Digital Circuits: Part 2

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* A "minimal" expression has a minimum number of terms, each with a minimum number of variables. (For some functions, it is possible to have more than one minimal expressions, i.e., more than one expressions with the same complexity.)
* A minimal expression can be implemented with fewer gates.

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0				
1				

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0				
1	1			

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		
1	1			

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1			

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

$\Rightarrow$

C \ AB	00	01	11	10
0	0	1	0	X
1	1	0	1	0

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

$\Rightarrow$

C \ AB	00	01	11	10
0	0	1	0	X
1	1	0	1	0

* A K-map is the same as the truth table of a function except for the way the entries are arranged.

K-maps

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	X
1	0	1	0
1	1	0	0
1	1	1	1

C \ AB	00	01	11	10
0		1		X
1	1		1	

$\Rightarrow$

C \ AB	00	01	11	10
0	0	1	0	X
1	1	0	1	0

* A K-map is the same as the truth table of a function except for the way the entries are arranged.
* In a K-map, the adjacent rows or columns differ only in *one* variable. For example, in going from the column $AB = 01$ to $AB = 11$, there is only one change, viz., $A = 0 \rightarrow A = 1$.

K-maps: example with four variables

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	X
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	X
11	1	0	1	1
10	1	0	0	0

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_1 = A\,B\,\overline{C}\,\overline{D}$$

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_1 = A\,B\,\overline{C}\,\overline{D}$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$$X_2 = \overline{A}\,\overline{C}\,D$$

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_1 = A\,B\,\overline{C}\,\overline{D}$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$$X_2 = \overline{A}\,\overline{C}\,D$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

$$X_3 = A\,C$$

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_1 = A\,B\,\overline{C}\,\overline{D}$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$$X_2 = \overline{A}\,\overline{C}\,D$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

$$X_3 = A\,C$$

*

No. of variables	No. of 1's
4	2^0
3	2^1
2	2^2

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$X_1 = A\,B\,\overline{C}\,\overline{D}$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$X_2 = \overline{A}\,\overline{C}\,D$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

$X_3 = A\,C$

*

No. of variables	No. of 1's
4	2^0
3	2^1
2	2^2

* The 1's can be enclosed by a rectangle in each case.

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_1 = A\,B\,\overline{C}\,\overline{D}$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$$X_2 = \overline{A}\,\overline{C}\,D$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

$$X_3 = A\,C$$

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$X_1 = A\,B\,\overline{C}\,\overline{D}$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$X_2 = \overline{A}\,\overline{C}\,D$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

$X_3 = A\,C$

CD \ AB	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	1
10	0	0	1	1

$Y = X_1 + X_2 + X_3$

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_1 = A\,B\,\overline{C}\,\overline{D}$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$$X_2 = \overline{A}\,\overline{C}\,D$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

$$X_3 = A\,C$$

CD \ AB	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	1
10	0	0	1	1

$$Y = X_1 + X_2 + X_3$$

* We are interested in identifying a *minimal* expression from the given K-map.

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$X_1 = A\,B\,\overline{C}\,\overline{D}$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$$X_2 = \overline{A}\,\overline{C}\,D$$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

$$X_3 = A\,C$$

CD \ AB	00	01	11	10
00	0	0	1	0
01	1	1	0	0
11	0	0	1	1
10	0	0	1	1

$$Y = X_1 + X_2 + X_3$$

* We are interested in identifying a *minimal* expression from the given K-map.
* Minimal: smallest number of terms, smallest number of variables in each term
  $\rightarrow$ smallest number of rectangles containing 2^k 1's, each as large as possible

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

What is the logical function (Y) represented by this K-map?

K-maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

What is the logical function (Y) represented by this K-map?

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

What is the logical function (Y) represented by this K-map?

* There are 2^1 1's forming a rectangle $\rightarrow$ we can combine them.

K-maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

What is the logical function (Y) represented by this K-map?

* There are 2^1 1's forming a rectangle $\rightarrow$ we can combine them.
* The product term is 1 if $B=1$, and $C=0$.

K-maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

What is the logical function (Y) represented by this K-map?

* There are 2^1 1's forming a rectangle $\rightarrow$ we can combine them.
* The product term is 1 if $B = 1$, and $C = 0$.
* The product term does not depend on A.

K-maps

C \ AB	00	01	11	10
0	0	1	1	0
1	0	0	0	0

What is the logical function (Y) represented by this K-map?

* There are 2^1 1's forming a rectangle $\rightarrow$ we can combine them.
* The product term is 1 if $B=1$, and $C=0$.
* The product term does not depend on A.

$\rightarrow$ $Y = B\,\overline{C}$

C \ AB	00	01	11	10
0	0	0	1	0
1	0	0	0	1

Can the 1s shown in the K-map be combined?

C \ AB	00	01	11	10
0	0	0	1	0
1	0	0	0	1

Can the 1s shown in the K-map be combined?

Although the number of 1's is a power of 2 (2^1), they cannot be combined because they are not adjacent (i.e., they do not form a rectangle).

C \ AB	00	01	11	10
0	0	0	1	0
1	0	0	0	1

Can the 1s shown in the K-map be combined?

Although the number of 1's is a power of 2 (2^1), they cannot be combined because they are not adjacent (i.e., they do not form a rectangle).

$\rightarrow$ the function $(A\,B\,\overline{C} + A\,\overline{B}\,C)$ cannot be minimized.

K-maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

Can the 1's shown in the K-map be combined?

K-maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

Can the 1's shown in the K-map be combined?

Let us redraw the K-map by changing the order of the columns cyclically.

K-maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1's shown in the K-map be combined?

Let us redraw the K-map by changing the order of the columns cyclically.

K-maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1's shown in the K-map be combined?

Let us redraw the K-map by changing the order of the columns cyclically.

The two 1's are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

K-maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1's shown in the K-map be combined?

Let us redraw the K-map by changing the order of the columns cyclically.

The two 1's are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

K-maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1's shown in the K-map be combined?

Let us redraw the K-map by changing the order of the columns cyclically.

The two 1's are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

$\rightarrow$ Columns $A\,B = 00$ and $A\,B = 10$ in the K-map on the left are indeed "logically adjacent" (although they are not geometrically adjacent) since they differ only in one variable (A).

K-maps

C \ AB	00	01	11	10
0	1	0	0	1
1	0	0	0	0

C \ AB	10	00	01	11
0	1	1	0	0
1	0	0	0	0

Can the 1's shown in the K-map be combined?

Let us redraw the K-map by changing the order of the columns cyclically.

The two 1's are, in fact, adjacent and can be combined to give $\overline{B}\,\overline{C}$.

$\rightarrow$ Columns $A\,B = 00$ and $A\,B = 10$ in the K-map on the left are indeed "logically adjacent" (although they are not geometrically adjacent) since they differ only in one variable (A).

We could have therefore combined the 1's without actually redrawing the K-map.

K-maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

K-maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$\Rightarrow \quad X_1 = \overline{B}\,\overline{D}$$

K-maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$X_1 = \overline{B}\,\overline{D}$$

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

K-maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$X_1 = \overline{B}\,\overline{D}$$

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

K-maps

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$X_1 = \overline{B}\,\overline{D}$$

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

$$X_2 = \overline{B}\,\overline{C}$$

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$$X_1 = \underline{A\,B\,\overline{C}\,\overline{D}} + \underline{A\,B\,\overline{C}\,D} + \underline{\overline{A}\,B\,\overline{C}\,D}$$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$$X_1 = \underline{A\,B\,\overline{C}\,\overline{D}} + \underline{A\,B\,\overline{C}\,D} + \underline{\overline{A}\,B\,\overline{C}\,D}$$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D \qquad \text{(using Y=Y+Y)} \\ &= A\,B\,\overline{C}\,(\overline{D} + D) + B\,\overline{C}\,D\,(A + \overline{A}) \\ &= A\,B\,\overline{C} + B\,\overline{C}\,D \end{aligned}$$

K-maps

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$$X_1 = A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D$$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D \qquad \text{(using Y=Y+Y)} \\ &= A\,B\,\overline{C}\,(\overline{D} + D) + B\,\overline{C}\,D\,(A + \overline{A}) \\ &= A\,B\,\overline{C} + B\,\overline{C}\,D \end{aligned}$$

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	1	1	0
11	0	0	0	0
10	0	0	0	0

Standard sum-of-products form:

$$X_1 = A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D$$

Since the number of minterms is not a power of 2, they cannot be combined into a single term; however, they can be combined into two terms:

$$\begin{aligned} X_1 &= A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,\overline{C}\,D + \overline{A}\,B\,\overline{C}\,D \qquad \text{(using Y=Y+Y)} \\ &= A\,B\,\overline{C}\,(\overline{D} + D) + B\,\overline{C}\,D\,(A + \overline{A}) \\ &= A\,B\,\overline{C} + B\,\overline{C}\,D \end{aligned}$$

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K-maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K-maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K-maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K-maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

K-maps

X_1:

CD \ AB	00	01	11	10
00	1	1	0	1
01	1	1	0	1
11	0	0	1	0
10	0	0	0	1

$$\Rightarrow \quad X_1 = \overline{A}\,\overline{C} + \overline{B}\,\overline{C} + A\,B\,C\,D + A\,\overline{B}\,\overline{D}$$

K-maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

Since X represents a "don't care" condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.

K-maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

$\Rightarrow$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	1	1	1

Since X represents a “don’t care” condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.

K-maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

$\Rightarrow$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	1	1	1

Since X represents a "don't care" condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.

K-maps

Z:

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	0	0
11	0	0	0	0
10	1	X	1	1

$\Rightarrow$

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	1	1	1

$\Rightarrow$

$$Z = C\,\overline{D} + \overline{A}\,\overline{C}\,D$$

Since X represents a “don’t care” condition, we can assign 0 or 1 to the corresponding minterm to arrive at a minimal expression.