Chapter 1

Some useful techniques

1.1 Background

Thevenin's theorem: Consider a network consisting of dc independent sources (current/voltage sources), dependent sources (linear), and resistors. We are interested in the "port" behaviour of this circuit, i.e., in a simplified description of the circuit as seen from the port AB (see Fig. 1.1 (a)). Thevenin's theorem gives us this simplified description (see Fig. 1.1 (b)) in terms of a single voltage source V_{Th} and a single resistor R_{Th}.

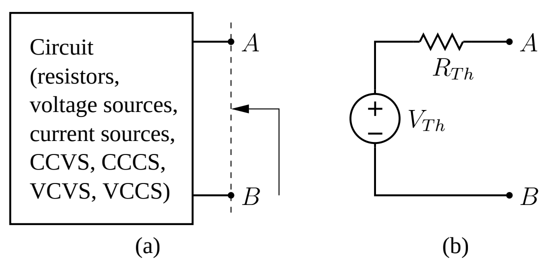

Figure 1.1: (a) An electrical network, (b) Thevenin equivalent circuit as seen from AB.

To determine V_{Th} for a network, we simply find the open-circuit voltage V_{oc} across AB, i.e., the voltage $V_A - V_B$ when nothing is connected on the right side in Fig. 1.1 (a); then, we have $V_{Th} = V_{oc}$.
To find R_{Th}, we can use two methods:

(a) Deactivating the independent sources in the given network is equivalent to deactivating V_{Th} in its Thevenin equivalent circuit (see Fig. 1.2 (a)), leaving only R_{Th}. Therefore, R_{Th} is simply the resistance seen from AB in the original network (Fig. 1.1 (a)) with all independent sources deactivated. Note that deactivating a voltage source amounts to making $V_s = 0\,V$, i.e., replacing it with a short circuit. Similarly, deactivating a current source, i.e., making $I_s = 0\,V$, is equivalent to replacing it with an open circuit. Note also that, in the above procedure, the *dependent* sources are to be left untouched, i.e., they should *not* be deactivated.

Once the independent sources in the network are deactivated, the resistance seen from the port of interest can often be found by inspection. In some cases, we may need to connect a test voltage source V_0, find the current I_0 through it (see Fig. 1.2 (b)), and

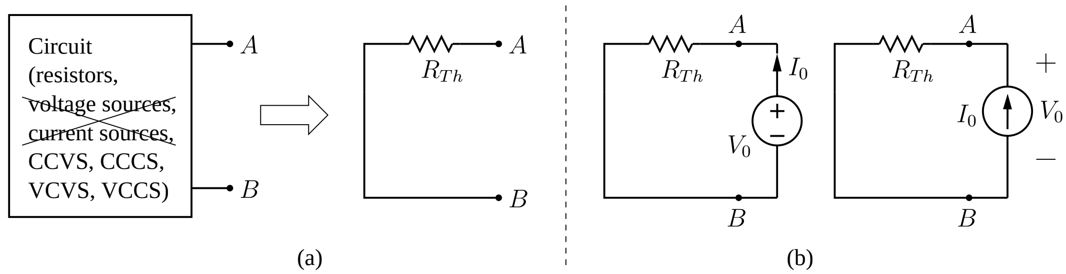

Figure 1.2: (a) Deactivating independent sources and its effect on the Thevenin equivalent circuit, (b) Computation of R_{Th}.

then use $R_{Th} = V_0/I_0$. Alternatively, we could connect a test current source I_0, find the voltage V_0 across it, and use $R_{Th} = V_0/I_0$.

(b) If the port AB is shorted, then the short-circuit current I_{sc} is V_{Th}/R_{Th} (see Figs. 1.3 (a) and (b)). This gives us an alternate method to find R_{Th}: Obtain the open-circuit voltage V_{oc} (which is equal to V_{Th}, as seen earlier) and the short-circuit current I_{sc}. Then, $R_{Th} = V_{Th}/I_{sc}$.

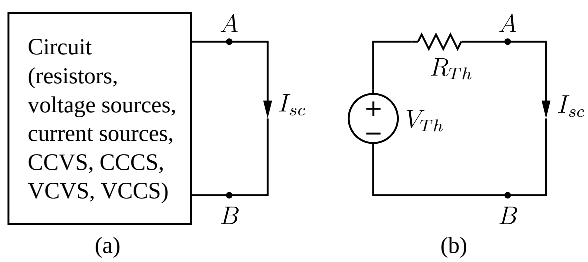

Figure 1.3: Short-circuit current I_{sc} (a) for the original network, (b) for the Thevenin equivalent.

Source transformation: Consider the circuit shown in Fig. 1.4 (a). The Thevenin voltage for this circuit is, $V_{Th} = V_{oc} = I_s\, R_p$. To find R_{Th}, we deactivate the current source (i.e., replace it with an open circuit) and view the circuit from AB to get $R_{Th} = R_p$. In other words, the circuit in Fig. 1.4 (a) is equivalent to that in Fig. 1.4 (b), which is already in the Thevenin form, if $R_s = R_p$ and $V_s = I_s\, R_p$.

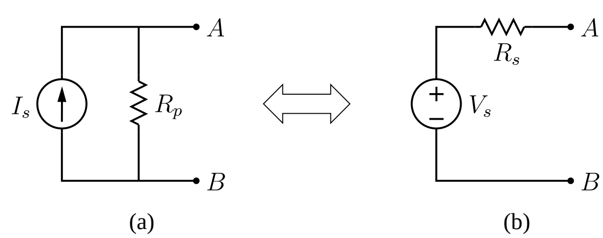

Figure 1.4: Illustration of source transformation.

The "source transformation" described above can be used to convert a Thevenin equivalent circuit into a "Norton equivalent circuit," which is of the form shown in Fig. 1.4 (a).

Superposition: The system of equations that applies to a circuit with resistors, dc independent sources, and dependent sources is a linear system, and therefore we can use the principle of superposition to analyse the circuit by performing the following steps.

(a) Denote the *independent* sources in the circuit by S_1, S_2, $\cdots$, S_N.

(b) For each source S_i, compute the variables of interest (currents or voltages), say y_1, y_2, $\cdots$, y_n, with all other *independent* sources deactivated. Denote the values of the variables by, $y_1^{(i)}$, $y_2^{(i)}$, $\cdots$, $y_n^{(i)}$.

(c) The net values of y_1, y_2, $\cdots$, y_n are then given by $y_j^{\text{net}} = \sum_{i=1}^{N} y_j^{(i)}$.

Maximum power transfer: Consider the power absorbed by R_L in the circuit shown in Fig. 1.5 (a) where V_{Th}, R_{Th} represent the Thevenin equivalent of a general network containing dc voltage or current sources, dependent sources, and resistors. The power absorbed by R_L is

$$P_L = i_L^2\, R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L\,. \tag{1.1}$$

By differentiating P_L with respect to R_L and equating the derivative to zero, it can be shown that P_L is maximum when $R_L = R_{Th}$ (see Fig. 1.5 (b)).

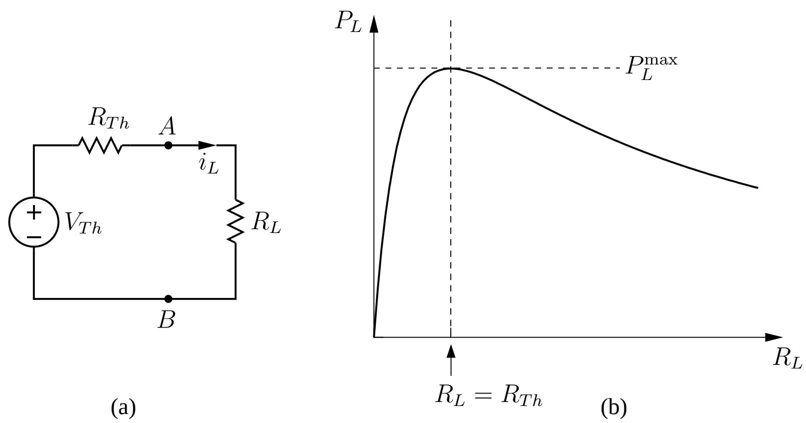

Figure 1.5: Illustration of maximum power transfer.

1.2 Examples

1. For the circuit shown in Fig. 1.6 (a), find the currents i_1 and i_2 using source transformation.

   Converting the circuit with I_s and R_2 into an equivalent form (voltage source in series with a resistance), we get the circuit shown in Fig. 1.6 (b). The computation of i_1 now becomes trivial, viz., $i_1 = \dfrac{5\,V - 6\,V}{7\,\Omega} = -0.143\,A$. Coming back to the original circuit

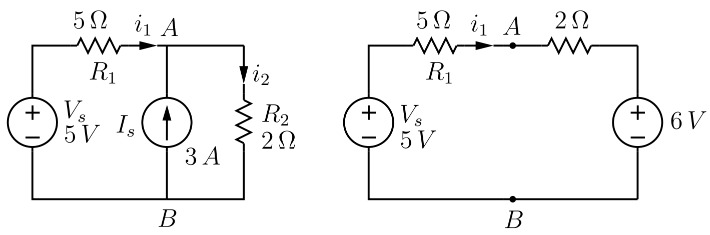

Figure 1.6: (a) Circuit for Example 1, (b) Equivalent circuit after source transformation.

(Fig. 1.6 (a)), we get $i_2 = i_1 + I_s = 2.857\,A$. Note that the source I_s is delivering power while V_s is absorbing power. The reader is encouraged to verify power balance (i.e., the total power absorbed is equal to the total power delivered).

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2. For the circuit shown in Fig. 1.6 (a), find the current i_2 by superposition.

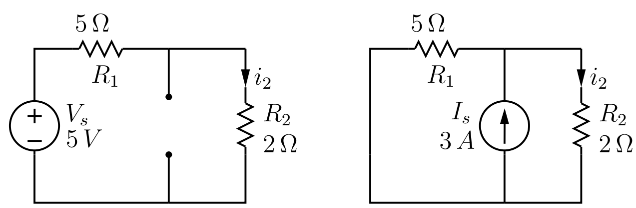

Figure 1.7: Circuit of Fig. 1.6 (a): (a) with V_s active and I_s deactivated, (b) with I_s active and V_s deactivated.

Since there are two independent sources (V_s and I_s) in the circuit, we consider two cases:

(a) V_s active, I_s deactivated: In this case, $i_2^{(1)} = \dfrac{V_s}{R_1 + R_2} = 0.714\,A$ (see Fig. 1.7 (a)).

(b) I_s active, V_s deactivated: In this case, by current division, we get,
$i_2^{(2)} = I_s \times \dfrac{R_1}{R_1 + R_2} = 2.143\,A$ (see Fig. 1.7 (b)).

Adding the individual contributions, we get, $i_2 = i_2^{(1)} + i_2^{(2)} = 2.857\,A$.

3. For the circuit shown in Fig. 1.8, $R_1 = 4\,\Omega$, $R_2 = 2.5\,\Omega$, $R_3 = 1\,\Omega$, $R_4 = 4\,\Omega$, $V_{s1} = 5\,V$, $V_{s2} = 2\,V$, $I_s = 5\,A$. Find the current i_{R3} using superposition. How will i_{R3} change if the source values are changed to $V_{s1} = 30\,V$, $V_{s2} = 20\,V$, $I_s = 15\,A$.

We have the following three cases to consider, corresponding to the three independent sources, V_{s1}, V_{s2}, I_s.

(a) Only V_{s1} active (Fig. 1.9 (a)): $i_{R3}^{(1)} = -\dfrac{V_{s1}}{R_1 + R_2 + R_3} = -0.67\,A$.

(b) Only V_{s2} active (Fig. 1.9 (b)): $i_{R3}^{(2)} = 0\,A$.

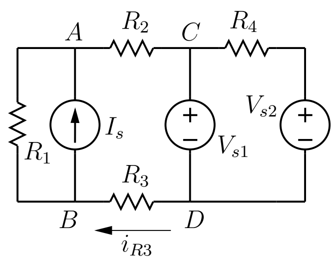

Figure 1.8: Circuit for Example 3.

(c) Only I_s active (Fig. 1.9 (c)): $i_{R3}^{(3)} = I_s \times \dfrac{R_1}{R_1 + R_2 + R_3} = 2.67\,A$ (by current division between two resistors in parallel).

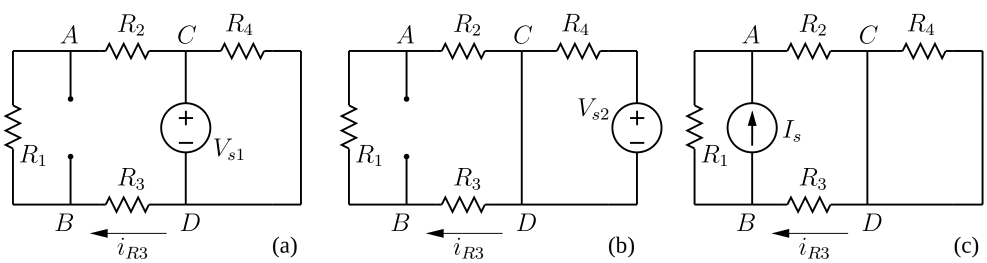

Figure 1.9: Circuit of Fig. 1.8: (a) only V_{s1} active, (b) only V_{s2} active, (c) only I_s active.

The net value of i_{R3} is therefore $i_{R3} = i_{R3}^{(1)} + i_{R3}^{(2)} + i_{R3}^{(3)} = 2\,A$.

When the source values are changed to $V_{s1} = 30\,V$, $V_{s2} = 20\,V$, $I_s = 15\,A$, we can find the new i_{R3} by using linearity of the circuit. Since V_{s1}, V_{s2}, I_s have been scaled (with respect to their earlier values) by factors of 6, 10, 3, respectively, we have
$i_{R3} = 6\,i_{R3}^{(1)} + 10\,i_{R3}^{(2)} + 3\,i_{R3}^{(3)} = 4\,A$.

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4. In the R-$2R$ ladder network shown in Fig. 1.10 (a), find V_o using superposition.

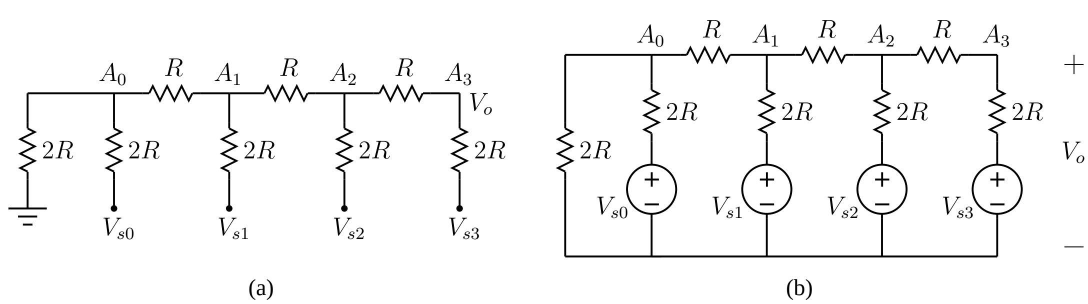

Figure 1.10: (a) Circuit for Example 4, (b) Circuit redrawn with sources shown explicitly.

We take up each of the voltage sources separately and then add their contributions:

(a) V_{s3} only (Fig. 1.11 (a)): Using Thevenin's theorem, the circuit can be simplified to that shown in Fig. 1.11 (b), and we get $V_o^{(1)} = V_{s3}/2$.

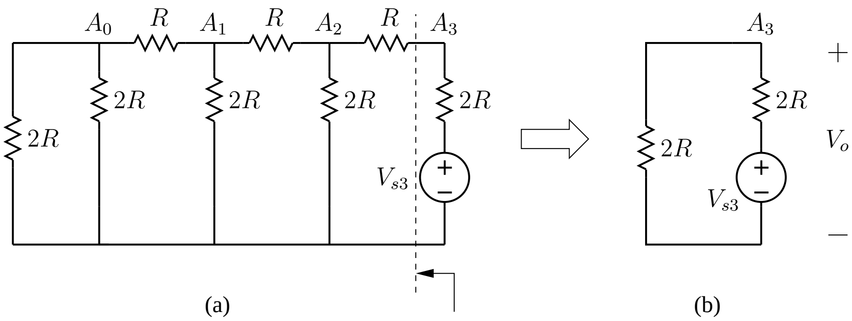

Figure 1.11: Circuit of Fig. 1.10 (a) with only V_{s3} active.

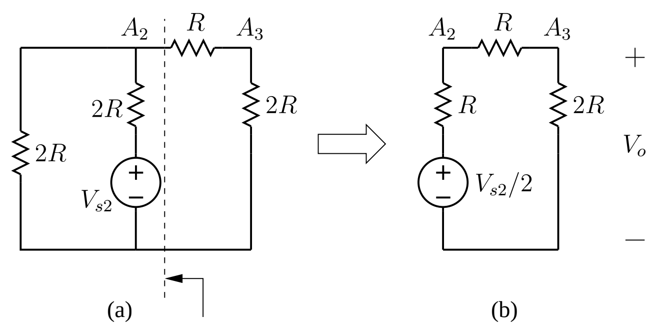

Figure 1.12: Circuit of Fig. 1.10 (a) with only V_{s2} active.

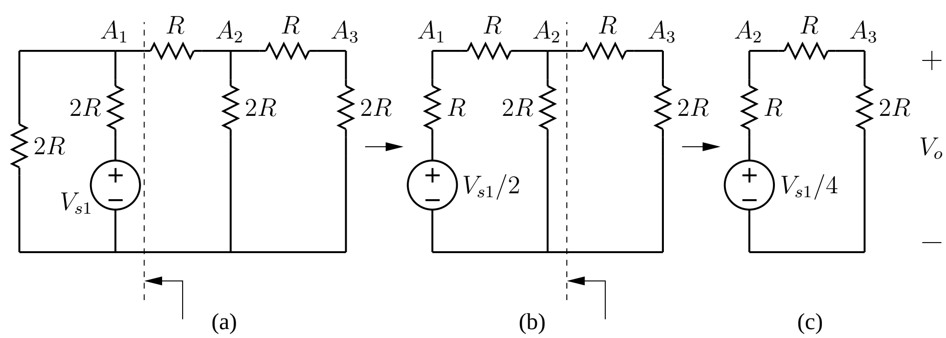

Figure 1.13: Circuit of Fig. 1.10 (a) with only V_{s1} active.

(b) V_{s2} only: Using Thevenin's theorem, this case can be simplified to the circuit shown in Fig. 1.12 (a) and can be further simplified to that shown in Fig. 1.12 (b), giving $V_o^{(2)} = V_{s2}/4$.

(c) V_{s1} only: Using Thevenin's theorem, this case can be progressively simplified to the circuits shown in Figs. 1.13 (a), (b), (c), giving $V_o^{(3)} = V_{s1}/8$.

(d) V_{s0} only: Using the above procedure, the reader can show that $V_o^{(4)} = V_{s0}/16$.

The net value of V_o is therefore

$$V_o = \frac{1}{16}\left(2^0\, V_{s0} + 2^1\, V_{s1} + 2^2\, V_{s2} + 2^3\, V_{s3}\right)\,.$$

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5. For the circuit shown in Fig. 1.14 $R_1 = 4\,\Omega$, $R_2 = 2\,\Omega$, $R_3 = 4\,\Omega$, $I_s = 3\,A$, $V_{s1} = 4\,V$. Find the current i_{R2} by superposition.

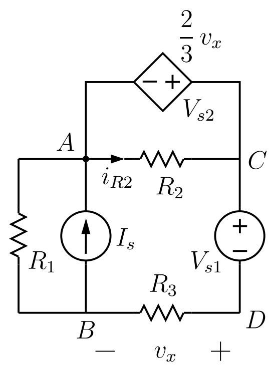

Figure 1.14: Circuit for Example 5.

There are two independent sources in the circuit, and we will consider each of them separately. Note that the dependent source must be retained in each circuit and *not* deactivated.

(a) I_s only (Fig. 1.15 (a)): This case is simplified considerably with source transformation (see Fig. 1.15 (b)). Writing KVL for the simplified circuit, we obtain,

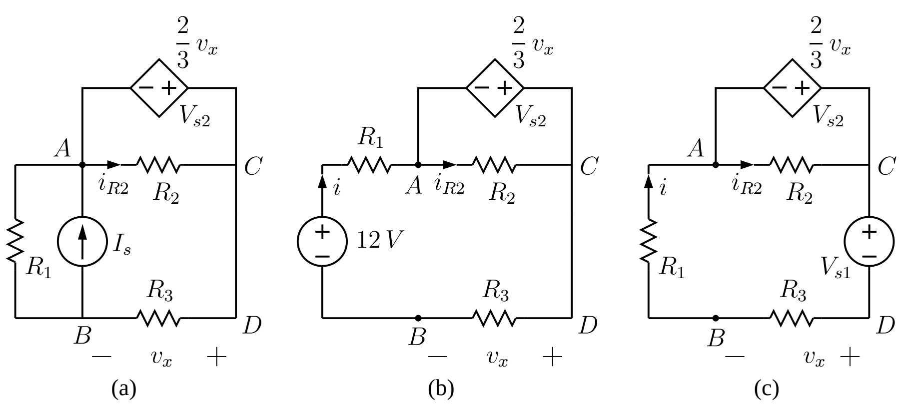

Figure 1.15: (a) Circuit of Fig. 1.14 with only I_s active, (b) Circuit of (a) after source transformation, (c) Circuit of Fig. 1.14 with only V_{s1} active.

$$-12\,V + (R_1 + R_3)\,i - \frac{2}{3}(R_3\,i) = 0\,, \tag{1.2}$$

which gives $i = 2.25\,A$, and $i_{R2}^{(1)} = -3\,A$.

(b) V_{s1} only (Fig. 1.15 (c)): Writing KVL for this circuit, we get

$$V_{s1} + (R_1 + R_3)\, i - \frac{2}{3}(R_3\, i) = 0\,, \tag{1.3}$$

which gives, $i = -0.75\,A$, and $i_{R2}^{(2)} = 1\,A$.

The net value of i_{R2} is therefore $-2\,A$.

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6. Simplify the circuit shown in Fig. 1.16 (a) using Thevenin's theorem and find the current i_L.

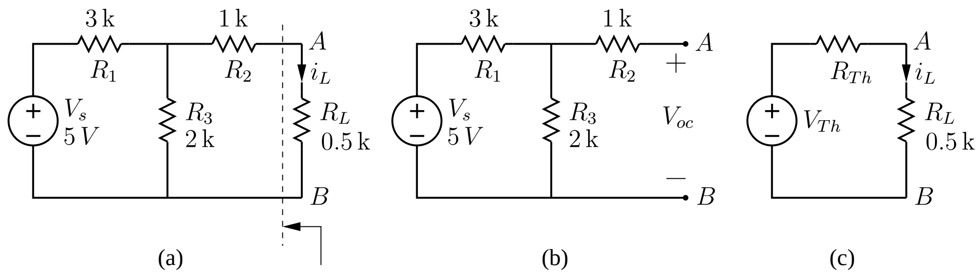

Figure 1.16: (a) Circuit for Example 6, (b) Computation of V_{oc}, (c) Simplified circuit diagram.

To obtain the Thevenin equivalent circuit as seen from AB (see Fig. 1.16 (a)), we deactivate (short) the voltage source and obtain $R_{Th} = (R_1 \parallel R_3) + R_2 = 2.2\,\text{k}$. $V_{Th} = V_{oc}$ is obtained from Fig. 1.16 (b) as $V_{oc} = \dfrac{R_3}{R_1 + R_3}\, V_s = 2\,V$. The simplified circuit is shown in Fig. 1.16 (c), giving $i_L = \dfrac{V_{Th}}{R_{Th} + R_L} = 0.74\,\text{m}A$.

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7. For the circuit shown in Fig. 1.17 (a), find the Thevenin's equivalent circuit as seen from AB.

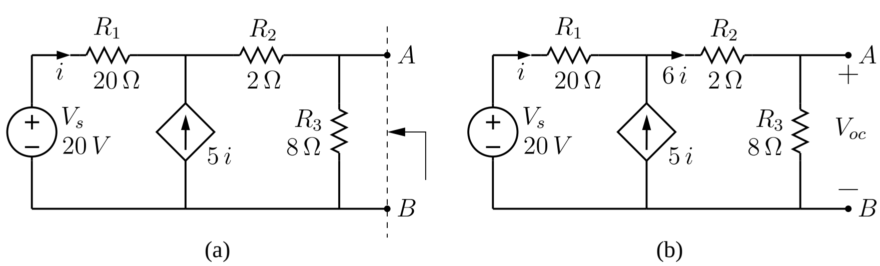

Figure 1.17: (a) Circuit for Example 7, (b) Computation of $V_{Th} = V_{oc}$.

The open-circuit voltage V_{oc} is found from Fig. 1.17 (b), by writing KVL:

$$V_s = R_1\, i + 6\, i\, (R_2 + R_3)\,, \tag{1.4}$$

giving $i = 0.25\,A$ and $V_{Th} = V_{oc} = 6\,i\,R_3 = 12\,V$.

R_{Th} can be obtained in two ways:

(a) We find the short-circuit current I_{sc} as shown in Fig. 1.18 (a). KVL gives

$$V_s = R_1\,i + 6\,i\,R_2 \rightarrow i = \frac{5}{8}\,A\,, \tag{1.5}$$

leading to $I_{sc} = 6\,i = 3.75\,A$, and $R_{Th} = V_{oc}/I_{sc} = 3.2\,\Omega$.

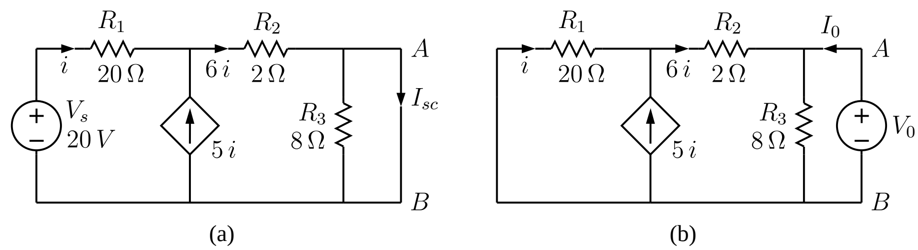

Figure 1.18: Computation of R_{Th} for the circuit of Fig. 1.17: (a) by computing the short-circuit current, (b) by using a test source.

(b) We deactivate the independent source (see Fig. 1.18 (b)) and use a test source V_0 to obtain R_{Th}. KVL gives

$$V_0 = (6\,i + I_0)\,R_3 = -(i\,R_1 + 6\,i\,R_2)\,. \tag{1.6}$$

Eliminating i from Eq. 1.6 gives $R_{Th} = V_0/I_0 = 3.2\,\Omega$.

How do we check our results with circuit simulation? Consider the circuit in Fig. 1.19 in which the Thevenin equivalent of a network has been represented by V_{Th} and R_{Th}. KVL gives

$$V_s = V_{Th} - I_s\,R_{Th}\,. \tag{1.7}$$

If V_s is plotted against I_s, the y-intercept gives V_{Th}, and the x-intercept gives $Isc = V_{Th}/R_{Th}$. These values can be checked against our computed results for the network.

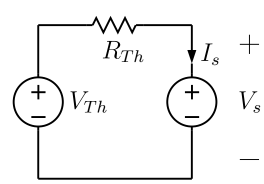

Figure 1.19: A general Thevenin equivalent circuit with a test source.

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8. Find the Thevenin equivalent of the network shown in Fig. 1.20 (a), as seen from AB.

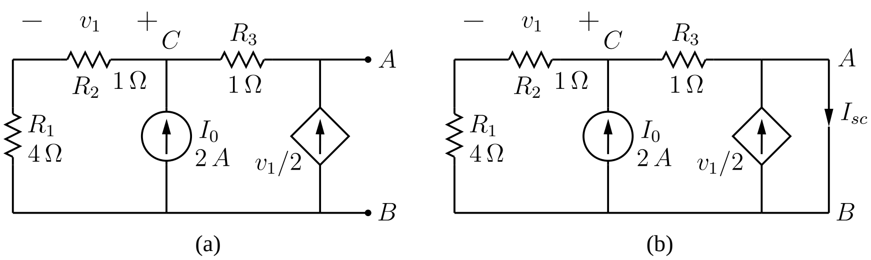

Figure 1.20: (a) Circuit for Example 8, (b) Computation of I_{sc}.

(a) Computation of V_{oc}: Taking node B as the reference node (i.e., V_B as $0\,V$), KCL at node C gives,

$$\frac{V_C}{R_1+R_2} - I_0 - \frac{1}{2}\frac{R_2}{R_1+R_2}V_C = 0\,, \tag{1.8}$$

which gives $V_C = 20\,V$. V_{oc} is then given by $V_{oc} = V_A = V_C + \frac{v_1}{2}R_3 = 22\,V$.

(b) Computation of I_{sc}: Again, taking $V_B = 0\,V$, KCL at node C gives,

$$\frac{V_C}{R_1+R_2} - I_0 + \frac{V_C}{R_3} = 0\,, \tag{1.9}$$

giving $V_C = 1.67\,V$. The current I_{sc} is then given by KCL at node A:

$$I_{sc} = \frac{V_C}{R_3} + \frac{v_1}{2} = 1.83\,A\,. \tag{1.10}$$

R_{Th} is therefore equal to $V_{oc}/I_{sc} = 12\,\Omega$.

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9. For the circuit shown in Fig. 1.21 (a), $R_1 = 20\,\Omega$, $R_2 = 40\,\Omega$, $R_L = 4\,\Omega$, $V_s = 50\,V$. Find i_L using Thevenin's theorem.

(a) Computation of V_{oc}: Writing KVL for the circuit in Fig. 1.21 (b), we get

$$-\frac{R_2\,i_1}{4} + (R_1+R_2)\,i_1 + V_s = 0\,, \tag{1.11}$$

giving $i_1 = -1\,A$, and $V_{oc} = R_2\,i_1 + V_s = 10\,V$.

(b) Computation of I_{sc}: The short-circuit current can be found from Fig. 1.21 (c). In this case, we have $v_x = -V_s$, $i_1 = \frac{v_x}{4}\frac{1}{R_1} = -\frac{5}{8}\,A$, $i_2 = -\frac{V_s}{R_2} = -\frac{5}{4}\,A$, and $I_s = i_1 - i_2 = \frac{5}{8}\,A$.

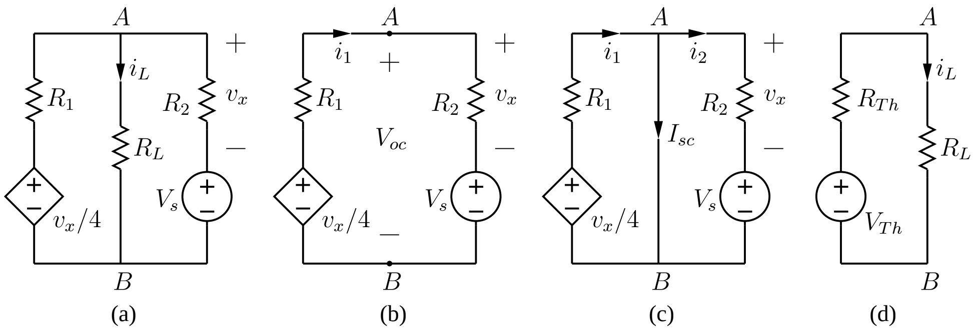

Figure 1.21: (a) Circuit for Example 9, (b) Computation of V_{oc}, (c) Computation of I_{sc}, (d) Computation of i_L.

From the above values, we get $R_{Th} = V_{oc}/I_{sc} = 16\,\Omega$. The original circuit in Fig. 1.21 (a) is thus equivalent to that shown in Fig. 1.21 (d), giving $i_L = \dfrac{V_{Th}}{R_{Th} + R_L} = 0.5\,A$.

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1.3 Exercise Set:

1. In the circuit shown in Fig. 1.22 (a), find the current i by superposition. Verify your result with simulation.

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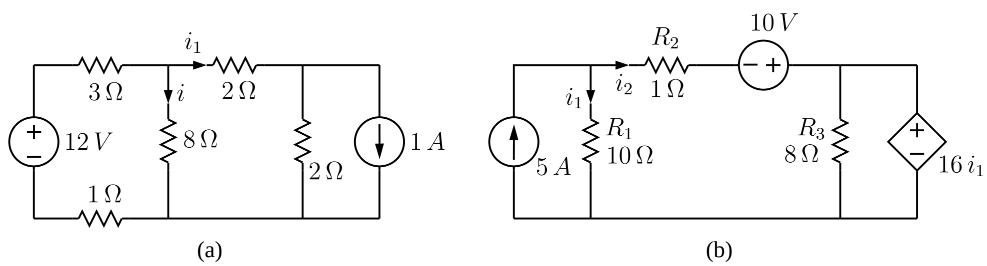

Figure 1.22: (a) Circuit for Exercises 1 and 2, (b) Circuit for Exercise 3.

2. In the circuit shown in Fig. 1.22 (a), find the current i_1 by source transformation.

3. In the circuit shown in Fig. 1.22 (b), find the currents i_1 and i_2 by superposition. Verify your results with simulation.

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4. For the circuit shown in Fig. 1.23 (a),

   (a) Find the value of R_L for maximum power transfer.

   (b) Calculate P_L^{max}, the maximum power absorbed by R_L.

   (c) Obtain a plot P_L versus R_L by simulation, and verify the answers you obtained for (a) and (b).

(d) How will the above plot change if the voltage source is changed from $18\,V$ to $12\,V$?

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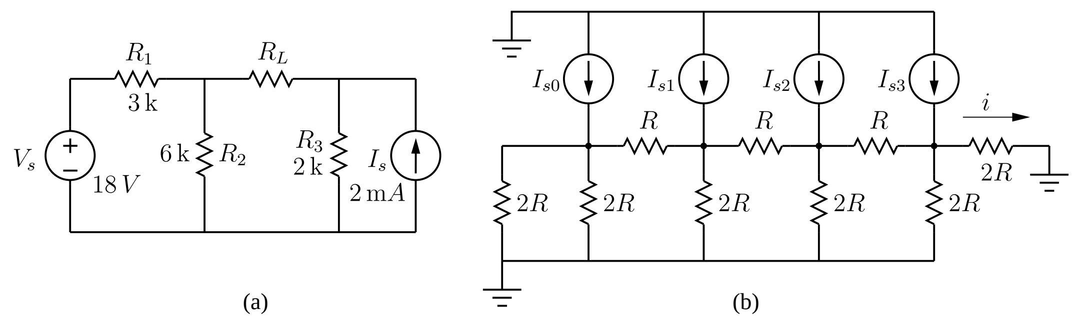

Figure 1.23: (a) Circuit for Exercises 4. (b) Circuit for Exercise 5.

5. Using superposition, obtain a general expression for the current i (see Fig. 1.23 (b)) in terms of the source currents I_{s0}, I_{s1}, I_{s2}, I_{s3}. For each of the following combinations, compute i using your expression, and verify with simulation results.

   (a) $I_{s1} = I_{s0} = 3\,\text{m}A$, others are zero.

   (b) $I_{s3} = I_{s1} = 3\,\text{m}A$, others are zero.

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