# Chapter 4

# Frequency response and Bode plots

## 4.1 Background

The transfer function[1] $H(s) = V_o(s)/V_i(s)$ of a system conveys important information about the gain and stability of the system. Bode plots provide an approximate picture of a given $H(s)$ from which a reasonable idea of the gain of the system and its stability properties can be obtained. The Bode magnitude and phase plots are graphs of $|H(j\omega)|$ and $\angle H(j\omega)$ versus $\log\omega$ (or $\log f$), respectively.
With Bode plots, we are interested in tracking variation of $\omega$ or $|H|$ over several orders of magnitude. Linear axes are not appropriate in such cases because all smaller values of $\omega$ or $|H|$ would then get compressed to such an extent that they cannot be resolved satisfactorily. As an example, consider the linear $\omega$ axis shown in Fig. 4.1. If we plot a function (e.g., $|H(\omega)|$) using

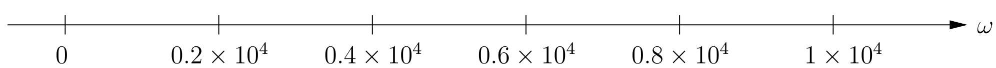


Figure 4.1: A linear $\omega$ axis.

this axis, it will be impossible to distinguish, for example, between the function values for $\omega = 8 \times 10^1$ rad/s and $\omega = 5 \times 10^2$ rad/s because they will both appear very close to the $\omega = 0$ rad/s point.
Fig. 4.2 shows a logarithmic axis for $\omega$ in various formats such that they are vertically aligned. For example, the points A, B, C represent the *same* frequency value, viz., $\omega = 1.8 \times 10^2$ rad/s or $f = 1.13 \times 10^3$ Hz. Note that each decade in frequency is well resolved with a log axis,

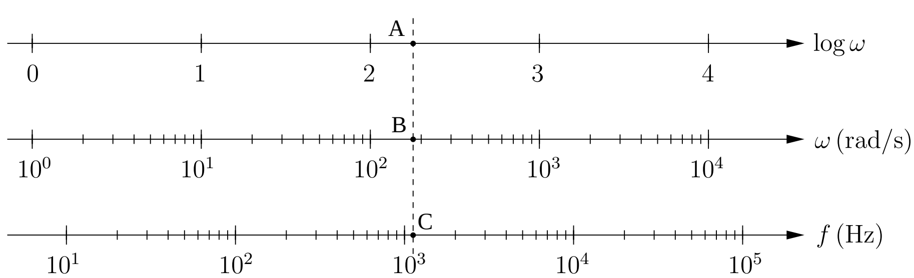


Figure 4.2: Logarithmic $\omega$ axes.

[1]We will use $H(s)$, $H(j\omega)$, and $H(\omega)$ interchangeably.

*irrespective* of whether the frequency values are low or high, allowing good resolution of the data ($|H|$ or $\angle H$) at all frequencies.
The magnitude of a typical transfer function $|H(s)|$ also varies by orders of magnitude with $\omega$, and therefore it makes sense to us a log axis for $|H(s)|$ as well. Fig. 4.3 shows two equivalent

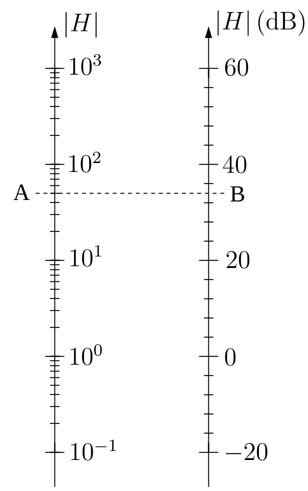


Figure 4.3: Logarithmic $|H|$ axes.

$|H|$ axes which are horizontally aligned. For example, the points A and B represent the *same* value of $|H|$, viz., $|H| = 50$ or $|H| = 34\,$dB.
We now discuss how Bode plots are obtained for a given $H(\omega)$. Consider $H(\omega) = H_1(\omega)H_2(\omega)$, where $H(\omega) = A(\omega)\,e^{j\phi(\omega)}$, $H_1(\omega) = A_1(\omega)\,e^{j\phi_1(\omega)}$, $H_2(\omega) = A_2(\omega)\,e^{j\phi_2(\omega)}$. Multiplying $H_1(\omega)$ and $H_2(\omega)$, we get,

$$H(\omega) = A(\omega)\,e^{j\phi(\omega)} = A_1(\omega)A_2(\omega)\,e^{j(\phi_1(\omega)+\phi_2(\omega))}, \tag{4.1}$$

resulting in $|H|(\omega) = |H_1(\omega)| \times |H_2(\omega)|$, i.e.,

$$20 \log |H(\omega)| = 20 \log |H_1(\omega)| + 20 \log |H_2(\omega)|\,. \tag{4.2}$$

In other words, the magnitude of $H(\omega)$ in dB is the sum of the magnitudes of $H_1(\omega)$ and $H_2(\omega)$ in dB. Also, from Eq. 4.1, it can be seen that

$$\phi(\omega) = \phi_1(\omega) + \phi_2(\omega)\,, \tag{4.3}$$

i.e., the angle (phase) of $H(\omega)$ is the sum of the angles of $H_1(\omega)$ and $H_2(\omega)$. These results enable us to construct the Bode plot for a given $H(\omega)$ in terms of the magnitude and angle plots of its factors, $H_1(\omega)$, $H_2(\omega)$, etc. We now look at a few typical factors.

1. $H(s) = K$ (a constant): For this function, the magnitude in dB is a constant, viz., $20 \log |K|$. The phase is 0 (if $K > 0$) or $\pi$ (if $K < 0$), irrespective of the frequency.

2. $H(s) = s = j\omega$: For this function, $|H| = \omega$, and $20 \log |H| = 20 \log \omega$. The following observations help us in plotting $|H(\omega)|$ and $\angle H(\omega)$.

   (i) For $\omega = 1\,$rad/s, $|H|$ (in dB)$=20 \log (1) = 0\,$dB.

(ii) Consider $\omega_1$ and $\omega_2$ such that $\omega_2 = 10 \times \omega_1$. Then
$|H(\omega_2)|\,(\text{dB}) = 20\,\log\,\omega_2 = 20\,\log\,10\omega_1 = 20 + 20\,\log\,\omega_1 = 20 + |H(\omega_1)|\,(\text{dB})$,
which means that, if the frequency is increased by one order of magnitude, $|H|$ increases by 20 dB. Equivalently, the graph of $|H|$ (in dB) versus $\log\omega$ has a slope of 20 dB/decade.

From (i) and (ii), we conclude that the graph of $|H|$ in dB versus log $\omega$ is a straight line passing through (1 rad/s, 0 dB), with a slope of 20 dB/dec, as shown in Fig. 4.4 (a).

(iii) Since $H(j\omega) = j\omega$, a purely imaginary number, $\angle H$ is always equal to $\pi/2$ (see Fig. 4.4 (b)).

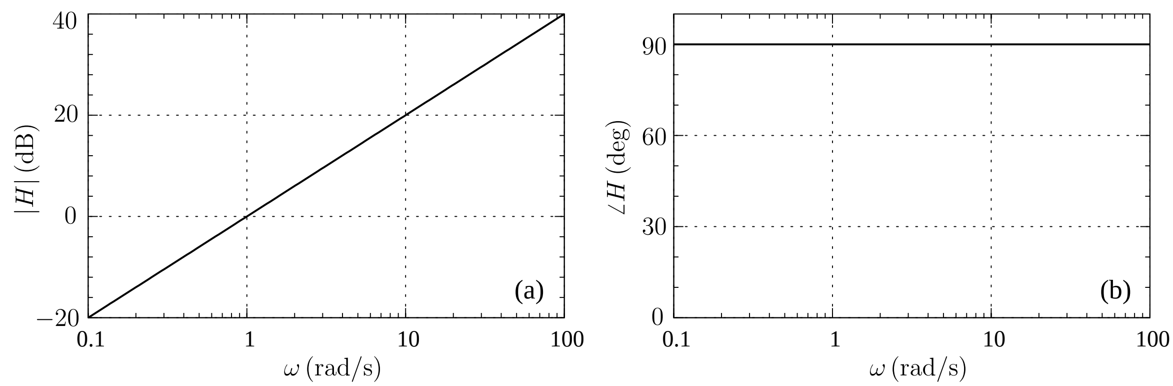


Figure 4.4: (a) $|H|$ (in dB) and (b) $\angle H$ (in degrees) versus log $\omega$ for $H(s) = s$.

3. $H(s) = 1/s = 1/j\omega$: For this function, $|H| = 1/\omega$, giving $20\,\log\,|H| = 20\,\log(1/\omega) = -20\,\log\omega$, and $\angle H$ is always $-\pi/2$, irrespective of the frequency. Note that $|H|$ is 0 dB for $\omega = 1$ rad/s and goes down by 20 dB as $\omega$ is increased by one order, which means that the plot of $|H|$ (dB) versus log $\omega$ is a straight line going through (1 rad/s, 0 dB), with a slope of $-20$ dB/dec. The magnitude and angle plots are shown in Figs. 4.5 (a) and (b), respectively.

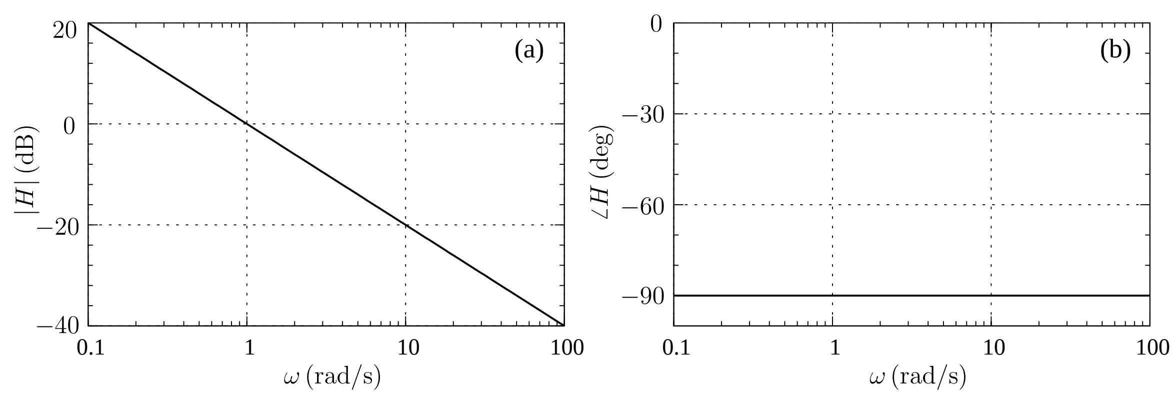


Figure 4.5: (a) $|H|$ (in dB) and (b) $\angle H$ (in degrees) versus log $\omega$ for $H(s) = 1/s$.

4. $H(s) = s^2 = -\omega^2$: For this function, the phase is always $\pi$ (equivalently, $-\pi$) rad. For the magnitude plot, we note that, for $\omega = 1$ rad/s, $|H| = 0$ dB, and

$|H|$ (dB) $= 20 \log \omega^2 = 40 \log \omega$. If $\omega$ is increased by a factor of 10, $|H|$ increases by 40 dB. The plot of $|H|$ (dB) versus $\log \omega$ is therefore a straight line passing through (1 rad/s, 0 dB), with a slope of 40 dB/dec. The magnitude and angle plots are shown in Figs. 4.6 (a) and (b), respectively.

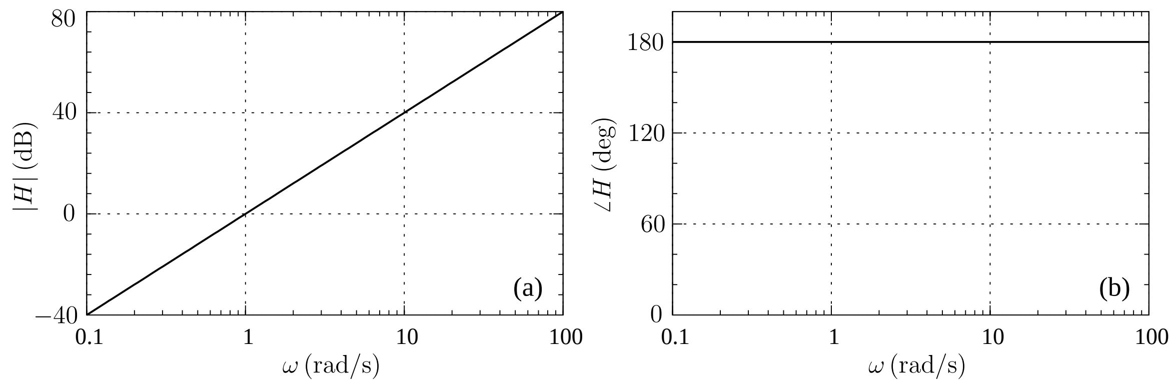


Figure 4.6: (a) $|H|$ (in dB) and (b) $\angle H$ (in degrees) versus $\log \omega$ for $H(s) = s^2$.

5. $H(s) = 1/s^2 = -1/\omega^2$ : For this function, the phase is always $\pi$ (equivalently, $-\pi$) rad. For the magnitude plot, we note that, for $\omega = 1$ rad/s, $|H| = 0$ dB, and $|H|$ (dB) $= 20 \log (1/\omega^2) = -40 \log \omega$. If $\omega$ is increased by a factor of 10, $|H|$ decreases by 40 dB. The plot of $|H|$ (dB) versus $\log \omega$ is therefore a straight line passing through (1 rad/s, 0 dB), with a slope of $-40$ dB/dec. The magnitude and angle plots are shown in Figs. 4.7 (a) and (b), respectively.

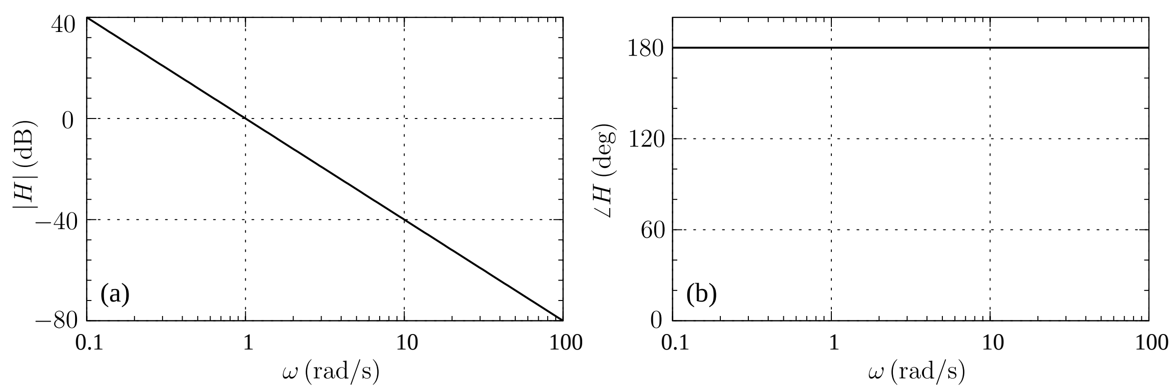


Figure 4.7: (a) $|H|$ (in dB) and (b) $\angle H$ (in degrees) versus $\log \omega$ for $H(s) = 1/s^2$.

6. $H(s) = a_0 + a_1 s$ : Let us first discuss the case where $a_0 > 0$, $a_1 > 0$, and rewrite $H(s) = a_0 H_1(s)$, where $H_1(s) = 1 + \dfrac{a_1}{a_0}\, s$, i.e., $H_1(j\omega) = 1 + j\,\dfrac{\omega}{\omega_0}$, with $\omega_0 = \dfrac{a_0}{a_1}$. Note that $\angle H(s) = \angle H_1(s)$, since $a_0$ is a positive constant. For the magnitude of $H(s)$, we have

$$|H(s)| = a_0\,|H_1(s)| = a_0 \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\,, \tag{4.4}$$

$$20 \log |H(s)| = 20 \log a_0 + 20 \log |H_1(s)|\,, \tag{4.5}$$

i.e., the $|H(s)|$ plot (in dB) is the same as the $|H_1(s)|$ plot (in dB) except for an upward shift[2] of $20\log a_0$. It suffices therefore to consider only $H_1(s)$ in the following.

$$|H_1(s)| = \sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2} \qquad \begin{cases} \approx 1 & \text{for } \omega \ll \omega_0 \\ \approx \omega/\omega_0 & \text{for } \omega \gg \omega_0 \end{cases} \tag{4.6}$$

Eq. 4.6 gives us the asymptotes for $|H_1(s)|$. In units of dB, $|H_1(s)|$ is given by

$$20\log|H_1(s)| \qquad \begin{cases} \approx 0\,\text{dB} & \text{for } \omega \ll \omega_0 \\ \approx 20\log\dfrac{\omega}{\omega_0} & \text{for } \omega \gg \omega_0 \end{cases} \tag{4.7}$$

The first asymptote is simply a horizontal line in the magnitude plot, the second is a straight line going through $(\omega_0, 0\,\text{dB})$ with a slope of 20 dB/dec. The net magnitude plot for $H_1(s)$ is obtained by using asymptote 1 for $\omega \le \omega_0$ and asymptote 2 for $\omega \ge \omega_0$ (see Fig. 4.8 (a)).

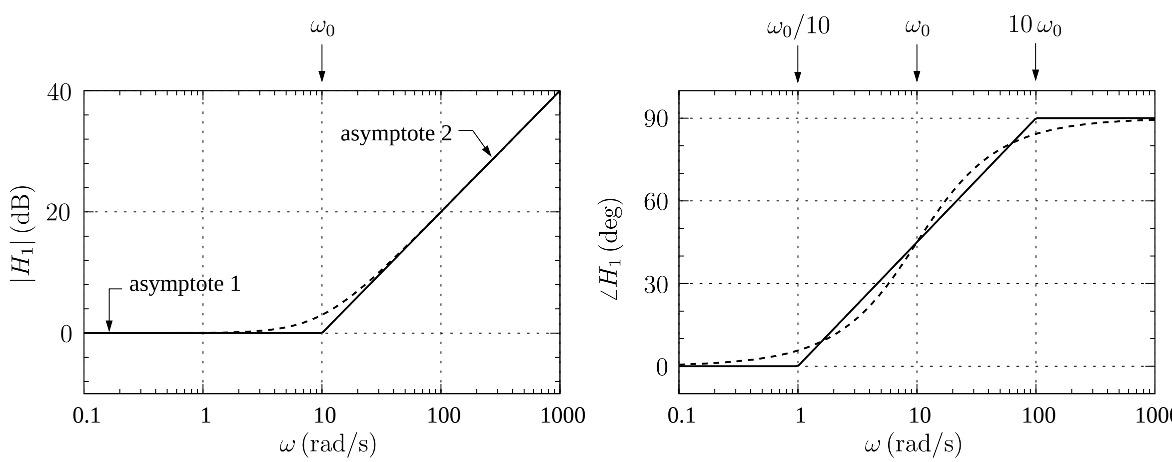


Figure 4.8: (a) $|H_1|$ (in dB) and (b) $\angle H_1$ (in degrees) versus $\log\omega$ for $H_1(s) = 1+(s/\omega_0)$, with $\omega_0 = 10$ rad/s. Solid line: Bode approximation, Dashed line: exact result

For the phase $H_1(j\omega) = 1 + j\,\dfrac{\omega}{\omega_0}$, we have

$$\angle H_1(j\omega) = \tan^{-1}\left(\frac{\omega}{\omega_0}\right). \tag{4.8}$$

In the limiting case of $\omega \ll \omega_0$, $\angle H_1 \approx 0$, and in the other limiting case of $\omega \gg \omega_0$, $\angle H_1 \approx \pi/2$. In the Bode approximation, we say that

$$\angle H_1 \qquad \begin{cases} = 0 \quad \text{for } \omega/\omega_0 \le 0.1 \\ = \pi/2 \quad \text{for } \omega/\omega_0 \ge 10 \\ \text{varies linearly with } \log\omega \text{ for } 0.1 < \omega/\omega_0 < 10 \end{cases} \tag{4.9}$$

[2]If $a_0 < 1$, $20\log a_0 < 0$, and the shift is actually downward.

For $\omega = \omega_0$, i.e., $\omega/\omega_0 = 1$, $\angle H_1(j\omega)$ is $\pi/4$ (see Eq. 4.8). The Bode approximation also gives $\pi/4$ at $\omega = \omega_0$ (show this). The phase plot is shown in Fig. 4.8 (b).

What happens if $H(s) = a_0 + a_1 s$ as before, but $a_0$ and $a_1$ are not both positive? Let us take an example of this case, $H(s) = -2 + 0.1\,s$, i.e., $H(j\omega) = 2(-1 + j\omega/20)$. For this function,

$$|H| = 2\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}\,, \tag{4.10}$$

which is identical in form to Eq. 4.4, and no special consideration is required to handle the negative sign of $a_0$. For the phase plot, we have

$$\angle H(j\omega) = \angle(-1 + j\omega/\omega_0). \tag{4.11}$$

The complex number in the brackets now falls in the second quadrant, giving the following Bode approximation:

$$\angle H \quad \begin{cases} = \pi \quad \text{for } \omega/\omega_0 \leq 0.1 \\ = \pi/2 \quad \text{for } \omega/\omega_0 \geq 10 \\ \text{varies linearly with } \log\omega \text{ for } 0.1 < \omega/\omega_0 < 10 \end{cases} \tag{4.12}$$

7. $H(s) = 1/(b_0 + b_1 s)$: Let us first discuss the case where $b_0 > 0$, $b_1 > 0$, and rewrite $H(s) = \frac{1}{b_0}\,H_1(s)$, where $H_1(s) = \dfrac{1}{1 + \frac{b_1}{b_0}\,s}$, i.e., $H_1(j\omega) = \dfrac{1}{1 + j\,\frac{\omega}{\omega_0}}$, with $\omega_0 = \frac{b_0}{b_1}$. Note that $\angle H(s) = \angle H_1(s)$, since $b_0$ is a positive constant. For the magnitude of $H(s)$, we have

$$|H(s)| = \frac{1}{b_0}\,|H_1(s)| = \frac{1}{b_0}\,\frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}\,, \tag{4.13}$$

$$20\,\log|H(s)| = -20\,\log b_0 + 20\,\log|H_1(s)|\,, \tag{4.14}$$

i.e., the $|H(s)|$ plot (in dB) is the same as the $|H_1(s)|$ plot (in dB) except for a downward shift[3] of $20\,\log b_0$. It suffices therefore to consider only $H_1(s)$ in the following.

$$|H_1(s)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}} \qquad \begin{cases} \approx 1 & \text{for } \omega \ll \omega_0 \\ \approx \omega_0/\omega & \text{for } \omega \gg \omega_0 \end{cases} \tag{4.15}$$

Eq. 4.15 gives us the asymptotes for $|H_1(s)|$. In units of dB, $|H_1(s)|$ is given by

$$20\,\log|H_1(s)| \qquad \begin{cases} \approx 0\,\text{dB} & \text{for } \omega \ll \omega_0 \\ \approx -20\,\log\dfrac{\omega}{\omega_0} & \text{for } \omega \gg \omega_0 \end{cases} \tag{4.16}$$

The first asymptote is simply a horizontal line in the magnitude plot, the second is a straight line going through $(\omega_0, 0\,\text{dB})$ with a slope of $-20\,\text{dB/dec}$. The net magnitude

[3]If $b_0 < 1$, $20\,\log b_0 < 0$, and the shift is actually upward.

plot for $H_1(s)$ is obtained by using asymptote 1 for $\omega \leq \omega_0$ and asymptote 2 for $\omega \geq \omega_0$ (see Fig. 4.9 (a)).

For the phase $H_1(j\omega) = 1/(1 + j\omega/\omega_0)$, we have

$$\angle H_1(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right) . \tag{4.17}$$

In the limiting case of $\omega \ll \omega_0$, $\angle H_1 \approx 0$, and in the other limiting case of $\omega \gg \omega_0$, $\angle H_1 \approx -\pi/2$. In the Bode approximation, we say that

$$\angle H_1 \quad \begin{cases} = 0 \;\; \text{for } \omega/\omega_0 \leq 0.1 \\ = \; -\pi/2 \;\; \text{for } \omega/\omega_0 \geq 10 \\ \text{varies linearly with } \log\omega \text{ for } 0.1 < \omega/\omega_0 < 10 \end{cases} \tag{4.18}$$

For $\omega = \omega_0$, i.e., $\omega/\omega_0 = 1$, $\angle H_1(j\omega)$ is $-\pi/4$ (see Eq. 4.17). The Bode approximation also gives $-\pi/4$ at $\omega = \omega_0$ (show this). The phase plot is shown in Fig. 4.9 (b).

The case where $b_0$ and $b_1$ are not both positive is left for the reader to work out.

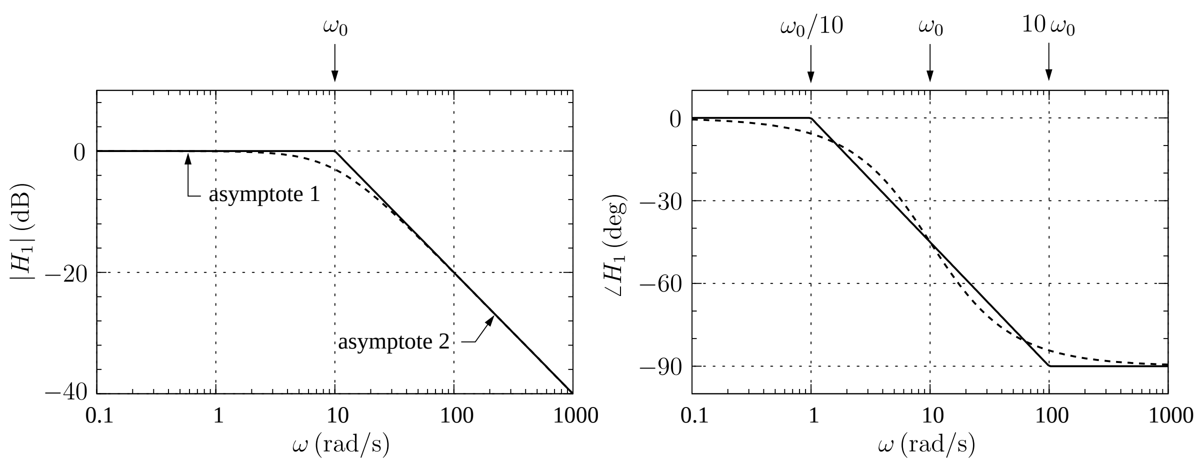


Figure 4.9: (a) $|H_1|$ (in dB) and (b) $\angle H_1$ (in degrees) versus $\log\omega$ for $H_1(s) = 1/(1 + (s/\omega_0))$, with $\omega_0 = 10$ rad/s. Solid line: Bode approximation, Dashed line: exact result

8. $H(s) = a_0 + a_1 s + a_2 s^2$: The nature of the Bode plots for this function depends on whether $\Delta = a_1^2 - 4a_0a_2$ is positive, zero, or negative. Let us consider these cases separately.

   (i) $\Delta > 0$: In this case, the roots of the quadratic $r_1$, $r_2$ are real and unequal, and we can write

   $$H(s) = a_2\,(s - r_1)(s - r_2) . \tag{4.19}$$

   The magnitude and phase Bode plots for $H(s)$ can be obtained by plotting the three terms separately and adding them up.

(ii) $\Delta = 0$: In this case, the roots are real and equal, i.e., $r_1 = r_2 = r$, and we have

$$H(s) = a_2\,(s-r)(s-r)\,. \tag{4.20}$$

Again, the magnitude and phase Bode plots for $H(s)$ can be obtained by plotting the three terms separately and adding them up.

(iii) $\Delta < 0$: For this condition, the roots are complex conjugates, and we can write $H(s)$ as

$$H(s) = a_0\left[1 + 2\,\zeta\,(s/\omega_0) + (s/\omega_0)^2\right] \equiv a_0 H_1(s)\,, \tag{4.21}$$

where $\omega_0 = \sqrt{a_0/a_2}$, and $\zeta = a_1\omega_0/(2a_0)$. Note that $0 \leq \zeta < 1$ since $\Delta < 0$ (show this).

(a) $\zeta = 0$: In this case, we have

$$H_1(s) = \left[1 + (s/\omega_0)^2\right] = \left[1 - (\omega/\omega_0)^2\right]\,. \tag{4.22}$$

For $\omega \ll \omega_0$, we get $|H_1| = 0\,\text{dB}$, a horizontal line. For $\omega \gg \omega_0$, we get $|H_1| = 20\,\log(\omega/\omega_0)^2$, a straight line passing through $(\omega_0, 0\,\text{dB})$ with a slope of $40\,\text{dB/dec}$. The phase of $H_1(s)$ is 0 if $\omega/\omega_0 < 1$ or $\pi$ if $\omega/\omega_0 > 1$, and no interpolation is required in the Bode approximation.

(b) $\zeta \neq 0$: In this case, we have from Eq. 4.21,

$$20\,\log|H_1(s)| = \left|1 - (\omega/\omega_0)^2 + j2\,\zeta\,(\omega/\omega_0)\right| \quad \begin{cases} \approx 0\,\text{dB} & \text{for } \omega \ll \omega_0 \\ \approx 40\,\log\dfrac{\omega}{\omega_0} & \text{for } \omega \gg \omega_0 \end{cases} \tag{4.23}$$

The first asymptote is simply a horizontal line in the magnitude plot, the second is a straight line going through $(\omega_0, 0\,\text{dB})$ with a slope of $40\,\text{dB/dec}$. For the phase, we note that

$$H_1(j\omega) = 1 - (\omega/\omega_0)^2 + j2\,\zeta\,(\omega/\omega_0)\,, \tag{4.24}$$

$$\angle H_1(j\omega) = \tan^{-1}\left(\frac{2\,\zeta\,\omega/\omega_0}{1 - (\omega/\omega_0)^2}\right)\,. \tag{4.25}$$

In the limiting case of $\omega \ll \omega_0$, $\angle H_1 \approx 0$, and in the other limiting case of $\omega \gg \omega_0$, $\angle H_1 \approx \pi$. In the Bode approximation, we say that

$$\angle H_1 \quad \begin{cases} = 0 & \text{for } \omega/\omega_0 \leq 0.1 \\ = \pi & \text{for } \omega/\omega_0 \geq 10 \\ \text{varies linearly with } \log\omega & \text{for } 0.1 < \omega/\omega_0 < 10 \end{cases} \tag{4.26}$$

For $\omega = \omega_0$, i.e., $\omega/\omega_0 = 1$, $\angle H_1(j\omega)$ is $\pi/2$ (see Eq. 4.25). The Bode approximation also gives $\pi/2$ at $\omega = \omega_0$ (show this).
Fig. 4.10 shows the magnitude and angle plots for $H_1(s)$ for $\zeta \neq 0$. The exact results are also shown for comparison.

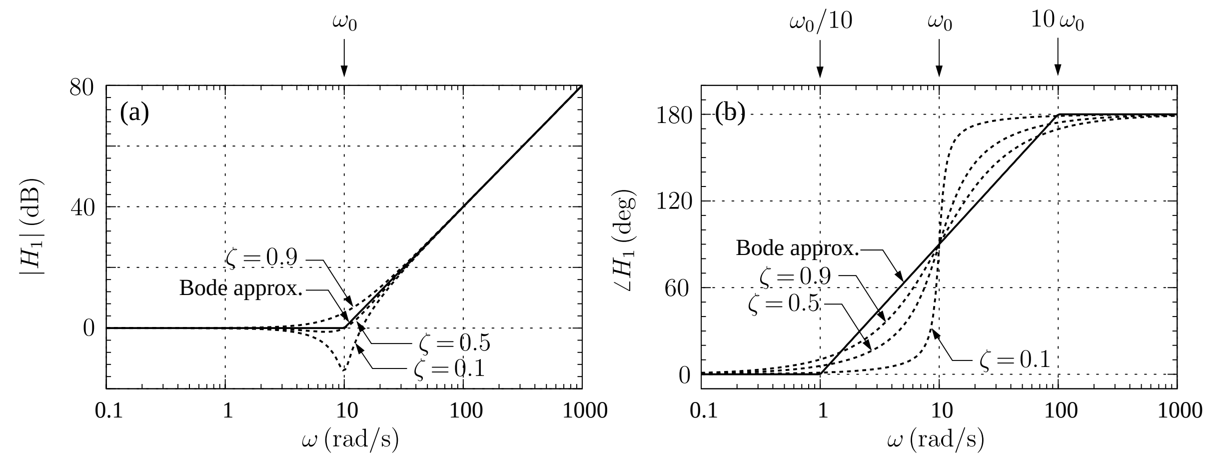


Figure 4.10: (a) $|H_1|$ (in dB) and (b) $\angle H_1$ (in degrees) versus $\log \omega$ for $H_1(s) = 1 + 2\,\zeta(s/\omega_0) + (s/\omega_0)^2$, with $\omega_0 = 10$ rad/s. Solid line: Bode approximation, Dashed lines: exact result

9. $H(s) = 1/(b_0 + b_1 s + b_2 s^2)$: The nature of the Bode plots for this function depends on whether $\Delta = b_1^2 - 4b_0 b_2$ is positive, zero, or negative. Let us consider these cases separately.

   (i) $\Delta > 0$: In this case, the roots of the quadratic $r_1$, $r_2$ are real and unequal, and we can write

   $$H(s) = \frac{1}{b_2}\,\frac{1}{(s - r_1)}\,\frac{1}{(s - r_2)}\,. \tag{4.27}$$

   The magnitude and phase Bode plots for $H(s)$ can be obtained by plotting the three terms separately and adding them up.

   (ii) $\Delta = 0$: In this case, the roots are real and equal, i.e., $r_1 = r_2 = r$, and we have

   $$H(s) = \frac{1}{b_2}\,\frac{1}{(s - r)}\,\frac{1}{(s - r)}\,. \tag{4.28}$$

   Again, the magnitude and phase Bode plots for $H(s)$ can be obtained by plotting the three terms separately and adding them up.

   (iii) $\Delta < 0$: For this condition, the roots are complex conjugates, and we can write $H(s)$ as

   $$H(s) = \frac{1}{b_0}\,\frac{1}{\left[1 + 2\,\zeta\,(s/\omega_0) + (s/\omega_0)^2\right]} \equiv \frac{1}{b_0}\,H_1(s)\,, \tag{4.29}$$

   where $\omega_0 = \sqrt{b_0/b_2}$, and $\zeta = b_1\omega_0/(2b_0)$. Note that $0 \leq \zeta < 1$ since $\Delta < 0$ (show this).

   (a) $\zeta = 0$: In this case, we have

   $$H_1(s) = \frac{1}{\left[1 + (s/\omega_0)^2\right]} = \frac{1}{\left[1 - (\omega/\omega_0)^2\right]}\,. \tag{4.30}$$

   For $\omega \ll \omega_0$, we get $|H_1| = 0$ dB, a horizontal line. For $\omega \gg \omega_0$, we get $|H_1| = -20\,\log(\omega/\omega_2)^2$, a straight line passing through $(\omega_0, 0\,\text{dB})$ with a slope of

$-40\,\text{dB/dec}$. The phase of $H_1(s)$ is 0 if $\omega/\omega_0 < 1$ or $\pi$ if $\omega/\omega_0 > 1$, and no interpolation is required in the Bode approximation.

(b) $\zeta \neq 0$: In this case, we have from Eq. 4.29,

$$20\,\log|H_1(s)| = \frac{1}{\left|1-(\omega/\omega_0)^2 + j2\,\zeta\,(\omega/\omega_0)\right|} \quad \begin{cases} \approx 0\,\text{dB} & \text{for } \omega \ll \omega_0 \\ \approx 40\,\log\dfrac{\omega_0}{\omega} & \text{for } \omega \gg \omega_0 \end{cases} \tag{4.31}$$

The first asymptote is simply a horizontal line in the magnitude plot, the second is a straight line going through $(\omega_0, 0\,\text{dB})$ with a slope of $-40\,\text{dB/dec}$. For the phase, we note that

$$H_1(j\omega) = \frac{1}{1-(\omega/\omega_0)^2 + j2\,\zeta\,(\omega/\omega_0)}\,, \tag{4.32}$$

$$\angle H_1(j\omega) = -\tan^{-1}\left(\frac{2\,\zeta\,\omega/\omega_0}{1-(\omega/\omega_0)^2}\right). \tag{4.33}$$

In the limiting case of $\omega \ll \omega_0$, $\angle H_1 \approx 0$, and in the other limiting case of $\omega \gg \omega_0$, $\angle H_1 \approx -\pi$. In the Bode approximation, we say that

$$\angle H_1 \quad \begin{cases} = 0 & \text{for } \omega/\omega_0 \leq 0.1 \\ = -\pi & \text{for } \omega/\omega_0 \geq 10 \\ \text{varies linearly with } \log\omega & \text{for } 0.1 < \omega/\omega_0 < 10 \end{cases} \tag{4.34}$$

For $\omega = \omega_0$, i.e., $\omega/\omega_0 = 1$, $\angle H_1(j\omega)$ is $-\pi/2$ (see Eq. 4.33). The Bode approximation also gives $-\pi/2$ at $\omega = \omega_0$ (show this).
Fig. 4.11 shows the magnitude and angle plots for $H_1(s)$ for $\zeta \neq 0$. The exact results are also shown for comparison.

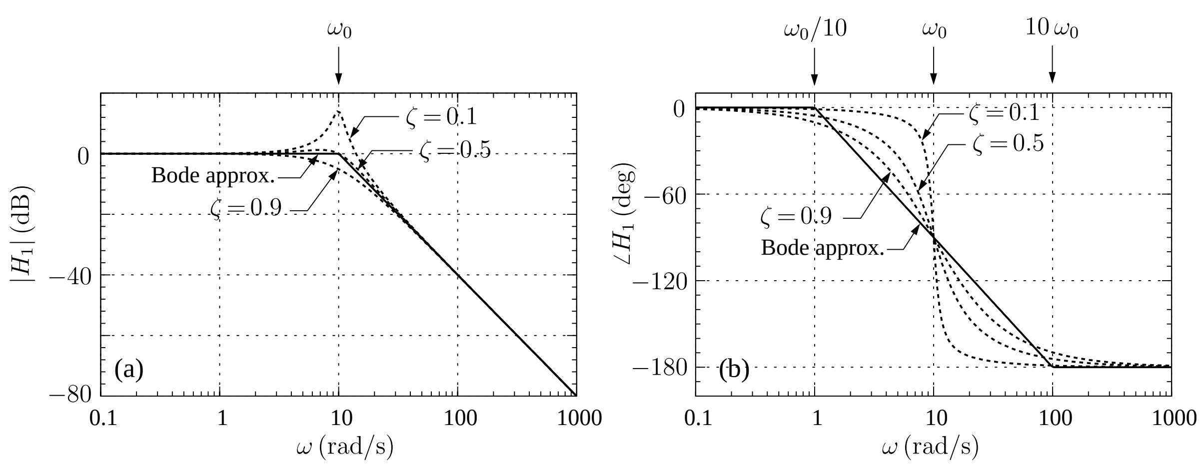


Figure 4.11: (a) $|H_1|$ (in dB) and (b) $\angle H_1$ (in degrees) versus $\log\omega$ for $H_1(s) = 1/(1+2\,\zeta(s/\omega_0)+(s/\omega_0)^2)$, with $\omega_0 = 10\,\text{rad/s}$. Solid line: Bode approximation, Dashed lines: exact result

## 4.2 Examples

1. Construct the magnitude and phase Bode plots for $H(s) = \dfrac{20\,(s+10)}{(s+100)}$.

   To get an idea of the corner frequencies, we rewrite $H(s)$ as

   $$H(s) = \frac{2(1+(s/10))}{(1+(s/100))} = 2(1+(s/10))\,\frac{1}{(1+(s/100))} \equiv H_1(s)H_2(s)\,. \tag{4.35}$$

   From this expression, it is clear that the corner frequencies are $\omega_{01} = 10$ rad/s and $\omega_{02} = 100$ rad/s. This is an important observation because it enables us to choose the range of frequencies which we must consider. In this case, frequencies much less than $\omega_{01}/10$ and much greater than $10\,\omega_{02}$ need not be considered since the magnitude and phase plots would become constant for those frequencies.

   Next, we construct the Bode plots for $H_1(s)$ and $H_2(s)$ separately, and then add them up to get the plots for $H(s)$, as shown in Fig. 4.12.

   **SEQUEL file:** `test_filter_6.sqproj`

2. Construct the magnitude and phase Bode plots for $H(s) = \dfrac{1000\,s^2}{(s^2+5s+100)}$.

   To get an idea of the corner frequency, we rewrite $H(s)$ as

   $$H(s) = \frac{10\,s^2}{(1+(s/20)+(s^2/100))} = (10\,s^2)\,\frac{1}{(1+(s/20)+(s^2/100))} \equiv H_1(s)H_2(s)\,. \tag{4.36}$$

   For $H_2(s)$, $\Delta$ is negative, resulting in complex conjugate roots, with $\omega_0 = 10$ rad/s and $\zeta = 0.25$. (see Eq. 4.29).

   Next, we construct the Bode plots for $H_1(s)$ and $H_2(s)$ separately, and then add them up to get the plots for $H(s)$, as shown in Fig. 4.13.

   **SEQUEL file:** `test_filter_7.sqproj`

## 4.3 Exercise Set:

Construct the magnitude and phase Bode plots for the following functions, and check your plots against simulation results. Note that the integer parameter `flag_asympt` provided in SEQUEL elements can be set to `1` to get asymptotic (Bode) plots for magnitude and phase. If the flag is set to `0`, the exact magnitude and phase are computed. Compare the Bode plots with the exact results in each case to judge the accuracy of the Bode approximations.

1. $H(s) = \dfrac{100000\,s}{(s+1)(s^2+20\,s+10000)}$.

   **SEQUEL file:** `test_filter_3.sqproj`

2. $H(s) = \dfrac{10\,(s+100)}{(s+1)(s^2+2\,s+100)}$.

   **SEQUEL file:** `test_filter_4.sqproj`

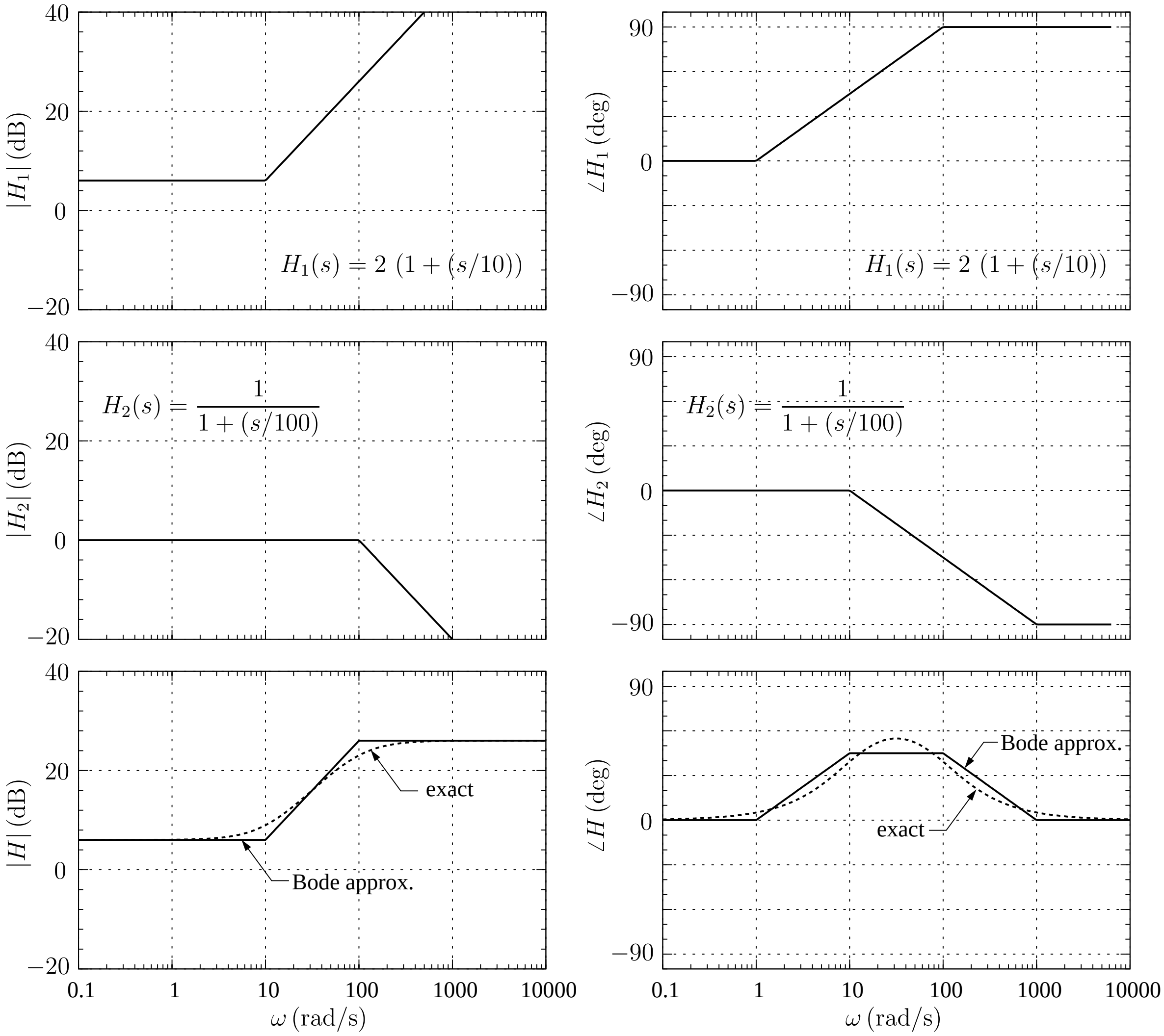


Figure 4.12: Construction of Bode plots for Example 1.

3. $H(s) = \dfrac{10\,(s+100)}{(s+1)(s^2+25\,s+100)}$.

   **SEQUEL file:** `test_filter_4a.sqproj`

4. $H(s) = \dfrac{20\,s\,(s+100)}{(s+2)(s+10)}$.

   **SEQUEL file:** `test_filter_5.sqproj`

5. $H(s) = \dfrac{5 \times 10^8\,s\,(s+100)}{(s+20)(s+1000)^3}$.

   **SEQUEL file:** `test_filter_8.sqproj`

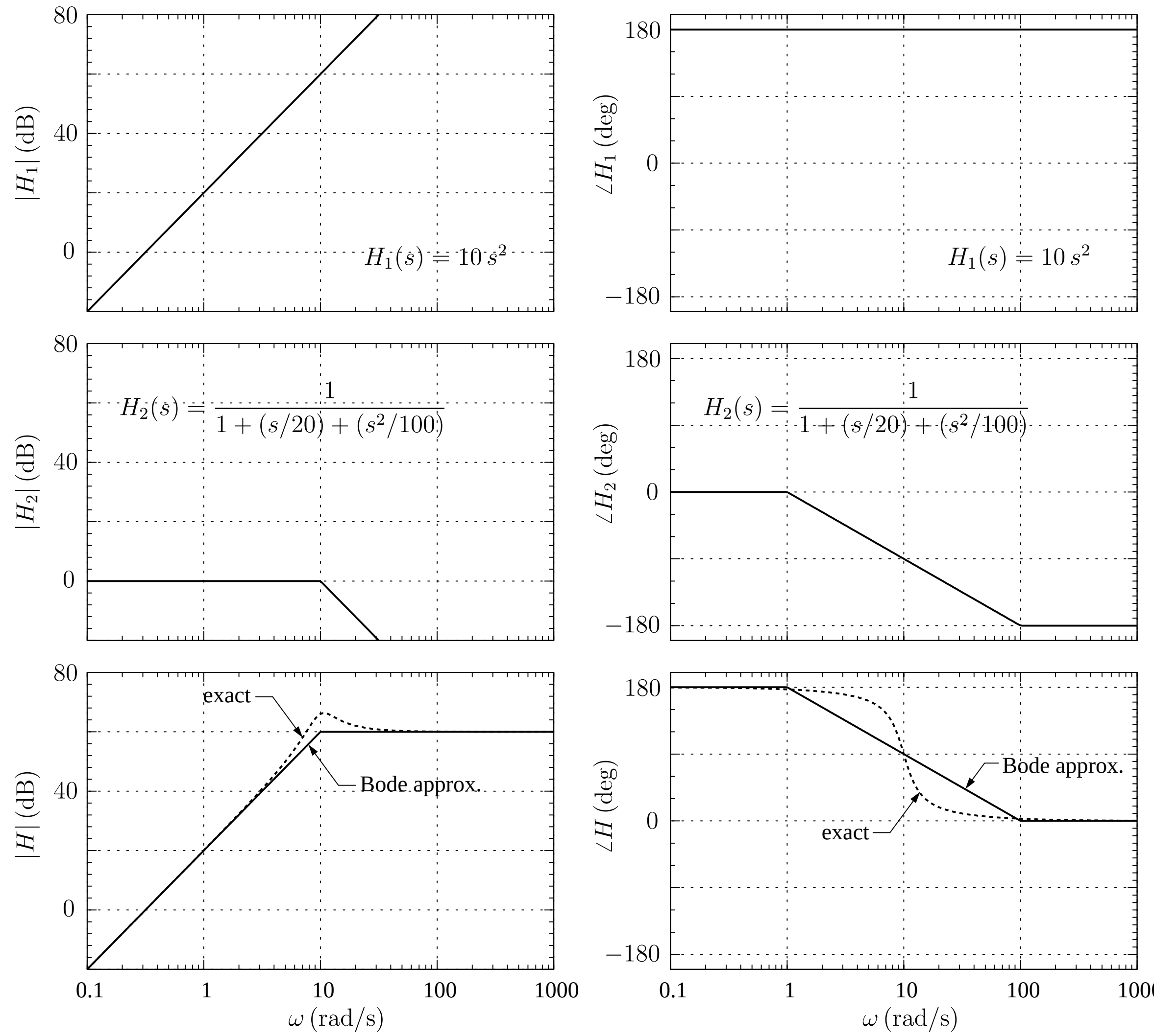


Figure 4.13: Construction of Bode plots for Example 2.