Digital Circuits: Part 1

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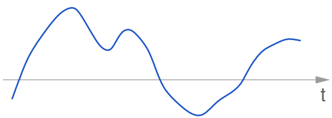
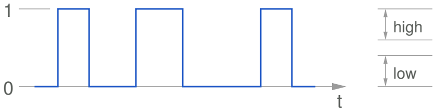
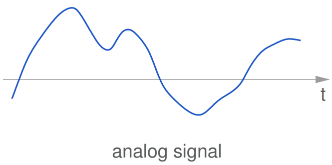
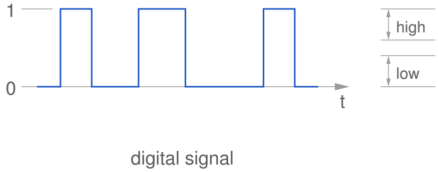


analog signal

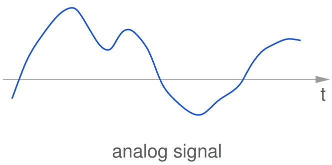


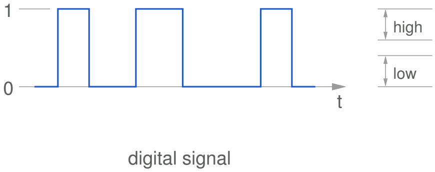
digital signal





* An analog signal $x(t)$ is represented by a real number at a given time point.





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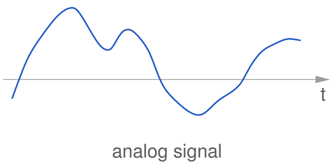
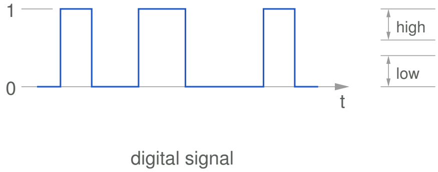
* A digital signal is "binary" in nature, i.e., it takes on only two values: low (0) or high (1).





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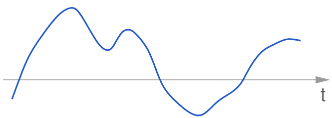
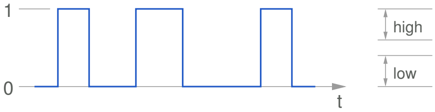
* Although we have shown 0 and 1 as constant levels, in reality, that is not required. Any value in the low (high) band will be interpreted as 0 (1) by digital circuits.

Digital circuits





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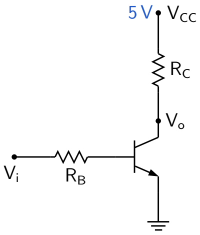
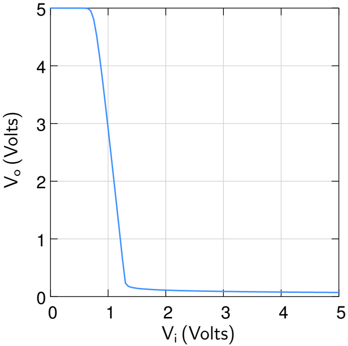
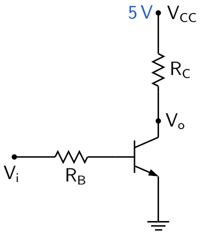
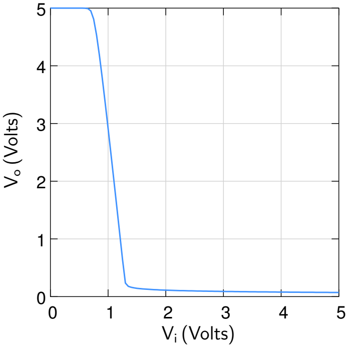
- A digital signal is "binary" in nature, i.e., it takes on only two values: low (0) or high (1).
- Although we have shown 0 and 1 as constant levels, in reality, that is not required. Any value in the low (high) band will be interpreted as 0 (1) by digital circuits.
- The definition of low and high bands depends on the technology used, e.g.,
  TTL (Transistor-Transistor Logic)
  CMOS (Complementary MOS)
  ECL (Emitter-Coupled Logic)


5 V
V_{CC}
R_C
V_o
V_i
R_B


V_o (Volts)
V_i (Volts)
0
1
2
3
4
5

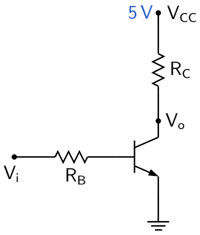
A simple digital circuit

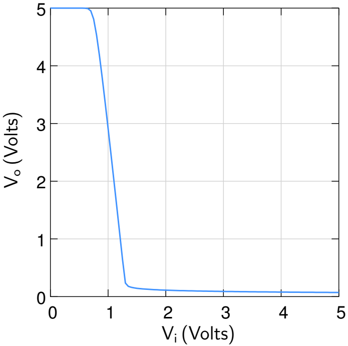




* If V_i is low ("0"), V_o is high ("1").
If V_i is high ("1"), V_o is low ("0").

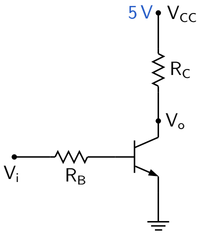
A simple digital circuit

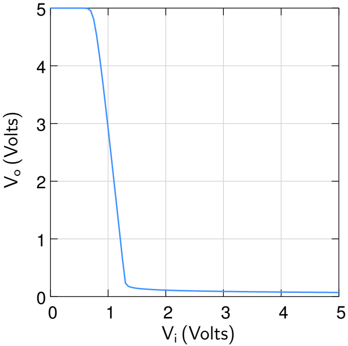




* If V_i is low ("0"), V_o is high ("1").
  If V_i is high ("1"), V_o is low ("0").
* The circuit is called an "inverter" because it inverts the logic level of the input. If the input is 0, it makes the output 1, and vice versa.

A simple digital circuit





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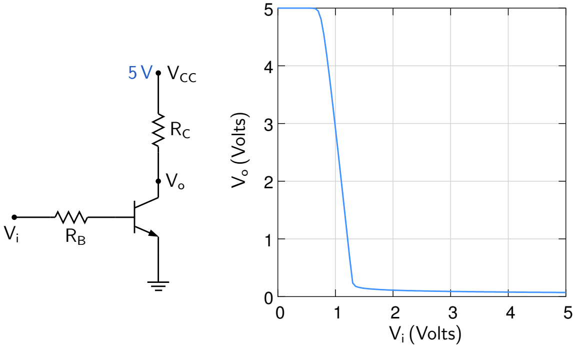
* Digital circuits are made using a variety of devices. The simple BJT inverter is only an illustration.

A simple digital circuit



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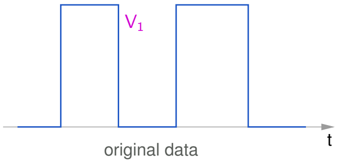
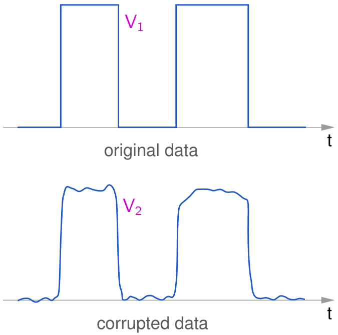
* Digital circuits are made using a variety of devices. The simple BJT inverter is only an illustration.
* Most of the VLSI circuits today employ the MOS technology because of the high packing density, high speed, and low power consumption it offers.

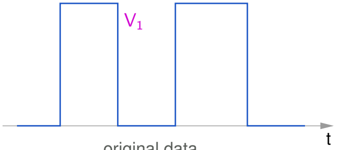
Digital circuits



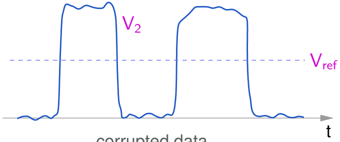
Digital circuits



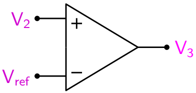
Digital circuits



original data

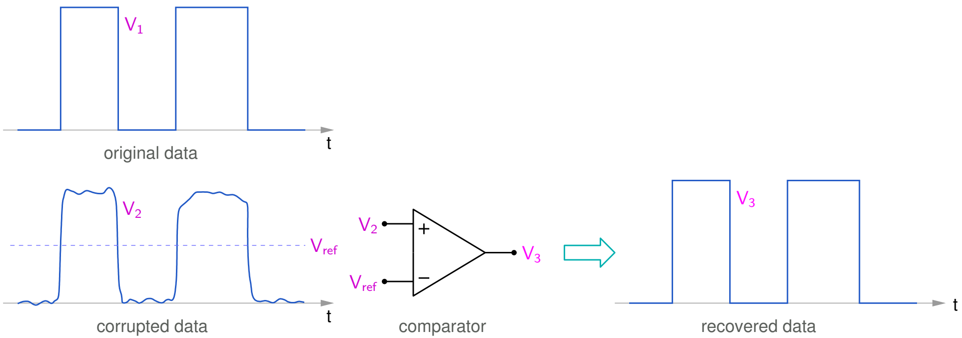


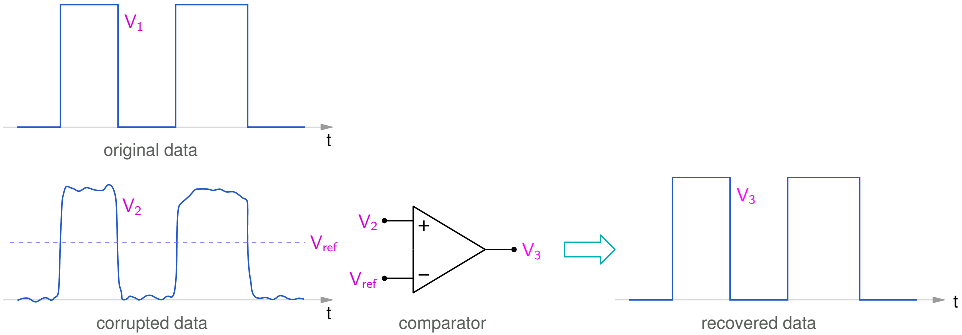
corrupted data



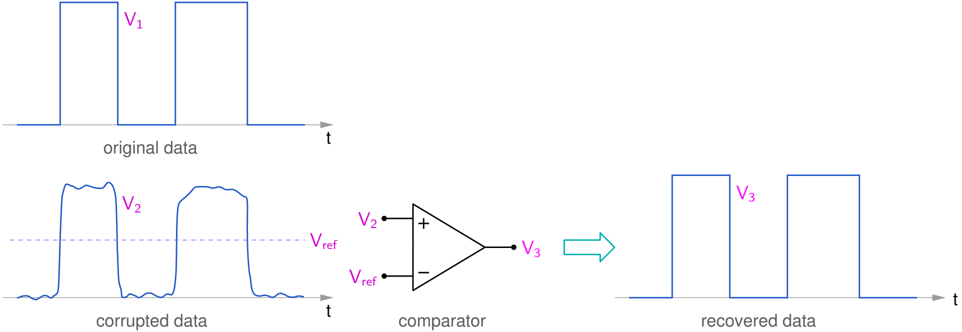
comparator

Digital circuits





* A major advantage of digital systems is that, even if the original data gets distorted (e.g., in transmitting through optical fibre or storing on a CD) due to noise, attenuation, etc., it can be retrieved easily.



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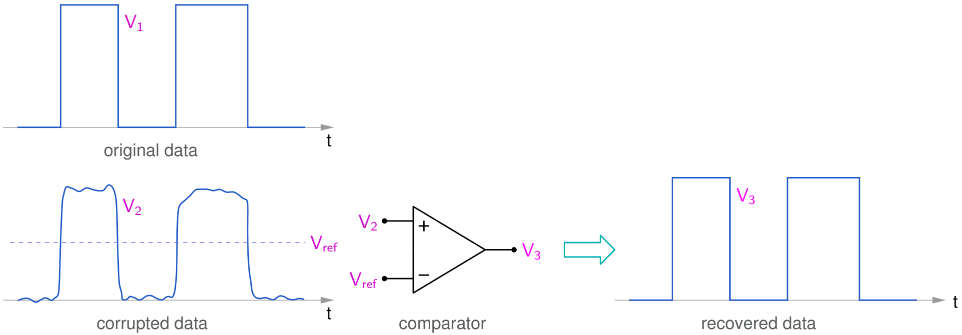
* There are several other benefits of using digital representation:



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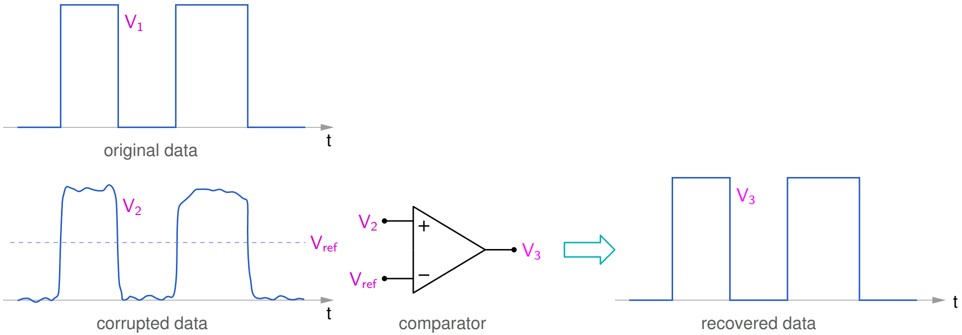
  - can use computers to process the data.



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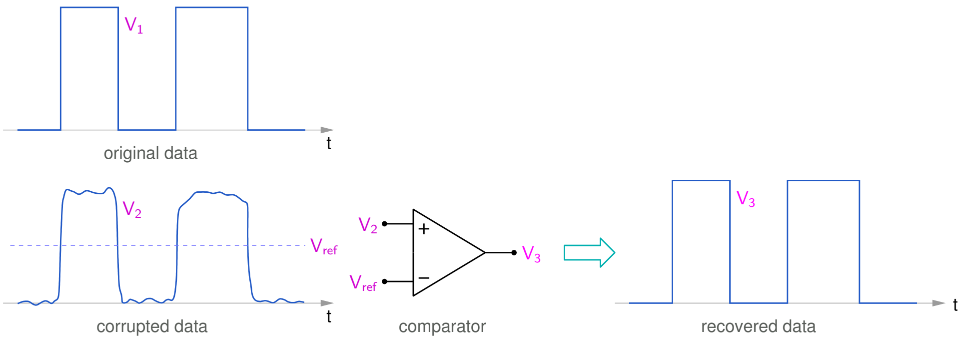
  - can store in a variety of storage media.

Digital circuits



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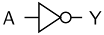
* There are several other benefits of using digital representation:
  - can use computers to process the data.
  - can store in a variety of storage media.
  - can *program* the functionality. For example, the behaviour of a digital filter can be changed simply by changing its coefficients.

Logical operations

Operation	NOT	AND	OR
Gate			
Truth table			
Notation			

Operation NOT AND OR

Gate



Truth table

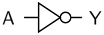
A	Y
0	1
1	0

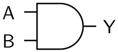
Notation $Y = \overline{A}$

Logical operations

Operation	NOT	AND	OR

Gate

NOT: 

AND: 

Truth table

NOT:

A	Y
0	1
1	0

AND:

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Notation

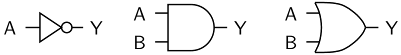
NOT: $Y = \overline{A}$

AND:

$$\begin{aligned} Y &= A \cdot B \\ &= A\,B \end{aligned}$$

Operation	NOT	AND	OR

Gate



Truth table

NOT:

A	Y
0	1
1	0

AND:

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Notation

NOT: $Y = \overline{A}$

AND: $Y = A \cdot B = A\,B$

OR: $Y = A + B$

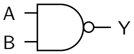
Logical operations

Operation	NAND	NOR	XOR
Gate			
Truth table			
Notation			

Logical operations

Operation	NAND	NOR	XOR

Gate



Truth table

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Notation

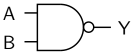
$$\begin{aligned} Y &= \overline{A \cdot B} \\ &= \overline{A\,B} \end{aligned}$$

Logical operations

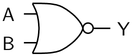
Operation	NAND	NOR	XOR

Gate

NAND:



NOR:



Truth table

NAND:

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR:

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Notation

NAND:

$$\mathrm{Y} = \overline{\mathrm{A} \cdot \mathrm{B}} = \overline{\mathrm{A}\,\mathrm{B}}$$

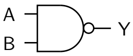
NOR:

$$\mathrm{Y} = \overline{\mathrm{A} + \mathrm{B}}$$

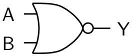
Operation: NAND | NOR | XOR

Gate

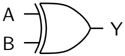
NAND:



NOR:



XOR:



Truth table

NAND:

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR:

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XOR:

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Notation

NAND: $Y = \overline{A \cdot B} = \overline{A\,B}$

NOR: $Y = \overline{A + B}$

XOR: $Y = A \oplus B = A\,\overline{B} + \overline{A}\,B$

* The AND operation is *commutative*.
  $\rightarrow A \cdot B = B \cdot A$.

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* The AND operation is *associative*.
  $\rightarrow (A \cdot B) \cdot C = A \cdot (B \cdot C)$.

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  $\rightarrow A + B = B + A$.

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* The OR operation is *commutative*.
  $\rightarrow A + B = B + A$.
* The OR operation is *associative*.
  $\rightarrow (A + B) + C = A + (B + C)$.

* Theorem: $\overline{\overline{A}} = A$.

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  The theorem can be proved by constructing a truth table:

A	$\overline{A}$	$\overline{\overline{A}}$
0	1	0
1	0	1

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  Therefore, for all possible values that A can take (i.e., 0 and 1), $\overline{\overline{A}}$ is the same as A.

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* Similarly, the following theorems can be proved:

$$
\begin{array}{ll}
A + 0 = A & A \cdot 1 = A \\
A + 1 = 1 & A \cdot 0 = 0 \\
A + A = A & A \cdot A = A \\
A + \overline{A} = 1 & A \cdot \overline{A} = 0
\end{array}
$$

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  The theorem can be proved by constructing a truth table:

A	$\overline{A}$	$\overline{\overline{A}}$
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1	0	1

  Therefore, for all possible values that A can take (i.e., 0 and 1), $\overline{\overline{A}}$ is the same as A.

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* Similarly, the following theorems can be proved:

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  $A + 1 = 1 \qquad A \cdot 0 = 0$

  $A + A = A \qquad A \cdot A = A$

  $A + \overline{A} = 1 \qquad A \cdot \overline{A} = 0$

  Note the duality: $(+ \longleftrightarrow \cdot)$ and $(1 \longleftrightarrow 0)$.

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0								
0	1								
1	0								
1	1								

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0							
0	1	1							
1	0	1							
1	1	1							

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1						
0	1	1	0						
1	0	1	0						
1	1	1	0						

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1					
0	1	1	0	1					
1	0	1	0	0					
1	1	1	0	0					

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1				
0	1	1	0	1	0				
1	0	1	0	0	1				
1	1	1	0	0	0				

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1			
0	1	1	0	1	0	0			
1	0	1	0	0	1	0			
1	1	1	0	0	0	0			

De Morgan's theorems

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$	$A \cdot B$	$\overline{A \cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0		
0	1	1	0	1	0	0	0		
1	0	1	0	0	1	0	0		
1	1	1	0	0	0	0	1		

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	
0	1	1	0	1	0	0	0	1	
1	0	1	0	0	1	0	0	1	
1	1	1	0	0	0	0	1	0	

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

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A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

* Comparing the truth tables for $\overline{A+B}$ and $\overline{A}\,\overline{B}$, we conclude that $\overline{A+B} = \overline{A}\,\overline{B}$.
* Similarly, $\overline{A\cdot B} = \overline{A}+\overline{B}$.

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

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* Similar relations hold for more than two variables, e.g.,

De Morgan's theorems

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

* Comparing the truth tables for $\overline{A+B}$ and $\overline{A}\,\overline{B}$, we conclude that $\overline{A+B}=\overline{A}\,\overline{B}$.
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  $\overline{A\cdot B\cdot C}=\overline{A}+\overline{B}+\overline{C}$,

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

* Comparing the truth tables for $\overline{A+B}$ and $\overline{A}\,\overline{B}$, we conclude that $\overline{A+B}=\overline{A}\,\overline{B}$.
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  $\overline{A\cdot B\cdot C}=\overline{A}+\overline{B}+\overline{C}$,

  $\overline{A+B+C+D}=\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$,

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$A\cdot B$	$\overline{A\cdot B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	1	0	0	1	0	0	1	1
1	1	1	0	0	0	0	1	0	0

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  $\overline{A+B+C+D}=\overline{A}\cdot\overline{B}\cdot\overline{C}\cdot\overline{D}$,

  $\overline{(A+B)\cdot C}=\overline{(A+B)}+\overline{C}=\overline{A}\cdot\overline{B}+\overline{C}$.

1. $A \cdot (B + C) = A\,B + A\,C.$

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A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

Distributive laws

1. $A \cdot (B + C) = A\,B + A\,C$.

A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0				
0	0	1	1				
0	1	0	1				
0	1	1	1				
1	0	0	0				
1	0	1	1				
1	1	0	1				
1	1	1	1				

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A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0	0			
0	0	1	1	0			
0	1	0	1	0			
0	1	1	1	0			
1	0	0	0	0			
1	0	1	1	1			
1	1	0	1	1			
1	1	1	1	1			

1. $A \cdot (B + C) = A\,B + A\,C$.

A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0	0	0		
0	0	1	1	0	0		
0	1	0	1	0	0		
0	1	1	1	0	0		
1	0	0	0	0	0		
1	0	1	1	1	0		
1	1	0	1	1	1		
1	1	1	1	1	1		

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A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0	0	0	0	
0	0	1	1	0	0	0	
0	1	0	1	0	0	0	
0	1	1	1	0	0	0	
1	0	0	0	0	0	0	
1	0	1	1	1	0	1	
1	1	0	1	1	1	0	
1	1	1	1	1	1	1	

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A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

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A	B	C	$B + C$	$A \cdot (B + C)$	$A\,B$	$A\,C$	$A\,B + A\,C$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

2. $A + B \cdot C = (A + B) \cdot (A + C)$.

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A	B	C	$B\,C$	$A + B\,C$	$A + B$	$A + C$	$(A + B)\,(A + C)$
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

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A	B	C	$B\,C$	$A + B\,C$	$A + B$	$A + C$	$(A + B)\,(A + C)$
0	0	0	0				
0	0	1	0				
0	1	0	0				
0	1	1	1				
1	0	0	0				
1	0	1	0				
1	1	0	0				
1	1	1	1				

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A	B	C	$B\,C$	$A + B\,C$	$A + B$	$A + C$	$(A + B)\,(A + C)$
0	0	0	0	0			
0	0	1	0	0			
0	1	0	0	0			
0	1	1	1	1			
1	0	0	0	1			
1	0	1	0	1			
1	1	0	0	1			
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0	0	0	0	0	0		
0	0	1	0	0	0		
0	1	0	0	0	1		
0	1	1	1	1	1		
1	0	0	0	1	1		
1	0	1	0	1	1		
1	1	0	0	1	1		
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0	0	0	0	0	0	0	
0	0	1	0	0	0	1	
0	1	0	0	0	1	0	
0	1	1	1	1	1	1	
1	0	0	0	1	1	1	
1	0	1	0	1	1	1	
1	1	0	0	1	1	1	
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0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
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0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
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$$
\begin{aligned}
A + A\,B &= A \cdot 1 + A \cdot B \\
&= A \cdot (1 + B) \\
&= A \cdot (1) \\
&= A
\end{aligned}
$$

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  Proof: $$\begin{aligned} A \cdot (A + B) &= A \cdot A + A \cdot B \\ &= A + A\,B \\ &= A \end{aligned}$$

$A + A\,B = A \quad \longleftrightarrow \quad A \cdot (A + B) = A.$

Note the duality between OR and AND.

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Dual of $A + (A\,B)$ (LHS): $A\,B \rightarrow A + B$
$A + A\,B \rightarrow A \cdot (A + B)$.

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Dual of LHS $= A \cdot \overline{A}$.

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Dual of RHS $= 0$.

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* $A + \overline{A}\,B = A + B$.

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Proof: $$\begin{aligned} A + \overline{A}\,B &= (A + \overline{A}) \cdot (A + B) \qquad \text{(by distributive law)} \\ &= 1 \cdot (A + B) \\ &= A + B \end{aligned}$$

Dual theorem: $A \cdot (\overline{A} + B) = A\,B$.

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Useful theorems

* $A + \overline{A}\,B = A + B$.

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  Proof: $$\begin{aligned} A\,B + A\,\overline{B} &= A \cdot (B + \overline{B}) \quad \text{(by distributive law)} \\ &= A \cdot 1 \\ &= A \end{aligned}$$

  Dual theorem: $(A + B) \cdot (A + \overline{B}) = A$.

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Let $T \equiv$ Tendulkar scores a century.
$S \equiv$ Sehwag scores a century.
$W \equiv$ Warne fails.
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Let $T \equiv$ Tendulkar scores a century.
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$$I = T + \overline{T}\,W + \overline{T}\,S$$

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$$
\begin{aligned}
I &= T + \overline{T}\,W + \overline{T}\,S \\
&= T + T + \overline{T}\,W + \overline{T}\,S \\
&= (T + \overline{T}\,W) + (T + \overline{T}\,S)
\end{aligned}
$$

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$$
\begin{aligned}
I &= T + \overline{T}\,W + \overline{T}\,S \\
&= T + T + \overline{T}\,W + \overline{T}\,S \\
&= (T + \overline{T}\,W) + (T + \overline{T}\,S) \\
&= (T + \overline{T}) \cdot (T + W) + (T + \overline{T}) \cdot (T + S)
\end{aligned}
$$

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$$
\begin{aligned}
I &= T + \overline{T}\,W + \overline{T}\,S \\
&= T + T + \overline{T}\,W + \overline{T}\,S \\
&= (T + \overline{T}\,W) + (T + \overline{T}\,S) \\
&= (T + \overline{T}) \cdot (T + W) + (T + \overline{T}) \cdot (T + S) \\
&= T + W + T + S \\
&= T + W + S
\end{aligned}
$$

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$$
\begin{aligned}
I &= T + \overline{T}\,W + \overline{T}\,S \\
&= T + T + \overline{T}\,W + \overline{T}\,S \\
&= (T + \overline{T}\,W) + (T + \overline{T}\,S) \\
&= (T + \overline{T}) \cdot (T + W) + (T + \overline{T}) \cdot (T + S) \\
&= T + W + T + S \\
&= T + W + S
\end{aligned}
$$

i.e., India will win if one or more of the following hold:

(a) Tendulkar strikes, (b) Warne fails, (c) Sehwag strikes.

Consider a function X of three variables A, B, C:

$$X = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

$$\equiv X_1 \quad + X_2 \quad + X_3 \quad + X_4$$

Consider a function X of three variables A, B, C:

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This form is called the "sum of products" form ("sum" corresponding to OR and "product" corresponding to AND).

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(2) Tabulate $X_1 = \overline{A}\,B\,\overline{C}$, etc. Note that X_1 is 1 only if $\overline{A} = B = \overline{C} = 1$ (i.e., $A = 0$, $B = 1$, $C = 0$), and 0 otherwise.

Consider a function X of three variables A, B, C:

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(3) Since $X = X_1 + X_2 + X_3 + X_4$,
X is 1 if any of X_1, X_2, X_3, X_4 is 1; else X is 0.
$\rightarrow$ tabulate X.

"Sum of products" form

$$X = X_1 + X_2 + X_3 + X_4 = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

A	B	C	X_1	X_2	X_3	X_4	X
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

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$$X = X_1 + X_2 + X_3 + X_4 = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

A	B	C	X_1	X_2	X_3	X_4	X
0	0	0					
0	0	1					
0	1	0	1				
0	1	1					
1	0	0					
1	0	1					
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A	B	C	X_1	X_2	X_3	X_4	X
0	0	0	0				
0	0	1	0				
0	1	0	1				
0	1	1	0				
1	0	0	0				
1	0	1	0				
1	1	0	0				
1	1	1	0				

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A	B	C	X_1	X_2	X_3	X_4	X
0	0	0	0				
0	0	1	0				
0	1	0	1				
0	1	1	0	1			
1	0	0	0				
1	0	1	0				
1	1	0	0				
1	1	1	0				

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0	0	0	0	0			
0	0	1	0	0			
0	1	0	1	0			
0	1	1	0	1			
1	0	0	0	0			
1	0	1	0	0			
1	1	0	0	0			
1	1	1	0	0			

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$$X = X_1 + X_2 + X_3 + X_4 = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

A	B	C	X_1	X_2	X_3	X_4	X
0	0	0	0	0			
0	0	1	0	0			
0	1	0	1	0			
0	1	1	0	1			
1	0	0	0	0	1		
1	0	1	0	0			
1	1	0	0	0			
1	1	1	0	0			

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$$X = X_1 + X_2 + X_3 + X_4 = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

A	B	C	X_1	X_2	X_3	X_4	X
0	0	0	0	0	0		
0	0	1	0	0	0		
0	1	0	1	0	0		
0	1	1	0	1	0		
1	0	0	0	0	1		
1	0	1	0	0	0		
1	1	0	0	0	0		
1	1	1	0	0	0		

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0	0	0	0	0	0		
0	0	1	0	0	0		
0	1	0	1	0	0		
0	1	1	0	1	0		
1	0	0	0	0	1		
1	0	1	0	0	0		
1	1	0	0	0	0	1	
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0	0	0	0	0	0	0	
0	0	1	0	0	0	0	
0	1	0	1	0	0	0	
0	1	1	0	1	0	0	
1	0	0	0	0	1	0	
1	0	1	0	0	0	0	
1	1	0	0	0	0	1	
1	1	1	0	0	0	0	

"Sum of products" form

$$X = X_1 + X_2 + X_3 + X_4 = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

A	B	C	X_1	X_2	X_3	X_4	X
0	0	0	0	0	0	0	
0	0	1	0	0	0	0	
0	1	0	1	0	0	0	1
0	1	1	0	1	0	0	1
1	0	0	0	0	1	0	1
1	0	1	0	0	0	0	
1	1	0	0	0	0	1	1
1	1	1	0	0	0	0	

$$X = X_1 + X_2 + X_3 + X_4 = \overline{A}\,B\,\overline{C} + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

A	B	C	X_1	X_2	X_3	X_4	X
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	1	0	0	1
1	0	0	0	0	1	0	1
1	0	1	0	0	0	0	0
1	1	0	0	0	0	1	1
1	1	1	0	0	0	0	0

Consider a function Y of three variables A, B, C:

$$\begin{aligned} Y &= (A+B+C)\cdot(A+B+\overline{C})\cdot(\overline{A}+B+\overline{C})\cdot(\overline{A}+\overline{B}+\overline{C}) \\ &\equiv Y_1 \cdot Y_2 \cdot Y_3 \cdot Y_4 \end{aligned}$$

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This form is called the "product of sums" form ("sum" corresponding to OR, and "product" corresponding to AND).

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Y is 0 if any of Y_1, Y_2, Y_3, Y_4 is 0; else Y is 1.
$\rightarrow$ tabulate Y.

“Product of sums” form

$$Y = Y_1\,Y_2\,Y_3\,Y_4 = (A + B + C)\,(A + B + \overline{C})\,(\overline{A} + B + \overline{C})\,(\overline{A} + \overline{B} + \overline{C})$$

A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					

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A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0				
0	0	1					
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A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0				
0	0	1	1				
0	1	0	1				
0	1	1	1				
1	0	0	1				
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0	0	0	0				
0	0	1	1	0			
0	1	0	1				
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1	0	0	1				
1	0	1	1				
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A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1			
0	0	1	1	0			
0	1	0	1	1			
0	1	1	1	1			
1	0	0	1	1			
1	0	1	1	1			
1	1	0	1	1			
1	1	1	1	1			

"Product of sums" form

$$Y = Y_1 Y_2 Y_3 Y_4 = (A + B + C)(A + B + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$$

A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1			
0	0	1	1	0			
0	1	0	1	1			
0	1	1	1	1			
1	0	0	1	1			
1	0	1	1	1	0		
1	1	0	1	1			
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A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1	1		
0	0	1	1	0	1		
0	1	0	1	1	1		
0	1	1	1	1	1		
1	0	0	1	1	1		
1	0	1	1	1	0		
1	1	0	1	1	1		
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A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1	1		
0	0	1	1	0	1		
0	1	0	1	1	1		
0	1	1	1	1	1		
1	0	0	1	1	1		
1	0	1	1	1	0		
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1	1	1	1	1	1	0	

"Product of sums" form

$$Y = Y_1\,Y_2\,Y_3\,Y_4 = (A + B + C)\,(A + B + \overline{C})\,(\overline{A} + B + \overline{C})\,(\overline{A} + \overline{B} + \overline{C})$$

A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1	1	1	
0	0	1	1	0	1	1	
0	1	0	1	1	1	1	
0	1	1	1	1	1	1	
1	0	0	1	1	1	1	
1	0	1	1	1	0	1	
1	1	0	1	1	1	1	
1	1	1	1	1	1	0	

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$$Y = Y_1\,Y_2\,Y_3\,Y_4 = (A + B + C)\,(A + B + \overline{C})\,(\overline{A} + B + \overline{C})\,(\overline{A} + \overline{B} + \overline{C})$$

A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1	1	1	0
0	0	1	1	0	1	1	0
0	1	0	1	1	1	1	
0	1	1	1	1	1	1	
1	0	0	1	1	1	1	
1	0	1	1	1	0	1	0
1	1	0	1	1	1	1	
1	1	1	1	1	1	0	0

"Product of sums" form

$$Y = Y_1\,Y_2\,Y_3\,Y_4 = (A + B + C)\,(A + B + \overline{C})\,(\overline{A} + B + \overline{C})\,(\overline{A} + \overline{B} + \overline{C})$$

A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1	1	1	0
0	0	1	1	0	1	1	0
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	1	0	1	0
1	1	0	1	1	1	1	1
1	1	1	1	1	1	0	0

"Product of sums" form

$$Y = Y_1\,Y_2\,Y_3\,Y_4 = (A + B + C)\,(A + B + \overline{C})\,(\overline{A} + B + \overline{C})\,(\overline{A} + \overline{B} + \overline{C})$$

A	B	C	Y_1	Y_2	Y_3	Y_4	Y
0	0	0	0	1	1	1	0
0	0	1	1	0	1	1	0
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	1	0	1	0
1	1	0	1	1	1	1	1
1	1	1	1	1	1	0	0

Note that Y is identical to X (seen two slides back). This is an example of how the same function can be written in two seemingly different forms (in this case, the sum-of-products form and the product-of-sums form).

Standard sum-of-products form

Consider a function X of three variables A, B, C:
$X = A\,B\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$

Standard sum-of-products form

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$X = A\,B\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$

This form is called the *standard* sum-of-products form, and each individual term (consisting of all three variables) is called a "minterm."

Consider a function X of three variables A, B, C:

$X = A\,B\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$

This form is called the *standard* sum-of-products form, and each individual term (consisting of all three variables) is called a "minterm."

In the truth table for X, the numbers of 1s is the same as the number of minterms, as we have seen in an example.

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X can be rewritten as,

$$\begin{aligned} X &= A\,B\,\overline{C} + \overline{A}\,B\,(C + \overline{C}) \\ &= A\,B\,\overline{C} + \overline{A}\,B. \end{aligned}$$

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$$\begin{aligned} X &= A\,B\,\overline{C} + \overline{A}\,B\,(C + \overline{C}) \\ &= A\,B\,\overline{C} + \overline{A}\,B. \end{aligned}$$

This is also a sum-of-products form, but not the standard one.

Consider a function X of three variables A, B, C:

$X = (A + B + C)\,(\overline{A} + B + C)\,(\overline{A} + \overline{B} + C)$

Consider a function X of three variables A, B, C:

$X = (A + B + C)\,(\overline{A} + B + C)\,(\overline{A} + \overline{B} + C)$

This form is called the *standard* product-of-sums form, and each individual term (consisting of all three variables) is called a "maxterm."

Consider a function X of three variables A, B, C:

$X = (A + B + C)\,(\overline{A} + B + C)\,(\overline{A} + \overline{B} + C)$

This form is called the *standard* product-of-sums form, and each individual term (consisting of all three variables) is called a "maxterm."

In the truth table for X, the numbers of 0s is the same as the number of maxterms, as we have seen in an example.

Consider a function X of three variables A, B, C:

$X = (A + B + C)\,(\overline{A} + B + C)\,(\overline{A} + \overline{B} + C)$

This form is called the *standard* product-of-sums form, and each individual term (consisting of all three variables) is called a "maxterm."

In the truth table for X, the numbers of 0s is the same as the number of maxterms, as we have seen in an example.

X can be rewritten as,

$$\begin{aligned}
X &= (A + B + C)\,(\overline{A} + B + C)\,(\overline{A} + \overline{B} + C) \\
&= (A + B + C)\,(\overline{A} + C + B)\,(\overline{A} + C + \overline{B}) \\
&= (A + B + C)\,(\overline{A} + C + B\,\overline{B}) \\
&= (A + B + C)\,(\overline{A} + C).
\end{aligned}$$

Consider a function X of three variables A, B, C:

$X = (A+B+C)\,(\overline{A}+B+C)\,(\overline{A}+\overline{B}+C)$

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X &= (A+B+C)\,(\overline{A}+B+C)\,(\overline{A}+\overline{B}+C)\\
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&= (A+B+C)\,(\overline{A}+C+B\,\overline{B})\\
&= (A+B+C)\,(\overline{A}+C).
\end{aligned}
$$

This is also a product-of-sums form, but not the standard one.

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Don't care conditions can often be used to get a more efficient implementation of a logical function.