t/\tau) = \frac{V_0}{R} \exp(-t/\tau)$$

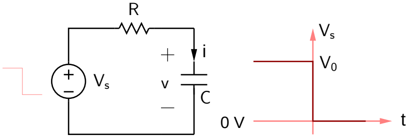
(B) Let $i(t) = A' \exp(-t/\tau) + B'$, $\quad t > 0$.

$t = 0^+$: $v = 0$, $V_S = V_0 \Rightarrow i(0^+) = V_0/R$.

$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = \frac{V_0}{R}, \; B' = 0 \Rightarrow i(t) = \frac{V_0}{R} \exp(-t/\tau)$$

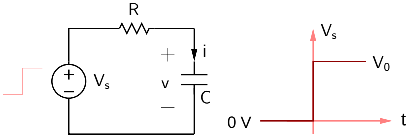


Compute $i(t)$, $t > 0$.

(A) $i(t) = C \dfrac{d}{dt} V_0 \left[\exp(-t/\tau)\right]$

$$= -\frac{CV_0}{\tau} \exp(-t/\tau) = -\frac{V_0}{R} \exp(-t/\tau)$$

RC circuits: charging and discharging transients



Compute $i(t)$, $t > 0$.

(A) $i(t) = C\dfrac{d}{dt}V_0\,[1 - \exp(-t/\tau)]$

$$= \frac{CV_0}{\tau}\exp(-t/\tau) = \frac{V_0}{R}\exp(-t/\tau)$$

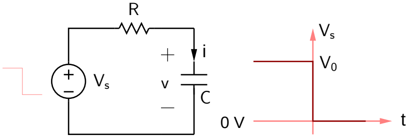
(B) Let $i(t) = A'\exp(-t/\tau) + B'$, $t > 0$.

$t = 0^+$: $v = 0$, $V_S = V_0 \Rightarrow i(0^+) = V_0/R$.

$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = \frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



Compute $i(t)$, $t > 0$.

(A) $i(t) = C\dfrac{d}{dt}V_0\,[\exp(-t/\tau)]$

$$= -\frac{CV_0}{\tau}\exp(-t/\tau) = -\frac{V_0}{R}\exp(-t/\tau)$$

(B) Let $i(t) = A'\exp(-t/\tau) + B'$, $t > 0$.

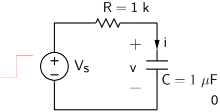
$t = 0^+$: $v = V_0$, $V_S = 0 \Rightarrow i(0^+) = -V_0/R$.

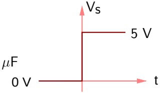
$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = -\frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

RC circuits: charging and discharging transients

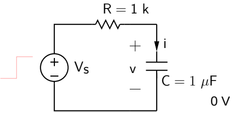


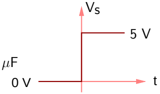


$$v(t) = V_0\left[1 - \exp(-t/\tau)\right]$$

$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$

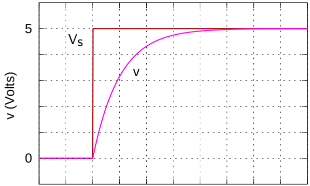
RC circuits: charging and discharging transients



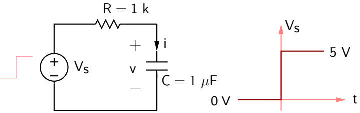


$$v(t) = V_0\left[1 - \exp(-t/\tau)\right]$$

$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$

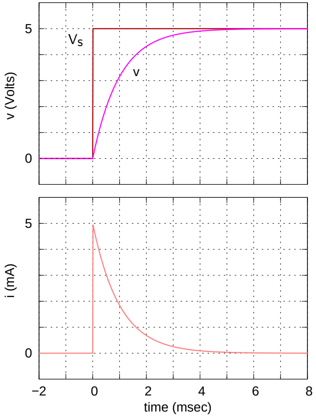


RC circuits: charging and discharging transients

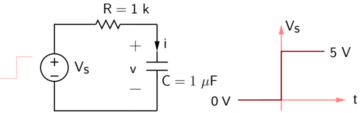


$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$

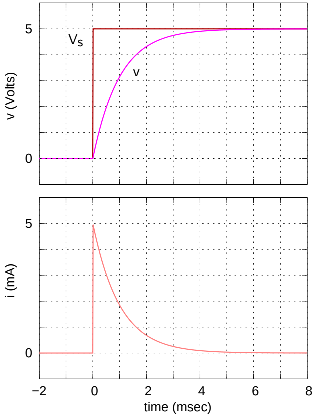


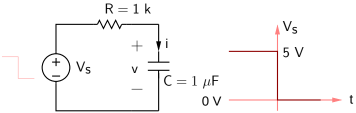
RC circuits: charging and discharging transients



$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$

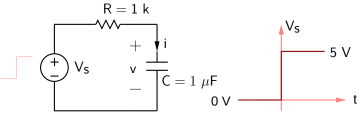




$$v(t) = V_0\exp(-t/\tau)$$

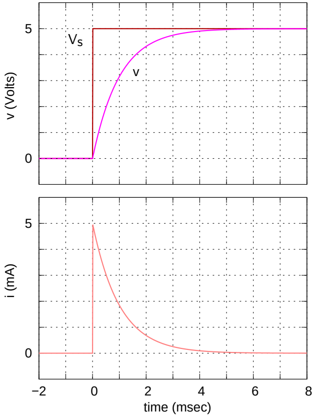
$$i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

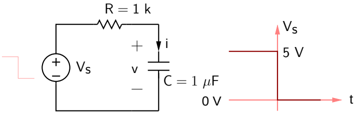
RC circuits: charging and discharging transients



$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

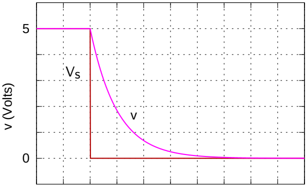
$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



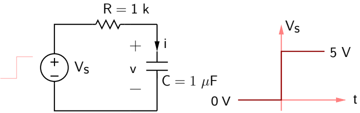


$$v(t) = V_0\exp(-t/\tau)$$

$$i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

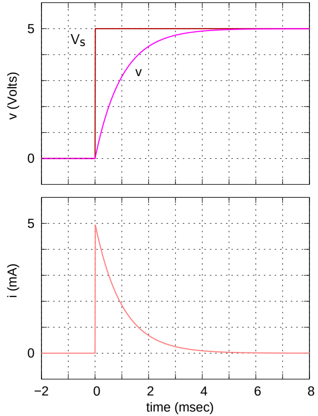


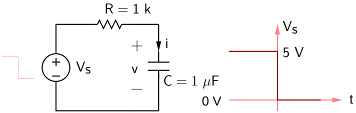
RC circuits: charging and discharging transients



$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

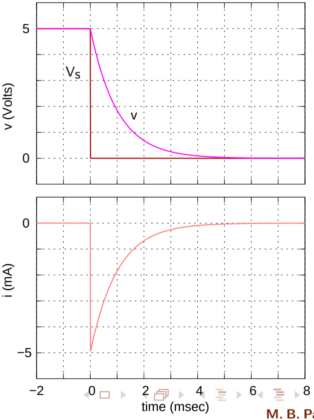
$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$





$$v(t) = V_0\exp(-t/\tau)$$

$$i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$



Significance of the time constant (τ)

x	e^{-x}	$1-e^{-x}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933

x	e^{-x}	$1 - e^{-x}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933

* For $x = 5$, $e^{-x} \simeq 0$, $1 - e^{-x} \simeq 1$.

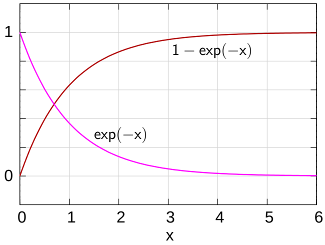
Significance of the time constant (τ)

x	e^{-x}	$1-e^{-x}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933

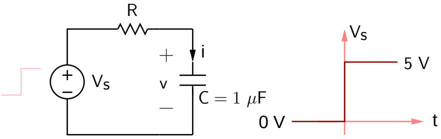
* For $x=5$, $e^{-x}\simeq 0$, $1-e^{-x}\simeq 1$.
* In *RC* circuits, $x=t/\tau \Rightarrow$ When $t=5\,\tau$, the charging (or discharging) process is almost complete.

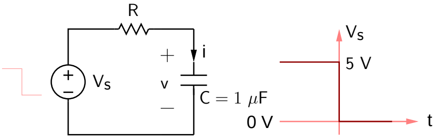
Significance of the time constant (τ)

x	e^{-x}	$1 - e^{-x}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933



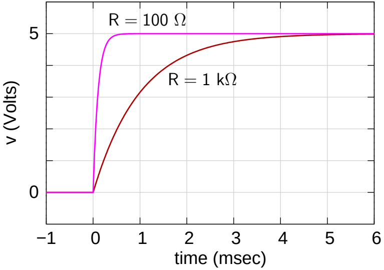
* For $x = 5$, $e^{-x} \simeq 0$, $1 - e^{-x} \simeq 1$.
* In *RC* circuits, $x = t/\tau \Rightarrow$ When $t = 5\,\tau$, the charging (or discharging) process is almost complete.

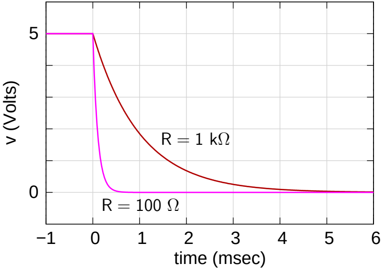




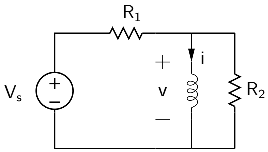
$v(t) = V_0\,[1 - \exp(-t/\tau)]$

$v(t) = V_0\,\exp(-t/\tau)$

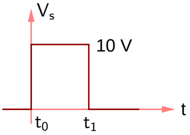




RL circuit: example



$i(0) = 0\,\text{A}$, Find $i(t)$.



$R_1 = 10\,\Omega$

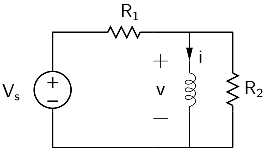
$R_2 = 40\,\Omega$

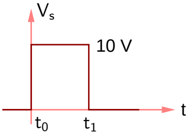
$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$

RL circuit: example





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$

$i(0) = 0\,\mathrm{A}$, Find $i(t)$.

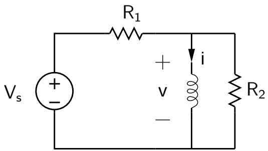
There are three intervals of constant V_s:

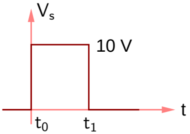
(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

RL circuit: example





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,H$

$t_0 = 0$

$t_1 = 0.1\,s$

$i(0) = 0\,A$, Find $i(t)$.

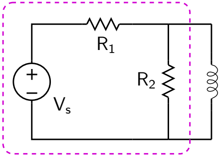
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

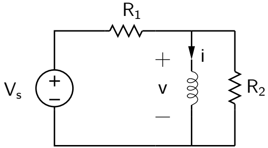
(3) $t > t_1$

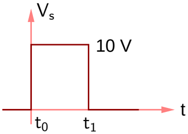
R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$$\tau = L/R_{Th}$$
$$= 0.8\text{ H}/8\,\Omega$$
$$= 0.1\text{ s}$$





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,H$

$t_0 = 0$

$t_1 = 0.1\,s$

$i(0) = 0\,A$, Find $i(t)$.

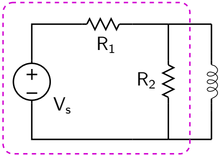
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

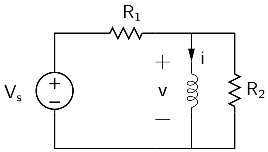
$= 0.8\ H/8\,\Omega$

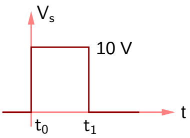
$= 0.1\ s$

At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V .

$\Rightarrow i(t_0^-) = 0\ A \Rightarrow i(t_0^+) = 0\ A$.

RL circuit: example





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$

$i(0) = 0\,\mathrm{A}$, Find $i(t)$.

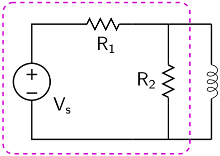
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

$= 0.8\ \mathrm{H}/8\,\Omega$

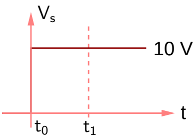
$= 0.1\ \mathrm{s}$

At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V .

$\Rightarrow i(t_0^-) = 0\ \mathrm{A} \Rightarrow i(t_0^+) = 0\ \mathrm{A}$.

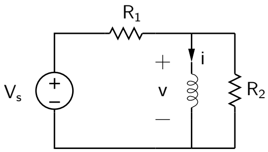
If V_s did not change at $t = t_1$,

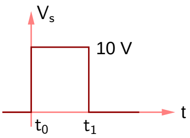
we would have



$v(\infty) = 0$ V, $i(\infty) = 10\ \mathrm{V}/10\ \Omega = 1$ A .

RL circuit: example





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$

$i(0) = 0\,\text{A}$, Find $i(t)$.

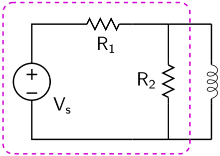
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

$= 0.8\text{ H}/8\,\Omega$

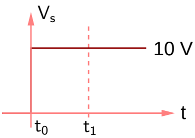
$= 0.1\text{ s}$

At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V .

$\Rightarrow i(t_0^-) = 0\text{ A} \Rightarrow i(t_0^+) = 0\text{ A}$.

If V_s did not change at $t = t_1$,

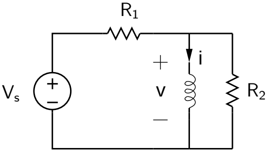
we would have

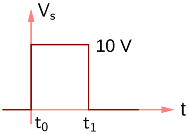


$v(\infty) = 0$ V, $i(\infty) = 10\text{ V}/10\,\Omega = 1\text{ A}$.

Using $i(t_0^+)$ and $i(\infty)$, we can obtain $i(t)$, $t > 0$ (See next slide).

RL circuit: example





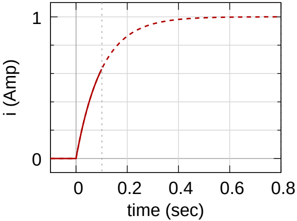
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

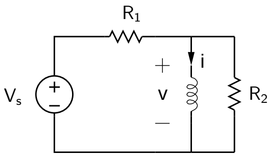
$L = 0.8\,\mathrm{H}$

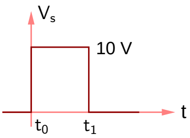
$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$



RL circuit: example





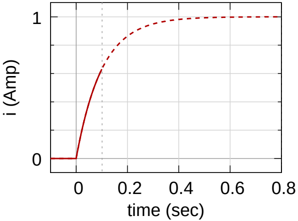
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

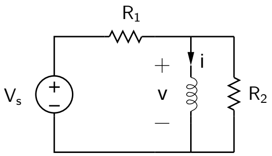
$L = 0.8\,\mathrm{H}$

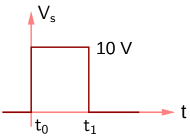
$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.





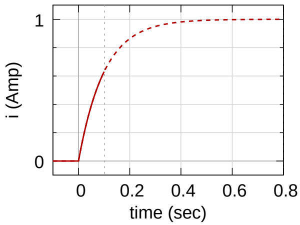
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$

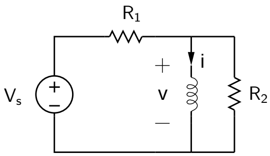


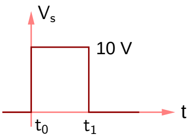
In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

RL circuit: example





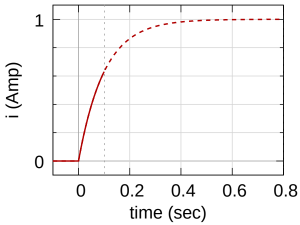
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,H$

$t_0 = 0$

$t_1 = 0.1\,s$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

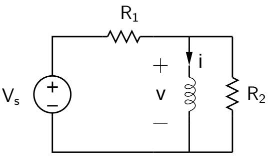
For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

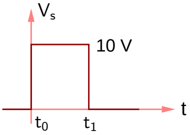
Consider $t > t_1$.

$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

$i(\infty) = 0$ A.

RL circuit: example





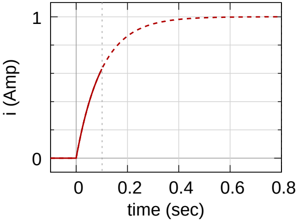
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

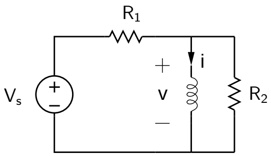
Consider $t > t_1$.

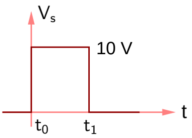
$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

$i(\infty) = 0$ A.

Let $i(t) = A\exp(-t/\tau) + B$.

RL circuit: example





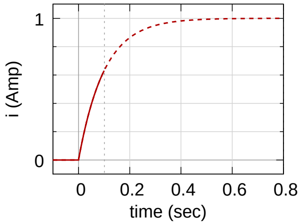
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

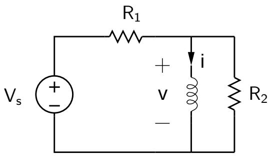
$i(\infty) = 0$ A.

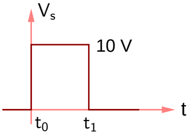
Let $i(t) = A\exp(-t/\tau) + B$.

It is convenient to rewrite $i(t)$ as

$i(t) = A' \exp[-(t - t_1)/\tau] + B$.

RL circuit: example





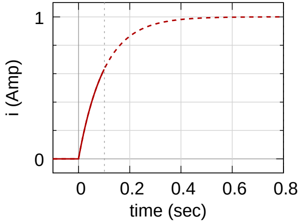
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,H$

$t_0 = 0$

$t_1 = 0.1\,s$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

$i(\infty) = 0$ A.

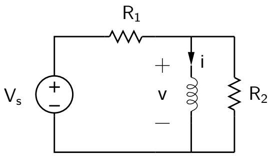
Let $i(t) = A\exp(-t/\tau) + B$.

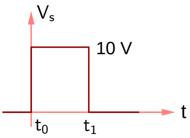
It is convenient to rewrite $i(t)$ as

$i(t) = A'\exp[-(t - t_1)/\tau] + B$.

Using $i(t_1^+)$ and $i(\infty)$, we get

$i(t) = 0.693\exp[-(t - t_1)/\tau]$ A.





$R_1 = 10\,\Omega$

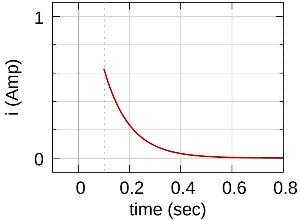
$R_2 = 40\,\Omega$

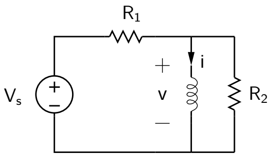
$L = 0.8\,H$

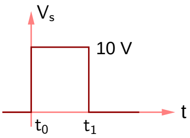
$t_0 = 0$

$t_1 = 0.1\,s$

$i(t) = 0.693\,\exp[-(t-t_1)/\tau]$ A.







$R_1 = 10\,\Omega$

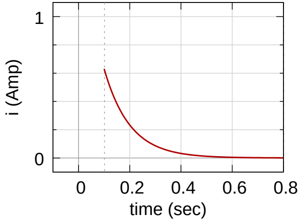
$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

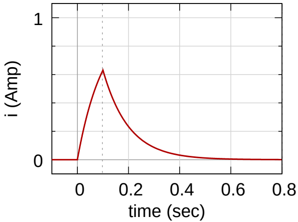
$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$

$i(t) = 0.693\,\exp[-(t-t_1)/\tau]$ A.

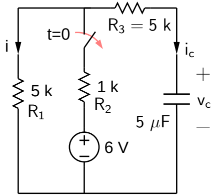


Combining the solutions for $t_0 < t < t_1$ and $t > t_1$, we get

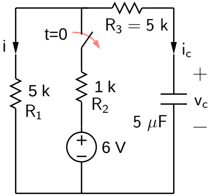


(SEQUEL file: ee101_rl1.sqproj)

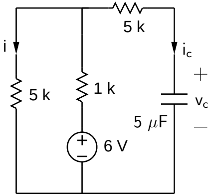
RC circuit: example



RC circuit: example

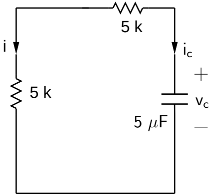


$\equiv$

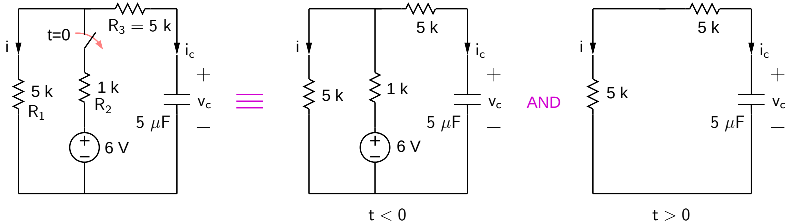


$t < 0$

AND

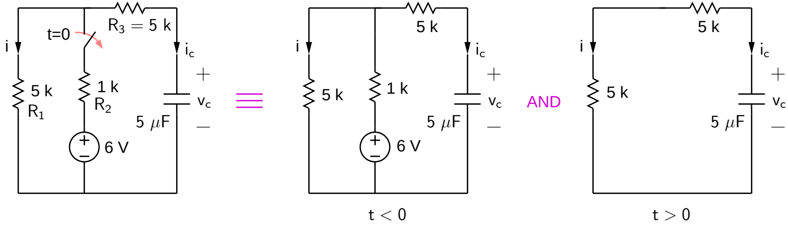


$t > 0$



$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\ \text{V}/(5\ \text{k} + 1\ \text{k}) = 1\ \text{mA}$.

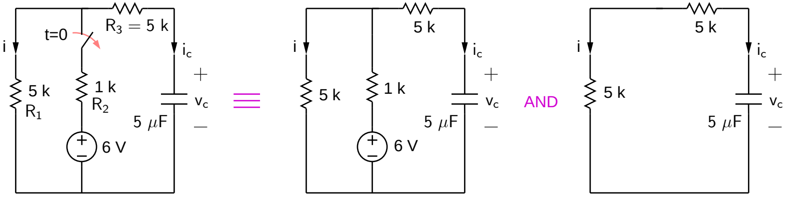
RC circuit: example



$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_c(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_c(0^+) = 5\text{ V}$.

RC circuit: example



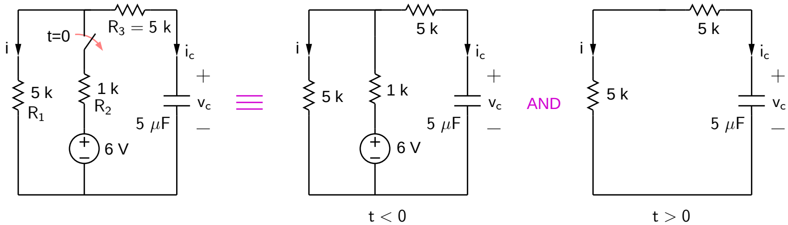
$t < 0$ $\qquad\qquad$ $t > 0$

$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\ \text{V}/(5\ \text{k} + 1\ \text{k}) = 1\ \text{mA}$.

$v_c(0^-) = 6\ \text{V} - 1\ \text{mA} \times R_2 = 5\ \text{V} \Rightarrow v_c(0^+) = 5\ \text{V}$.

$\Rightarrow i(0^+) = 5\ \text{V}/(5\ \text{k} + 5\ \text{k}) = 0.5\ \text{mA}$.

RC circuit: example



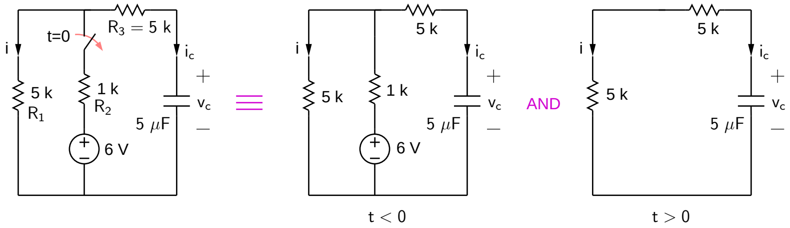
$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1$ mA.

$v_c(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_c(0^+) = 5$ V.

$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5$ mA.

Let $i(t) = A\exp(\text{-}t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50$ ms.

RC circuit: example

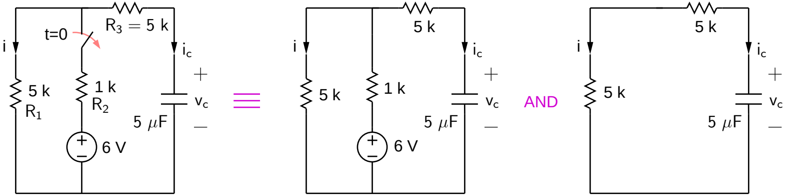


$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.
$v_c(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_c(0^+) = 5\text{ V}$.
$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

Let $i(t) = A\exp(\text{-}t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50\text{ ms}$.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get
$i(t) = 0.5\exp(\text{-}t/\tau)$ mA.

RC circuit: example



t < 0   t > 0

$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\ \mathrm{V}/(5\ \mathrm{k} + 1\ \mathrm{k}) = 1\ \mathrm{mA}$.

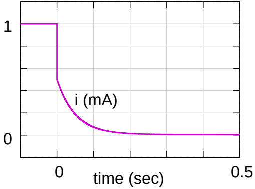
$v_c(0^-) = 6\ \mathrm{V} - 1\ \mathrm{mA} \times R_2 = 5\ \mathrm{V} \Rightarrow v_c(0^+) = 5\ \mathrm{V}$.

$\Rightarrow i(0^+) = 5\ \mathrm{V}/(5\ \mathrm{k} + 5\ \mathrm{k}) = 0.5\ \mathrm{mA}$.

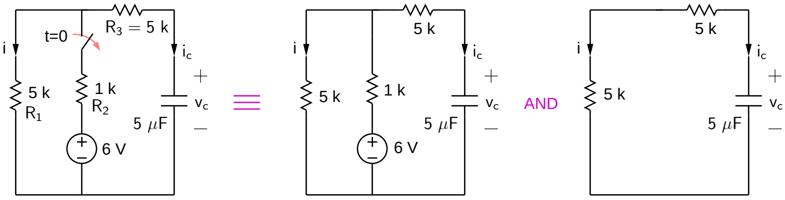
Let $i(t) = A \exp(\text{-}t/\tau) + B$ for $t > 0$, with $\tau = 10\ \mathrm{k} \times 5\ \mu\mathrm{F} = 50\ \mathrm{ms}$.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get

$i(t) = 0.5 \exp(\text{-}t/\tau)$ mA.



RC circuit: example

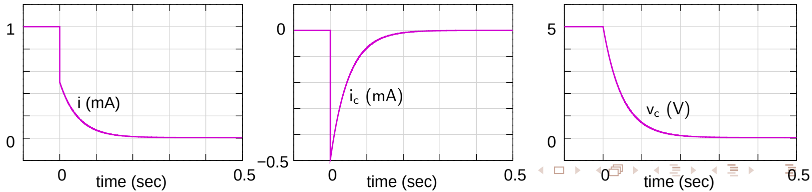


t < 0 t > 0

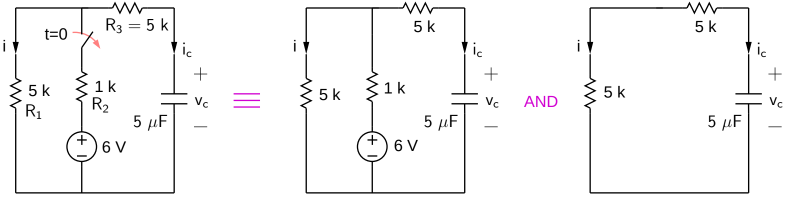
$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\ \text{V}/(5\ \text{k} + 1\ \text{k}) = 1$ mA.
$v_c(0^-) = 6\ \text{V} - 1\ \text{mA} \times R_2 = 5\ \text{V} \Rightarrow v_c(0^+) = 5$ V.
$\Rightarrow i(0^+) = 5\ \text{V}/(5\ \text{k} + 5\ \text{k}) = 0.5$ mA.

Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\ \text{k} \times 5\,\mu\text{F} = 50$ ms.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get
$i(t) = 0.5\exp(-t/\tau)$ mA.



RC circuit: example



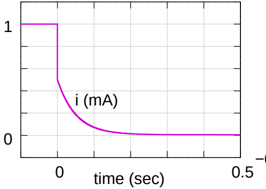
t < 0   t > 0

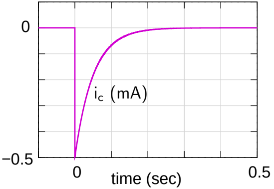
$t = 0^-$: capacitor is an open circuit, $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1$ mA.
$v_c(0^-) = 6\text{ V} - 1\text{ mA} \times R_2 = 5\text{ V} \Rightarrow v_c(0^+) = 5$ V.
$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5$ mA.

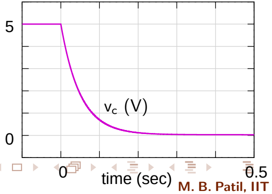
Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50$ ms.

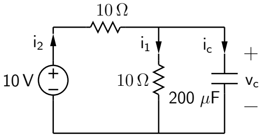
Using $i(0^+)$ and $i(\infty) = 0$ A, we get
$i(t) = 0.5\exp(-t/\tau)$ mA.

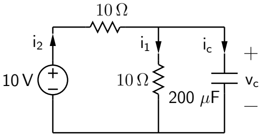
(SEQUEL file: ee101_rc2.sqproj)



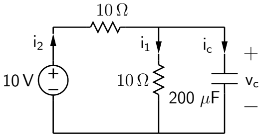




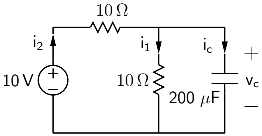

10 Ω
i2
i1
ic
+
10 V
10 Ω
vc
200 μF
−



* Given $v_c(0) = 0\ V$, find $v_c(t)$ for $t > 0$. Using this $v_c(t)$, find i_1, i_2, i_c for $t > 0$. Plot v_c, i_1, i_2, i_c versus t.

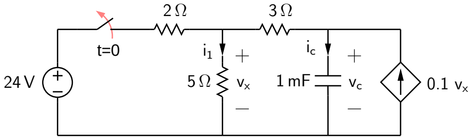


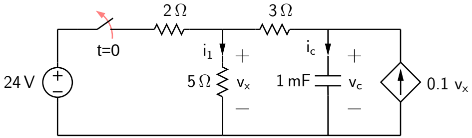
* Given $v_c(0) = 0\ V$, find $v_c(t)$ for $t > 0$. Using this $v_c(t)$, find i_1, i_2, i_c for $t > 0$. Plot v_c, i_1, i_2, i_c versus t.
* Find i_1, i_2, i_c directly (i.e., without getting v_c) by finding the initial and final conditions for each of them ($i_1(0^+)$ and $i_1(\infty)$, etc.) and then using them to compute the coefficients in the general expression, $x(t) = A\,\exp(-t/\tau) + B$.



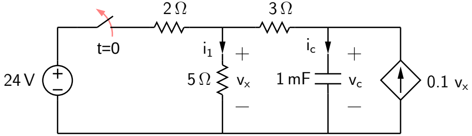
* Given $v_c(0) = 0\ V$, find $v_c(t)$ for $t > 0$. Using this $v_c(t)$, find i_1, i_2, i_c for $t > 0$. Plot v_c, i_1, i_2, i_c versus t.
* Find i_1, i_2, i_c directly (i.e., without getting v_c) by finding the initial and final conditions for each of them ($i_1(0^+)$ and $i_1(\infty)$, etc.) and then using them to compute the coefficients in the general expression, $x(t) = A\,\exp(-t/\tau) + B$.
* Verify your results with SEQUEL (file: `ee101_rc3.sqproj`).

RC circuits: home work

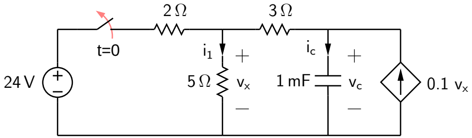




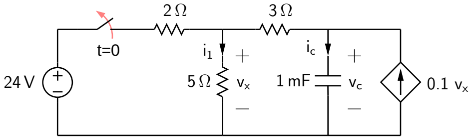
* Find $v_c(0^-)$, $v_c(\infty)$.



* Find $v_c(0^-)$, $v_c(\infty)$.
* Find R_{Th} as seen by the capacitor for $t > 0$.

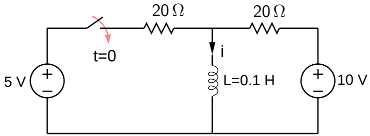


- Find $v_c(0^-)$, $v_c(\infty)$.
- Find R_{Th} as seen by the capacitor for $t > 0$.
- Solve for $v_c(t)$ and $i_1(t)$, $t > 0$.

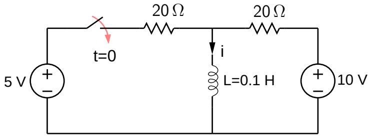


- Find $v_c(0^-)$, $v_c(\infty)$.
- Find R_{Th} as seen by the capacitor for $t > 0$.
- Solve for $v_c(t)$ and $i_1(t)$, $t > 0$.
- Verify your results with SEQUEL (file: `ee101_rc4.sqproj`).

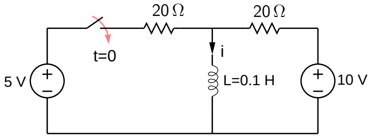
RL circuits: home work



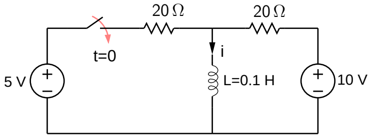
RL circuits: home work



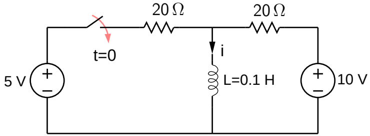
* Find $i(0^-)$, $i(\infty)$.



* Find $i(0^-)$, $i(\infty)$.
* Find R_{Th} as seen by the inductor for $t > 0$.



* Find $i(0^-)$, $i(\infty)$.
* Find R_{Th} as seen by the inductor for $t > 0$.
* Solve for $i(t)$, $t > 0$.



- Find $i(0^-)$, $i(\infty)$.
- Find R_{Th} as seen by the inductor for $t > 0$.
- Solve for $i(t)$, $t > 0$.
- Verify your results with SEQUEL (file: `ee101_rl2.sqproj`).