Op-Amp Circuits: Part 2

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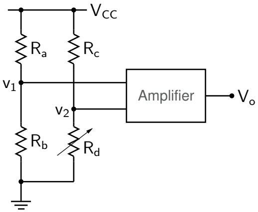
Common-mode and differential-mode voltages



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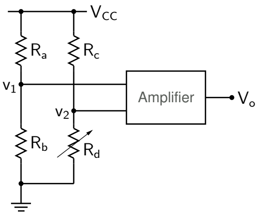

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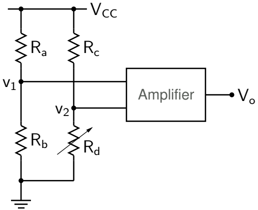

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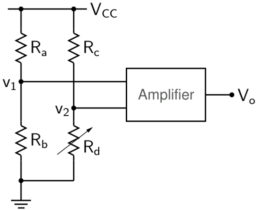

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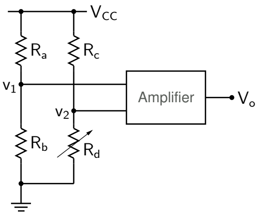

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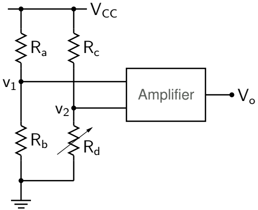

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where $x = \Delta R/R$.

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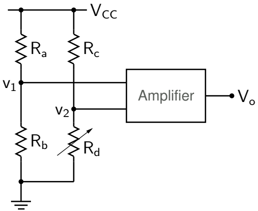

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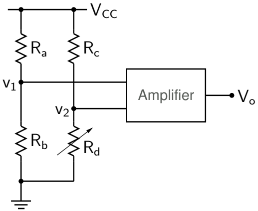
For example, with $V_{CC} = 15\ V$, $R = 1\,\text{k}$, $\Delta R = 0.01\,\text{k}$,

$v_1 = 7.5\ V$,

$v_2 = 7.5 + 0.0375\ V$.



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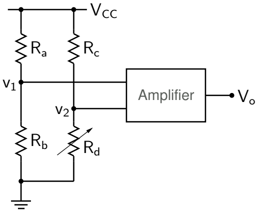
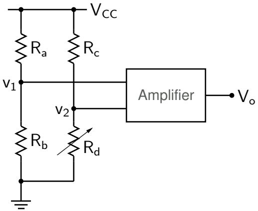


$v_1 = 7.5\ V$, $v_2 = 7.5 + 0.0375\ V$.

The amplifier should only amplify $v_2 - v_1 = 0.0375\ V$ (since that is the signal arising from ΔR).



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Given v_1 and v_2,

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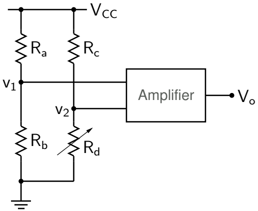

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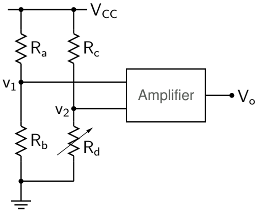

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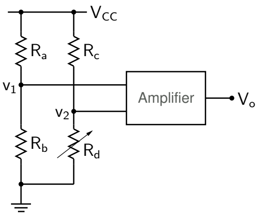

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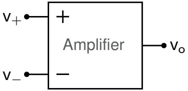
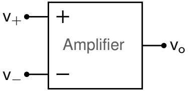
Note that the common-mode voltage is quite large compared to the differential-mode voltage.

This is a common situation in transducer circuits.



$$v_+ = v_c + v_d/2$$
$$v_- = v_c - v_d/2$$

An ideal amplifier would only amplify the difference $(v_+ - v_-)$, giving
$v_o = A_d\,(v_+ - v_-) = A_d\,v_d$,
where A_d is called the "differential gain" or simply the gain (A_V).



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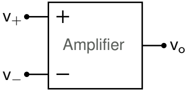

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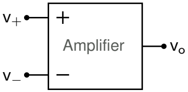
$$\boxed{\text{CMRR} = \frac{A_d}{A_c}}$$



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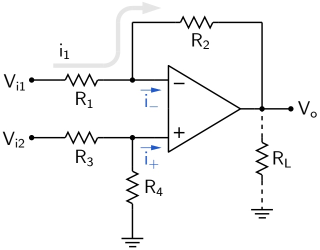
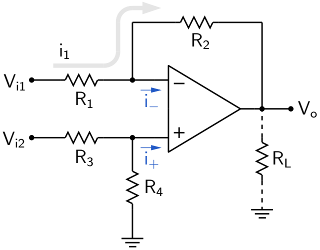
For the 741 op-amp, the CMRR is 90 dB ($\simeq$ 30,000), which may be considered to be infinite in many applications. In such cases, mismatch between circuit components will determine the overall common-mode rejection performance of the circuit.

Op-amp circuits (linear region)





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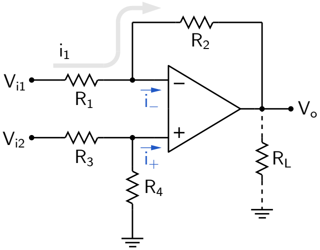
Large input resistance of op-amp → $i_+ = 0$, $V_+ = \dfrac{R_4}{R_3 + R_4}\, V_{i2}$.



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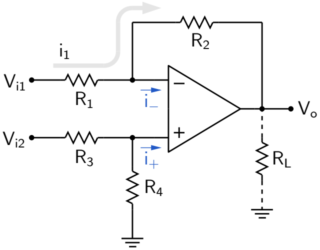

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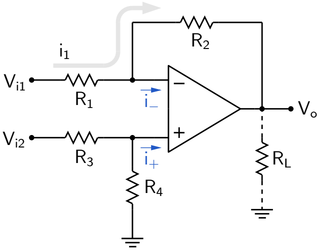
$i_- \approx 0 \rightarrow V_o = V_- - i_1 R_2 \approx V_+ - \dfrac{R_2}{R_1}\left(V_{i1} - V_+\right)$.



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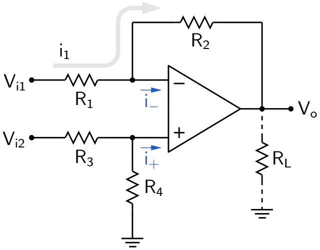

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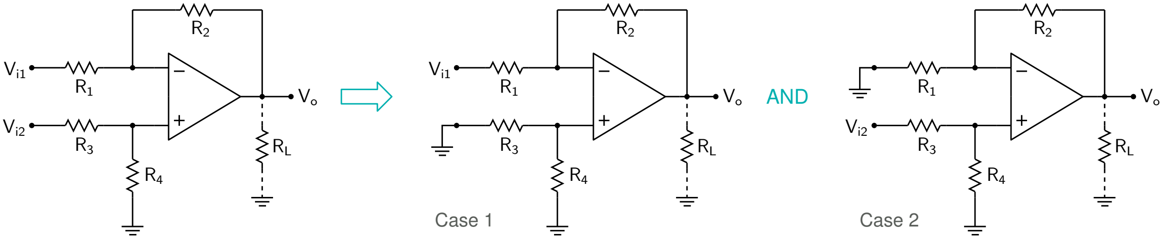
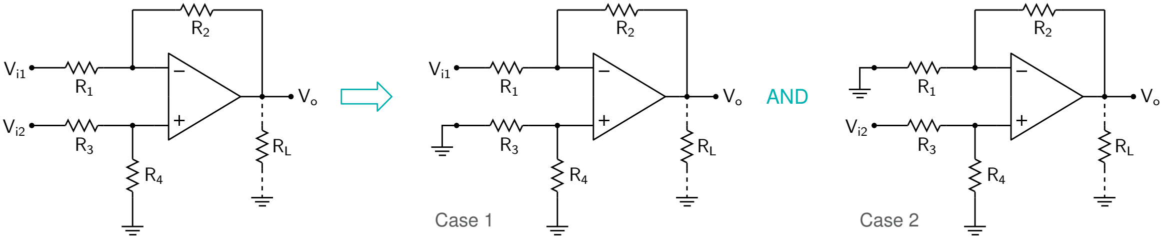
The circuit is a "difference amplifier."



<u>Method 2</u>:

Since the op-amp is operating in the linear region, we can use superposition:



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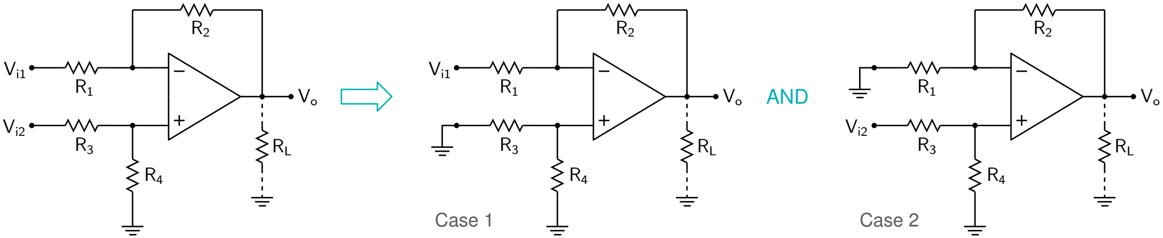
$$\rightarrow V_{o1} = -\frac{R_2}{R_1} V_{i1} \,.$$



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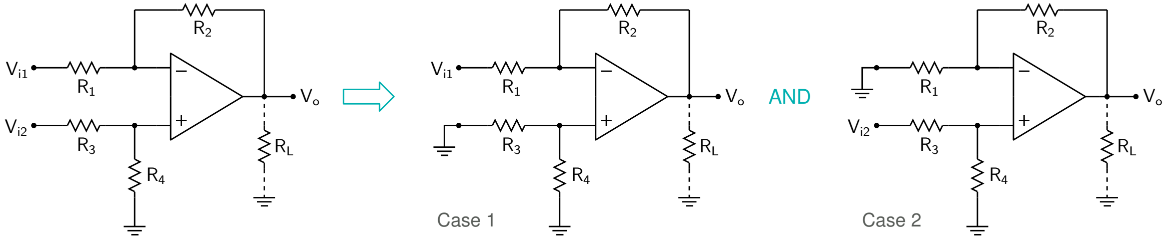
Case 2: Non-inverting amplifier, with $V_i = \dfrac{R_4}{R_3 + R_4}\, V_{i2}$.

$$\rightarrow V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2}\,.$$

Difference amplifier



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The net result is,

$$V_o = V_{o1} + V_{o2} = \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)V_{i2} - \frac{R_2}{R_1}\,V_{i1} = \frac{R_2}{R_1}\,(V_{i2}-V_{i1})\,,\ \text{if}\ \frac{R_4}{R_3} = \frac{R_2}{R_1}.$$

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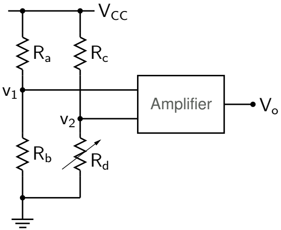
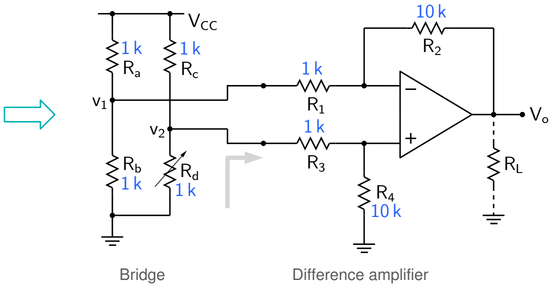
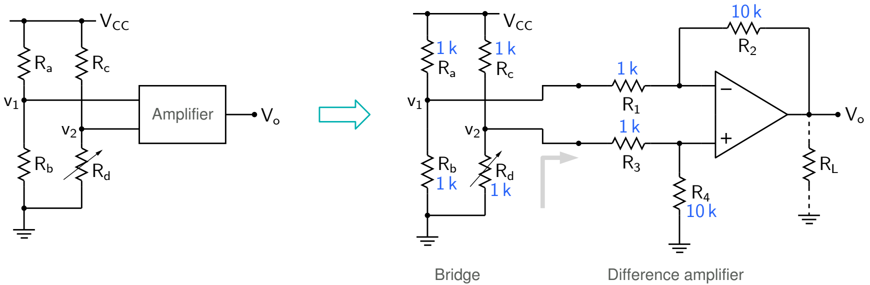


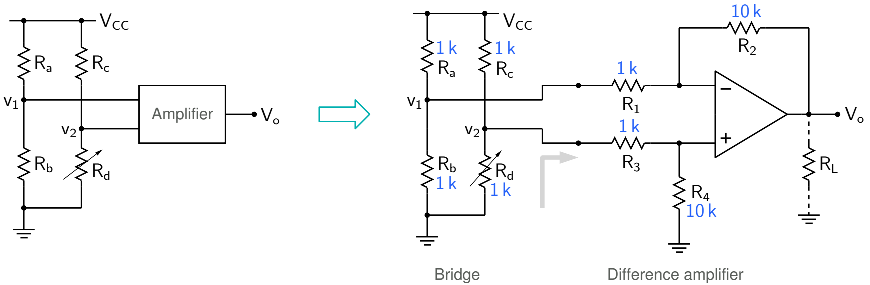




The resistance seen from v_2 is $(R_3 + R_4)$ which is small enough to cause v_2 to change.

This is not desirable.

Difference amplifier



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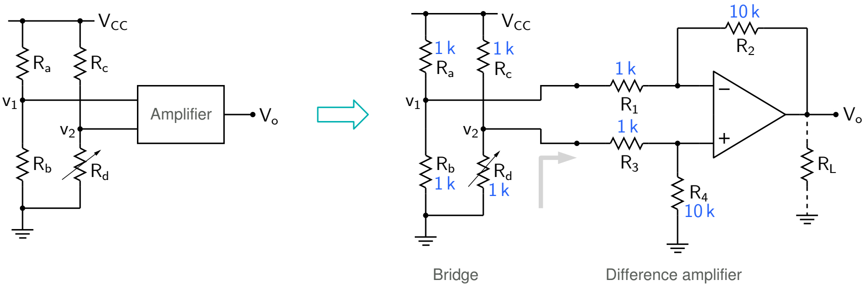

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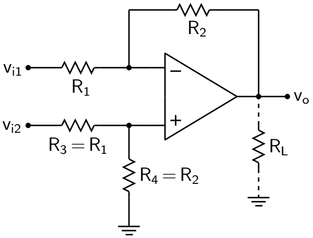
We will discuss an improved difference amplifier later. Before we do that, let us discuss another problem with the above difference amplifier which can be important for some applications (next slide).



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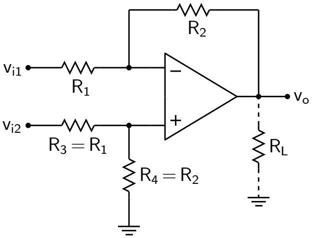

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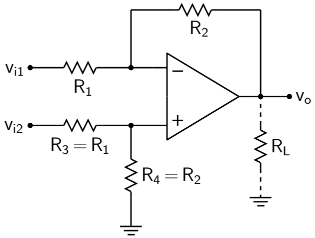
Consider the difference amplifier with $R_3 = R_1$, $R_4 = R_2 \rightarrow V_o = \dfrac{R_2}{R_1}\,(v_{i2} - v_{i1})$.

The output voltage depends only on the differential-mode signal $(v_{i2} - v_{i1})$,
i.e., A_c (common-mode gain) $= 0$.



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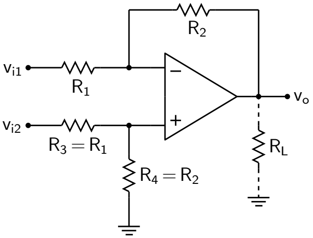
i.e., A_c (common-mode gain) $= 0$.

In practice, R_3 and R_1 may not be exactly equal. Let $R_3 = R_1 + \Delta R\,.$



$$v_{i1} = v_c - (v_d/2)$$
$$v_{i2} = v_c + (v_d/2)$$

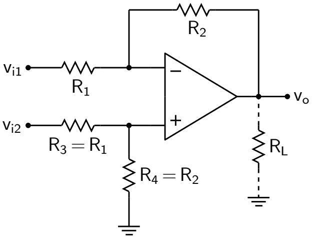
Consider the difference amplifier with $R_3 = R_1\,,\ R_4 = R_2 \rightarrow V_o = \dfrac{R_2}{R_1}\,(v_{i2} - v_{i1})\,.$

The output voltage depends only on the differential-mode signal $(v_{i2} - v_{i1})$,

i.e., A_c (common-mode gain) $= 0$.

In practice, R_3 and R_1 may not be exactly equal. Let $R_3 = R_1 + \Delta R\,.$

$$\begin{aligned} v_o &= \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right) v_{i2} - \frac{R_2}{R_1}\,v_{i1} = \frac{R_2}{R_1 + \Delta R + R_2}\left(1 + \frac{R_2}{R_1}\right) v_{i2} - \frac{R_2}{R_1}\,v_{i1} \\ &\simeq \frac{R_2}{R_1}(v_d - x\,v_c)\,, \text{with } x = \frac{\Delta R}{R_1 + R_2} \qquad \text{(show this)} \end{aligned}$$



$v_{i1} = v_c - (v_d/2)$

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Consider the difference amplifier with $R_3 = R_1$, $R_4 = R_2 \rightarrow V_o = \dfrac{R_2}{R_1}(v_{i2} - v_{i1})$.

The output voltage depends only on the differential-mode signal $(v_{i2} - v_{i1})$,

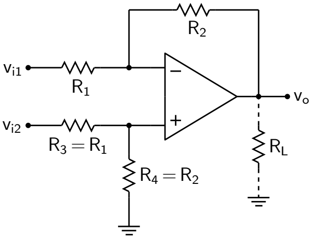
i.e., A_c (common-mode gain) = 0.

In practice, R_3 and R_1 may not be exactly equal. Let $R_3 = R_1 + \Delta R$.

$$v_o = \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right) v_{i2} - \frac{R_2}{R_1} v_{i1} = \frac{R_2}{R_1 + \Delta R + R_2}\left(1 + \frac{R_2}{R_1}\right) v_{i2} - \frac{R_2}{R_1} v_{i1}$$

$$\simeq \frac{R_2}{R_1}(v_d - x\, v_c)\,, \text{with } x = \frac{\Delta R}{R_1 + R_2} \quad \text{(show this)}$$

$$|A_c| = \frac{\Delta R}{R_1 + R_2}\,\frac{R_2}{R_1} \ll |A_d| = \frac{R_2}{R_1}.$$



$\mathrm{v_{i1}} = \mathrm{v_c} - (\mathrm{v_d}/2)$

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Consider the difference amplifier with $R_3 = R_1$, $R_4 = R_2 \rightarrow V_o = \dfrac{R_2}{R_1}\,(v_{i2} - v_{i1})$.

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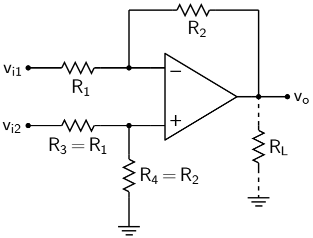
In practice, R_3 and R_1 may not be exactly equal. Let $R_3 = R_1 + \Delta R$.

$$v_o = \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right) v_{i2} - \frac{R_2}{R_1}\,v_{i1} = \frac{R_2}{R_1 + \Delta R + R_2}\left(1 + \frac{R_2}{R_1}\right) v_{i2} - \frac{R_2}{R_1}\,v_{i1}$$

$$\simeq \frac{R_2}{R_1}(v_d - x\,v_c)\,,\ \text{with } x = \frac{\Delta R}{R_1 + R_2} \qquad \text{(show this)}$$

$|A_c| = \dfrac{\Delta R}{R_1 + R_2}\,\dfrac{R_2}{R_1} \ll |A_d| = \dfrac{R_2}{R_1}$. However, since v_c can be large compared to v_d, the effect of A_c cannot be ignored.

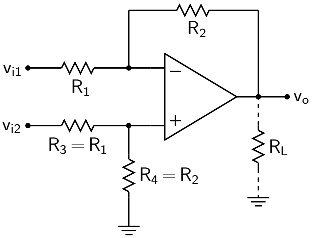
Difference amplifier



$v_{i1} = v_c - (v_d/2)$

$v_{i2} = v_c + (v_d/2)$

$|A_c| = x\,\dfrac{R_2}{R_1}$, $|A_d| = \dfrac{R_2}{R_1}$, where $x = \dfrac{\Delta R}{R_1 + R_2}$.



$v_{i1} = v_c - (v_d/2)$

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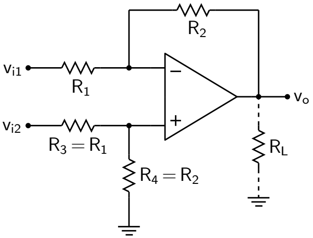
In our earlier example, $v_c = 7.5\,V$, $v_d = 0.0375\,V$.



$v_{i1} = v_c - (v_d/2)$

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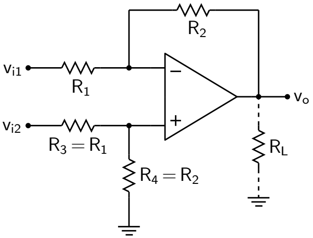
In our earlier example, $v_c = 7.5\,V$, $v_d = 0.0375\,V$.

With $R_1 = 1\,\text{k}$, $R_2 = 10\,\text{k}$, $x = \dfrac{0.01\,\text{k}}{11\,\text{k}} = 0.00091 \rightarrow |A_c| = 0.00091\,\dfrac{10\,\text{k}}{1\,\text{k}} = 0.0091$, $|A_d| = \dfrac{10\,\text{k}}{1\,\text{k}} = 10$.

$|v_o^c| = |A_c\,v_c| = 0.0091 \times 7.5 = 0.068\,V$.



$$v_{i1} = v_c - (v_d/2)$$
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$|A_c| = x\,\dfrac{R_2}{R_1}$, $|A_d| = \dfrac{R_2}{R_1}$, where $x = \dfrac{\Delta R}{R_1 + R_2}$.

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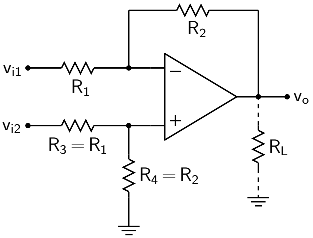
$|v_o^c| = |A_c\,v_c| = 0.0091 \times 7.5 = 0.068\,V$.

$|v_o^d| = |A_d\,v_d| = 10 \times 0.0375 = 0.375\,V$.



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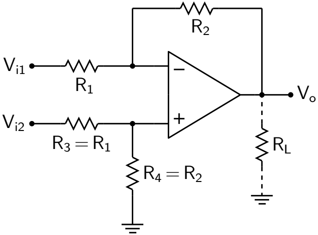
$|v_o^d| = |A_d\,v_d| = 10 \times 0.0375 = 0.375\,V$.

The (spurious) common-mode contribution is substantial.

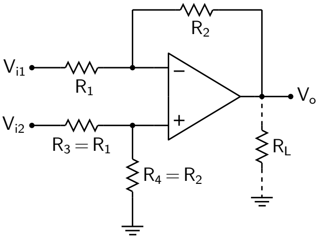
If we measure v_o, we will conclude that $v_d = \dfrac{v_o}{A_d}$, but in reality, it would be different.

$\rightarrow$ need a circuit which will drastically reduce the common-mode component at the output.



$$A_d = \frac{R_2}{R_1}$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1}\, V_{i1}$$



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Let $V_{i1} = V_{i2} = V_c \;\rightarrow\; A_c = \dfrac{V_o}{V_c}$.

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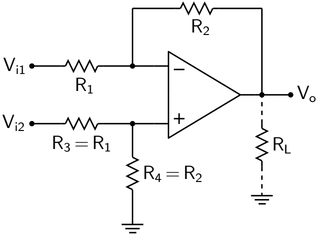

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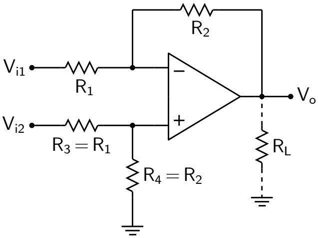
$$= \frac{R_4}{R_3+R_4}\left(1 - \frac{R_2}{R_1}\,\frac{R_3}{R_4}\right)$$



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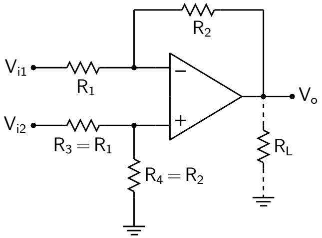
Assume ideal op-amp with $R_1 = R_1^0(1+x_1)$, etc. 1 % resistor $\rightarrow x = 0.01$.

Difference amplifier: resistance mismatch



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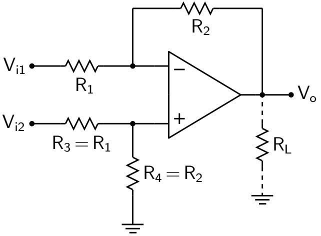
$$\rightarrow A_c = \frac{R_4}{R_3 + R_4}\left(1 - \frac{R_2^0\,(1 + x_2)}{R_1^0\,(1 + x_1)} \times \frac{R_3^0\,(1 + x_3)}{R_4^0\,(1 + x_4)}\right).$$



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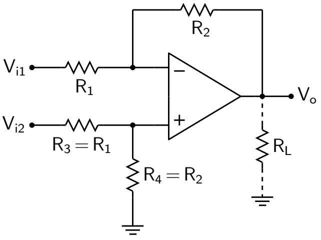
Using $(1 + u_1)\,(1 + u_2) \approx 1 + u_1 + u_2$ if $|u_1| \ll 1,\ |u_2| \ll 1,$



$A_d = \dfrac{R_2}{R_1}$

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$$= \frac{R_4}{R_3+R_4}\left(1-\frac{R_2}{R_1}\,\frac{R_3}{R_4}\right)$$

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Using $(1+u_1)\,(1+u_2) \approx 1+u_1+u_2$ if $|u_1| \ll 1$, $|u_2| \ll 1$,

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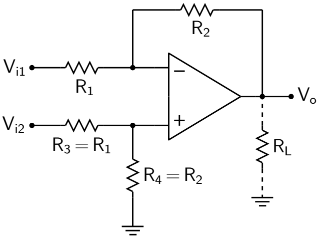

$$A_d = \frac{R_2}{R_1}$$

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$$= \frac{R_4}{R_3 + R_4}\left(1 - \frac{R_2}{R_1}\,\frac{R_3}{R_4}\right)$$

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Using $(1 + u_1)\,(1 + u_2) \approx 1 + u_1 + u_2$ if $|u_1| \ll 1$, $|u_2| \ll 1$,

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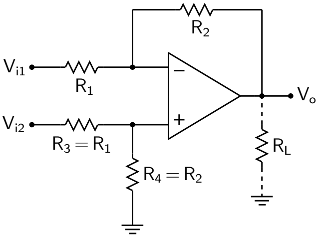
$$A_c = \frac{R_4}{R_3 + R_4}\,(x_1 - x_2 - x_3 + x_4).$$

Difference amplifier: resistance mismatch



$$A_d = \frac{R_2}{R_1}$$

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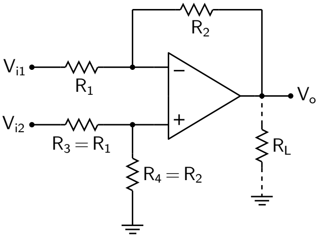

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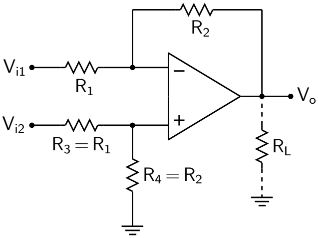
$$\frac{R_4}{R_3 + R_4} \approx \frac{R_4^0}{R_3^0 + R_4^0}.$$

Difference amplifier: resistance mismatch



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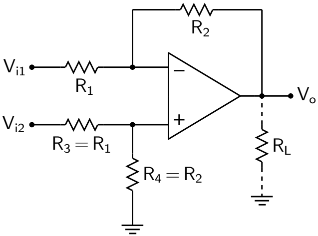
(1) $R_1^0 = R_2^0$ (i.e., $R_3^0 = R_4^0$)

Difference amplifier: resistance mismatch



$$A_d = \frac{R_2}{R_1}$$

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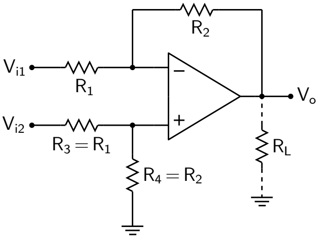
$$\frac{R_4}{R_3 + R_4} \approx \frac{R_4^0}{R_3^0 + R_4^0}.$$

(1) $R_1^0 = R_2^0$ (i.e., $R_3^0 = R_4^0$)

$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$

Difference amplifier: resistance mismatch



$$A_d = \frac{R_2}{R_1}$$

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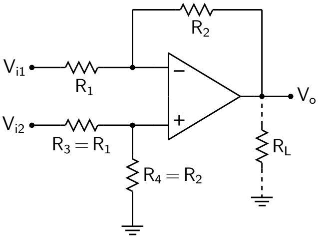
$$\frac{R_4}{R_3 + R_4} \approx \frac{R_4^0}{R_3^0 + R_4^0}.$$

$$(1)\quad R_1^0 = R_2^0 \;(\text{i.e., } R_3^0 = R_4^0)$$

$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$

$$= \frac{1}{2}\,4\,x = 2\,x \;(\text{worst case})$$



$$A_d = \frac{R_2}{R_1}$$

$$A_c = \frac{R_4}{R_3 + R_4}\,(x_1 - x_2 - x_3 + x_4).$$

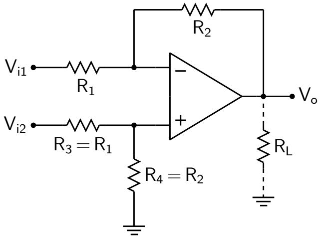
$$\frac{R_4}{R_3 + R_4} \approx \frac{R_4^0}{R_3^0 + R_4^0}.$$

(1) $R_1^0 = R_2^0$ (i.e., $R_3^0 = R_4^0$)

$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$

$$= \frac{1}{2}\,4\,x = 2\,x \text{ (worst case)}$$

(2) $R_1^0 \ll R_2^0$ (i.e., $R_3^0 \ll R_4^0$)



$$A_d = \frac{R_2}{R_1}$$

$$A_c = \frac{R_4}{R_3 + R_4}\,(x_1 - x_2 - x_3 + x_4).$$

$$\frac{R_4}{R_3 + R_4} \approx \frac{R_4^0}{R_3^0 + R_4^0}.$$

(1) $R_1^0 = R_2^0$ (i.e., $R_3^0 = R_4^0$)

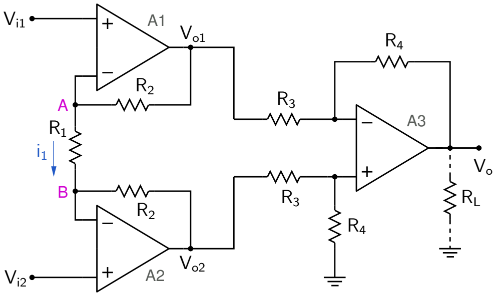
$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$

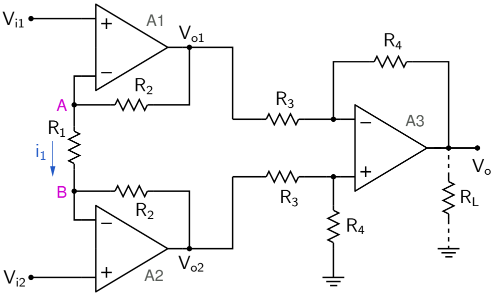
$$= \frac{1}{2}\,4\,x = 2\,x \text{ (worst case)}$$

(2) $R_1^0 \ll R_2^0$ (i.e., $R_3^0 \ll R_4^0$)

$$A_c = \frac{(R_4^0/R_3^0)}{1 + (R_4^0/R_3^0)}\,(x_1 - x_2 - x_3 + x_4) \approx 4\,x \text{ (worst case)}$$

Improved difference amplifier





$$V_+ \approx V_- \rightarrow V_A = V_{i1}\ ,\ \ V_B = V_{i2}\ , \rightarrow i_1 = \frac{1}{R_1}\left(V_{i1} - V_{i2}\right).$$

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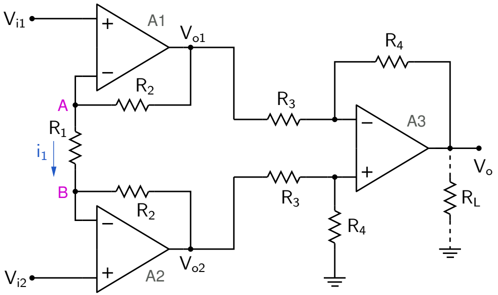

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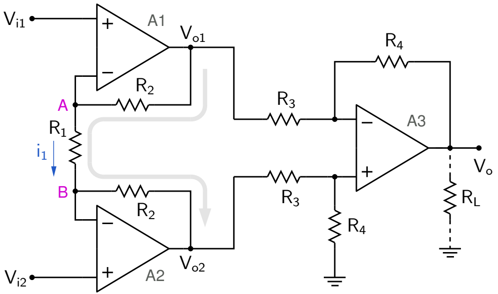

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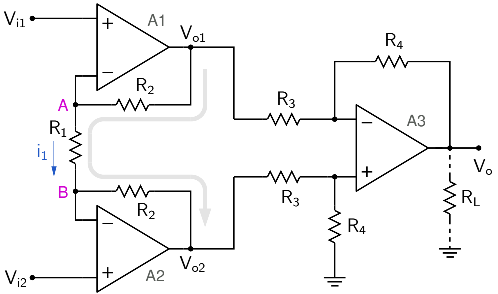

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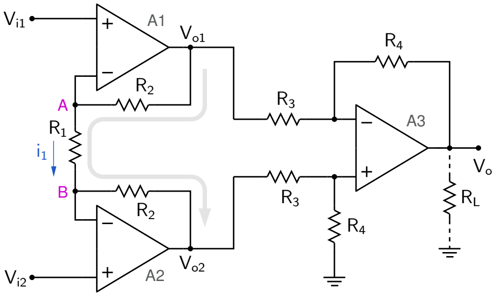

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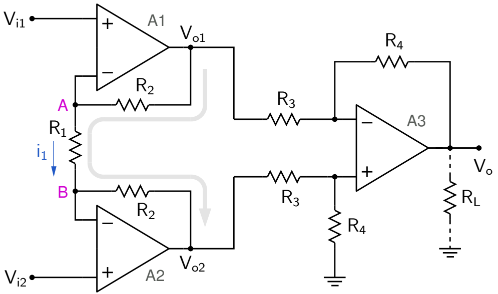

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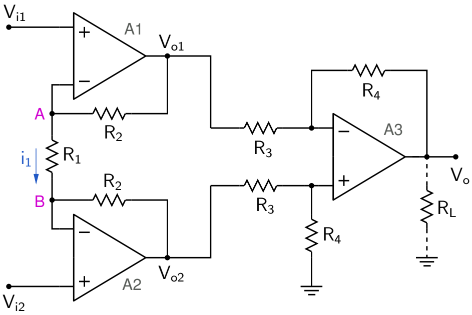
This circuit is known as the "instrumentation amplifier."

Instrumentation amplifier



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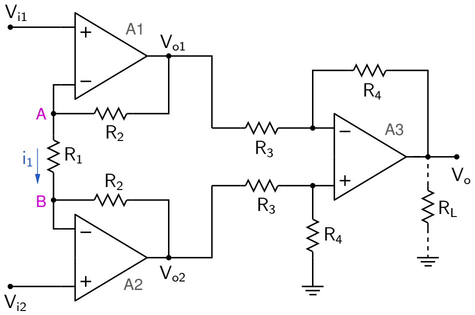

The input resistance seen from V_{i1} or V_{i2} is large (since an op-amp has a large input resistance).

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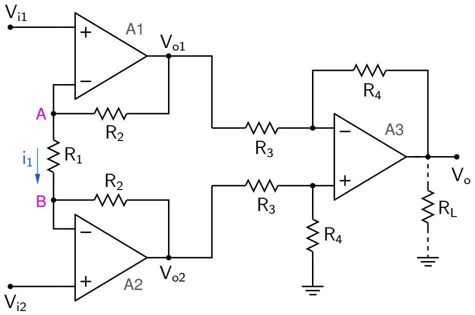

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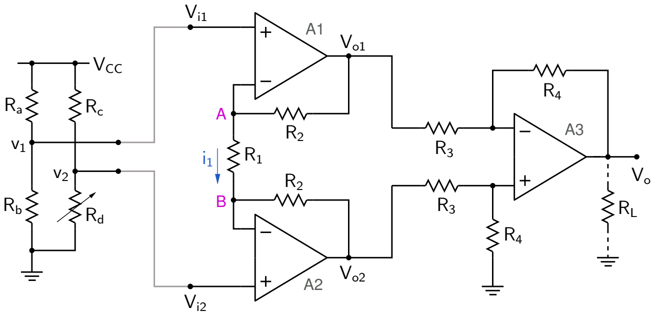

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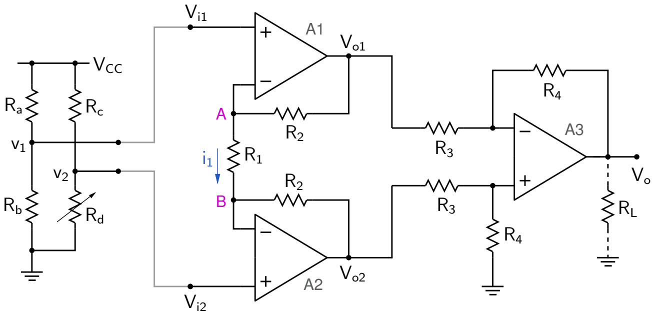

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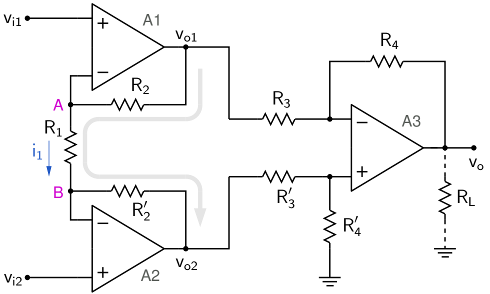
$\rightarrow$ the amplifier will not "load" the preceding stage, a desirable feature.
As a result, the voltages v_1 and v_2 in the bridge circuit will remain essentially the same when the bridge circuit is connected to the instrumentation amplifier.

Instrumentation amplifier: common-mode rejection



$$v_{i1} = v_c - (v_d/2)$$

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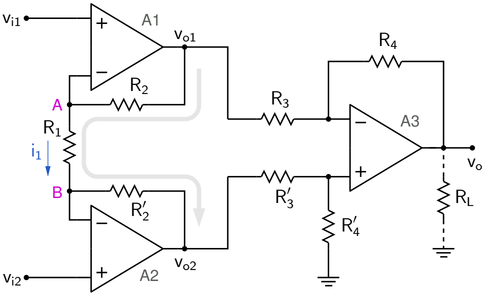
Instrumentation amplifier: common-mode rejection



$v_{i1} = v_c - (v_d/2)$

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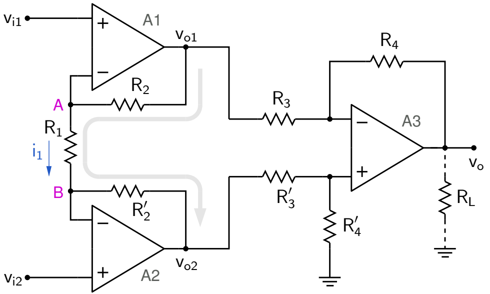
Note that v_{o1} serves as v_{i1} for the difference amplifier, and v_{o2} as v_{i2}. Let us find the differential-mode and common-mode components associated with v_{o1} and v_{o2}.

$$v'_{id} = v_{o2} - v_{o1}, \quad v'_{ic} = \frac{1}{2}\,(v_{o1} + v_{o2})$$



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$$v'_{id} = (R_2 + R'_2 + R_1)\,\frac{1}{R_1}\left[\left(v_c + \frac{v_d}{2}\right) - \left(v_c - \frac{v_d}{2}\right)\right] = \left(1 + \frac{R_2 + R'_2}{R_1}\right)\, v_d\,.$$

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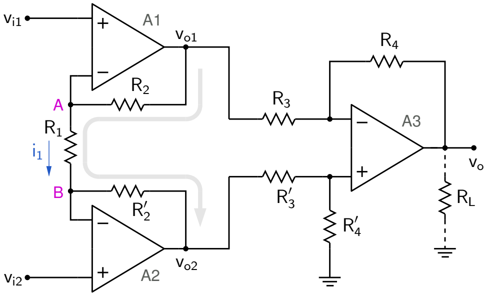

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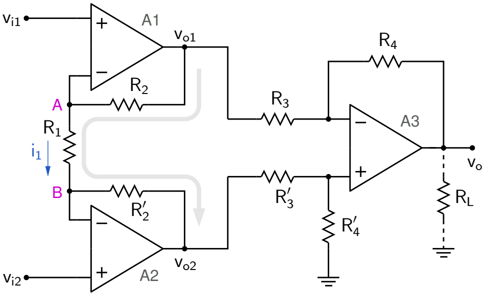

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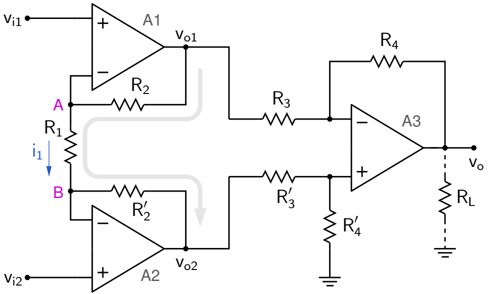
Instrumentation amplifier: common-mode rejection



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(Note that resistor mismatch in the second stage needs to be considered, but it will have a limited effect.)

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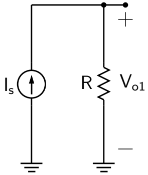

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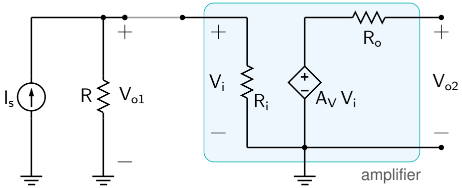
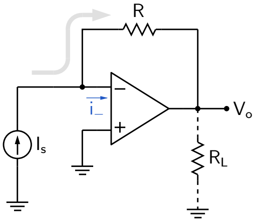


However, this simple approach will not work if the next stage in the circuit (such as an amplifier) has a finite R_i, since it will modify V_{o1} to $V_{o1} = I_s\, (R_i \parallel R)$, which is not desirable.

Current-to-voltage conversion



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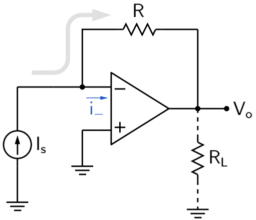

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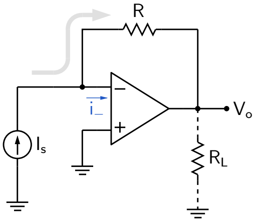

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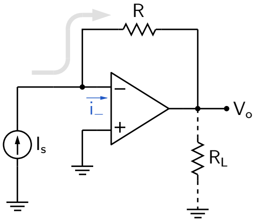
The output voltage is proportional to the source current, *irrespective* of the value of R_L, i.e., irrespective of the next stage.



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Example: a photocurrent detector.

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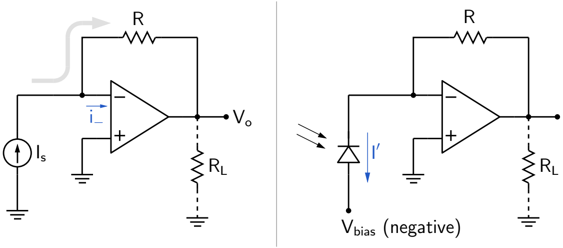

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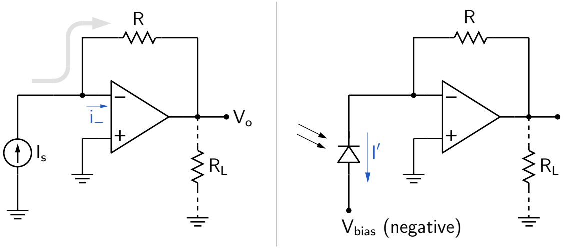

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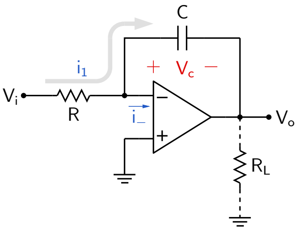
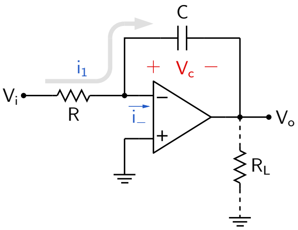
$V_o = I'\,R$. (Note: The diode is under a reverse bias, with $V_n = 0\;V$ and $V_p = V_{\text{bias}}$.)


C
i_1
$+$ V_c $-$
V_i
R
i_-
V_o
R_L



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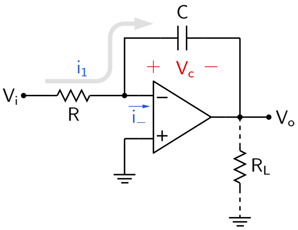

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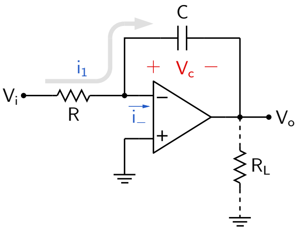

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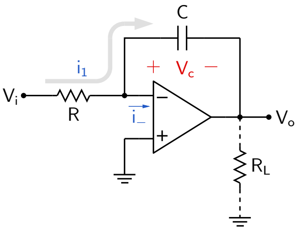

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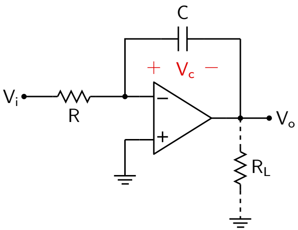
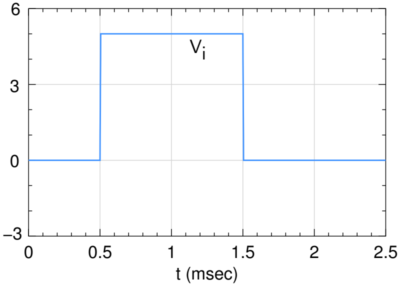
$$\boxed{V_o = -\frac{1}{RC}\int V_i\,dt}$$

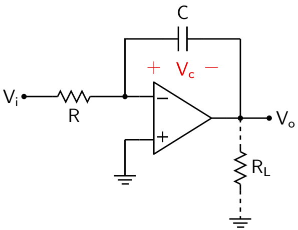
The circuit works as an integrator.

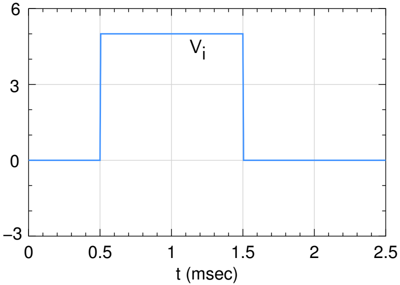




Given: $R = 10\,\text{k}$, $C = 0.2\,\mu\text{F}$.

If $V_o = 0\,\text{V}$ at $t = 0$, find $V_o(t)$ (Let $t_0 = 0.5\,\text{msec}$, $t_1 = 1.5\,\text{msec}$).





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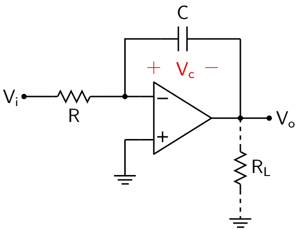
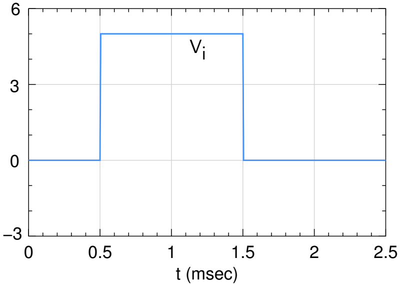
$$V_o = -\frac{1}{RC}\int V_i\,dt, \quad \tau \equiv RC = 2\,\text{msec}.$$

Integrator





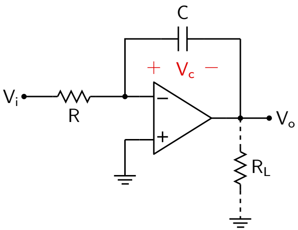
Given: $R = 10\,\mathrm{k}$, $C = 0.2\,\mu\mathrm{F}$.

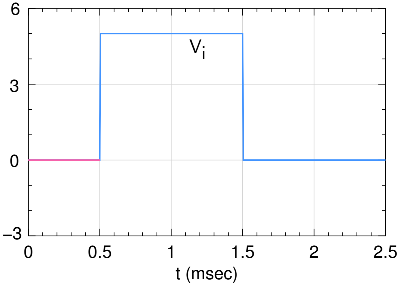
If $V_o = 0\,\mathrm{V}$ at $t = 0$, find $V_o(t)$ (Let $t_0 = 0.5\,\mathrm{msec}$, $t_1 = 1.5\,\mathrm{msec}$).

$$V_o = -\frac{1}{RC}\int V_i\,dt, \quad \tau \equiv RC = 2\,\mathrm{msec}.$$

* For $t < t_0$, $V_o(t) - V_o(0) = -\dfrac{1}{\tau}\displaystyle\int_0^t 0\,dt' = 0 \rightarrow V_o(t) = V_o(0) = 0\,\mathrm{V}$

Integrator





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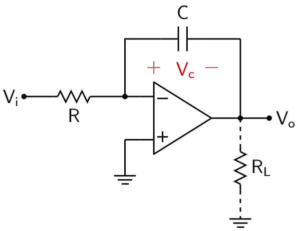

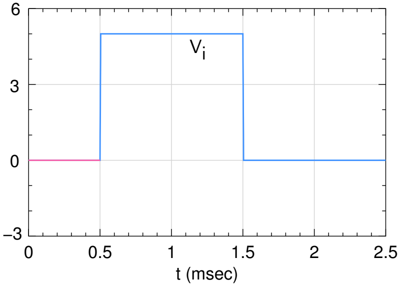

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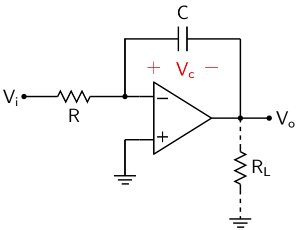

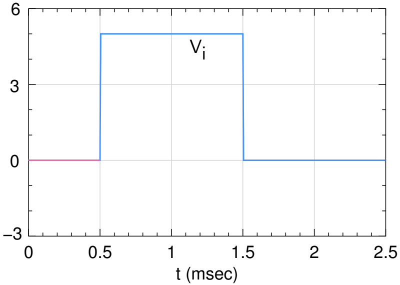

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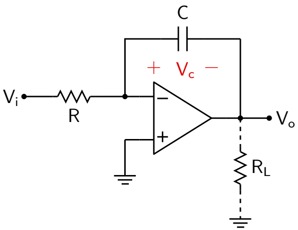

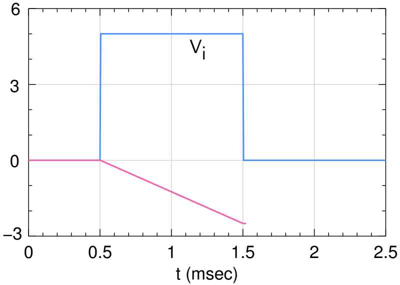

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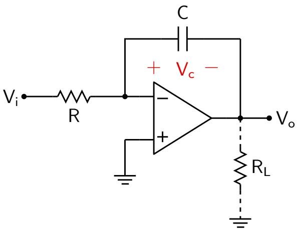

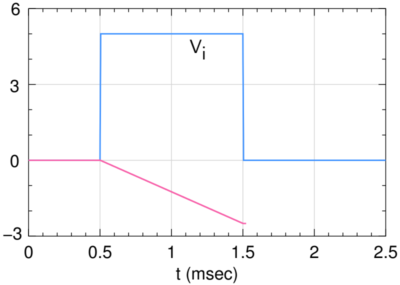

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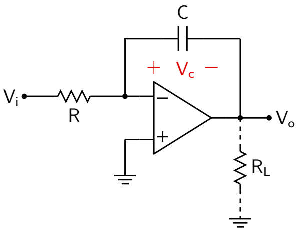

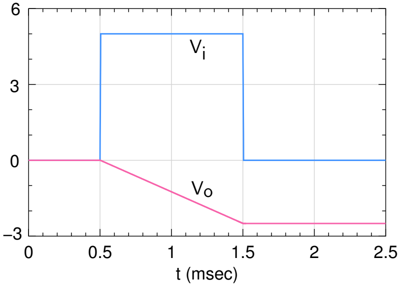

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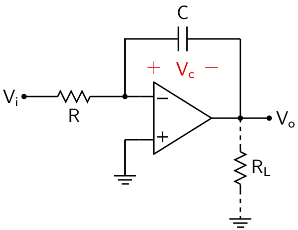

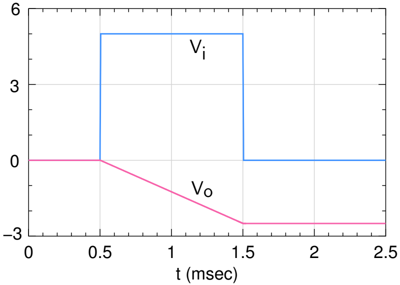

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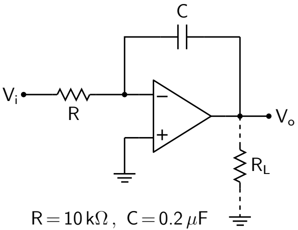
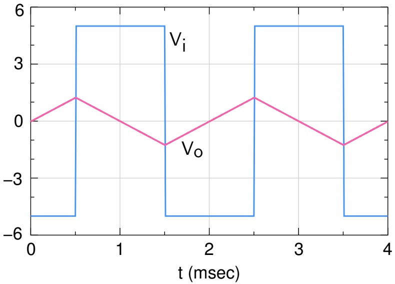
SEQUEL file: `ee101_integrator_1.sqproj`



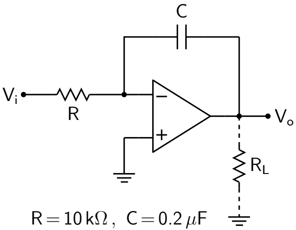
$R = 10\,\text{k}\Omega\,,\;\; C = 0.2\,\mu\text{F}$

$$V_o = -\frac{1}{RC}\int V_i\,dt$$



* An integrator can be used to convert a square wave to a triangle wave.

Integrator



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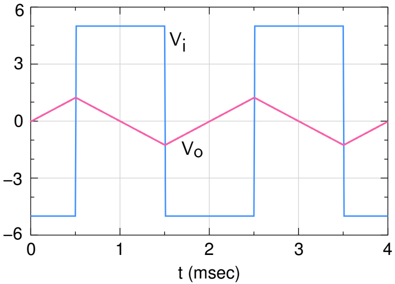
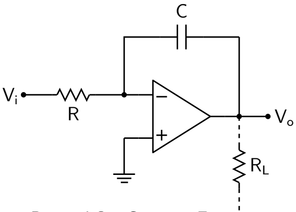


* An integrator can be used to convert a square wave to a triangle wave.
* In practice, the circuit needs a small modification, as discussed in the following.

Integrator



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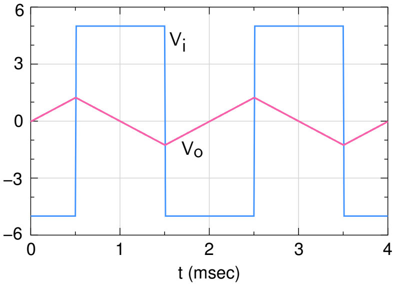

* An integrator can be used to convert a square wave to a triangle wave.
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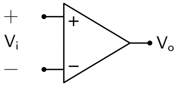
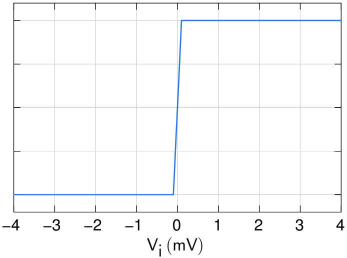
SEQUEL file: `ee101_integrator_2.sqproj`

Practical op-amps: Offset voltage





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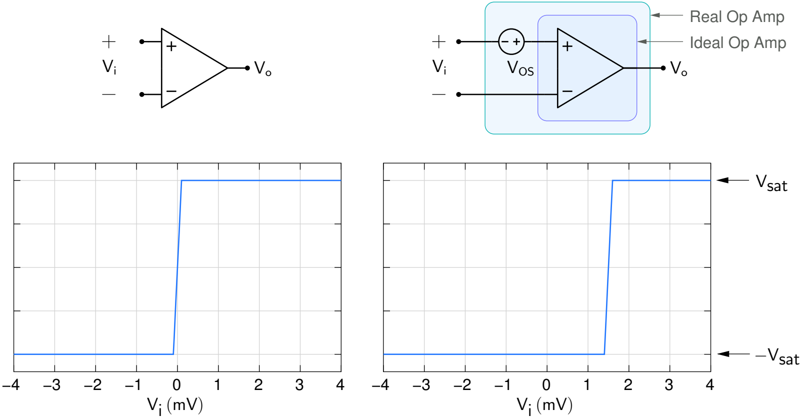

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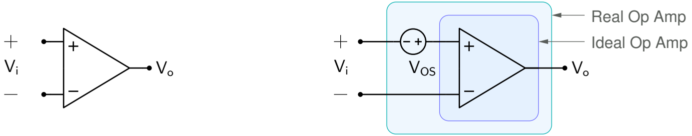

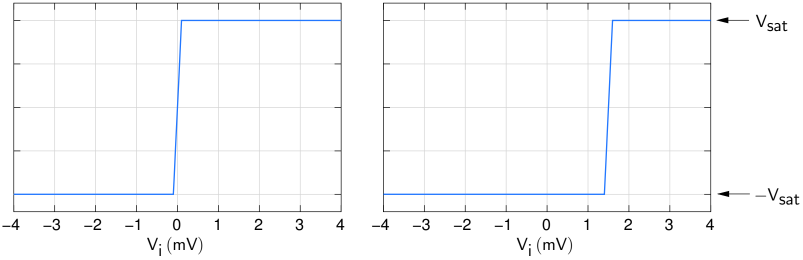

For the real op-amp, $V_o = A_V((V_+ + V_{OS}) - V_-)$.

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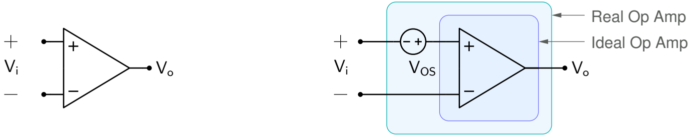

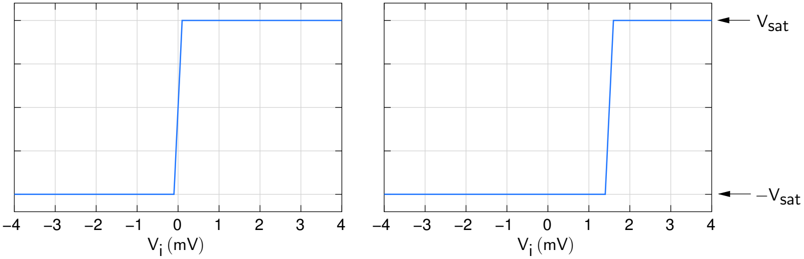

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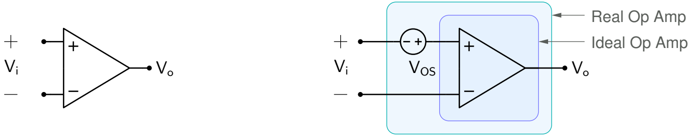

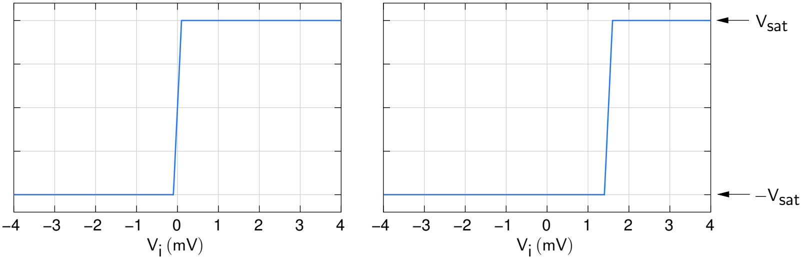

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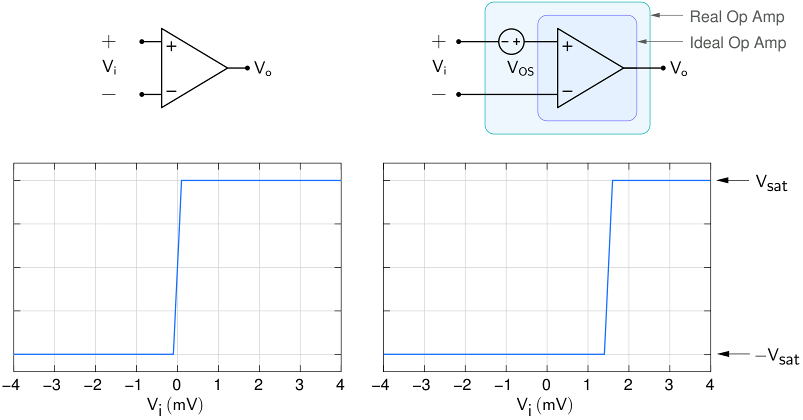

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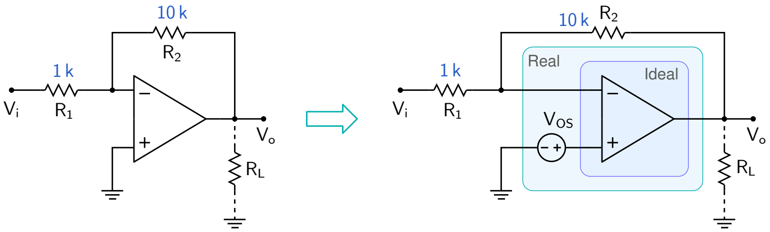
741: $-6\,\mathrm{m}V < V_{OS} < 6\,\mathrm{m}V$, OP-77: $-50\,\mu V < V_{OS} < 50\,\mu V$.

Effect of V_{OS}: inverting amplifier



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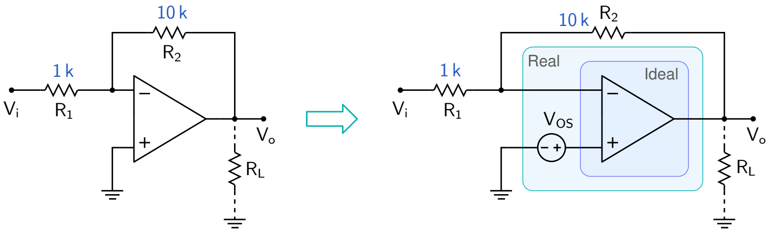
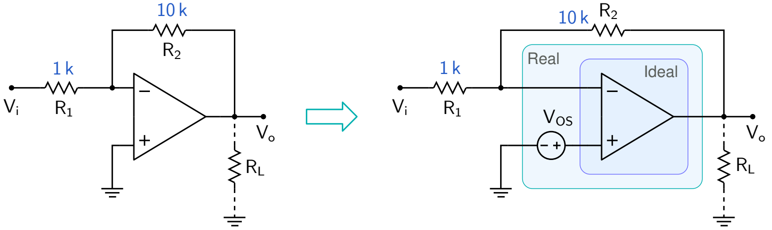


By superposition, $V_o = -\dfrac{R_2}{R_1}\, V_i + V_{OS}\left(1 + \dfrac{R_2}{R_1}\right)$.

Effect of V_{OS}: inverting amplifier



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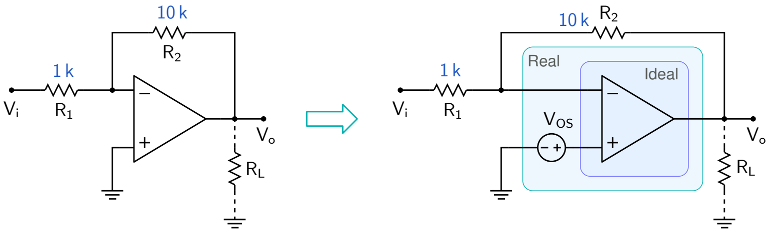

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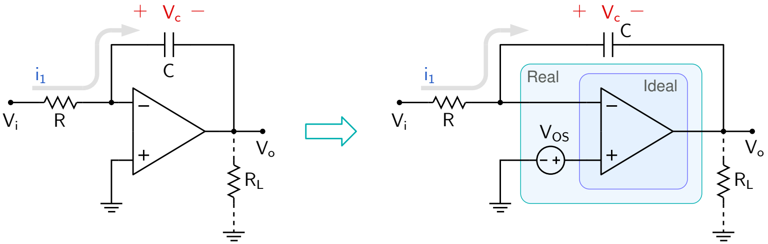
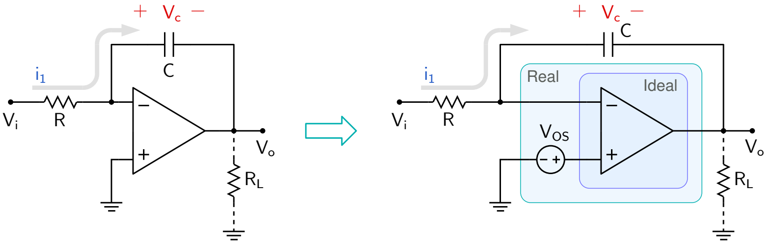
i.e., a DC shift of $22\,\mathrm{m}V$.

Effect of V_{OS}: integrator

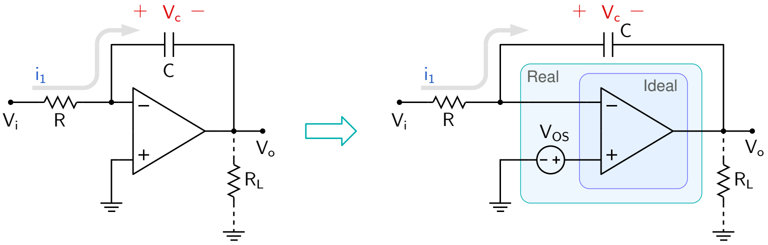


Effect of V_{OS}: integrator



$$V_- \approx V_+ = V_{OS} \rightarrow i_1 = \frac{1}{R}(V_i - V_{OS}) = C\,\frac{dV_c}{dt}\,.$$

Effect of V_{OS}: integrator



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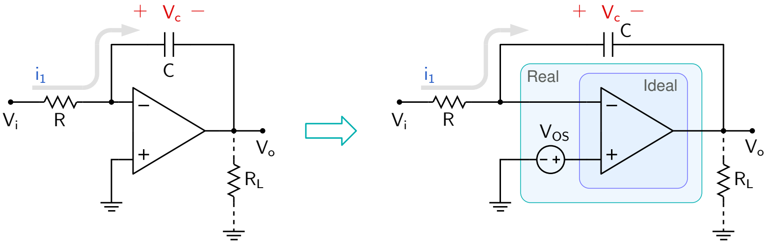

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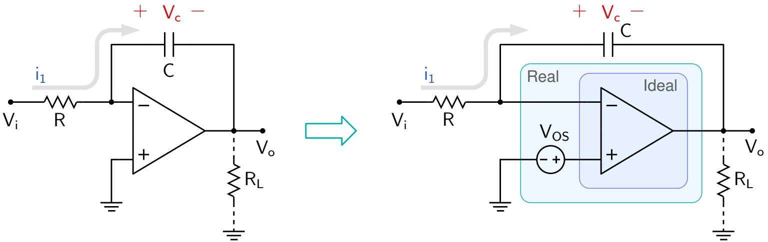
Even with $V_i = 0\,V$, V_c will keep rising or falling (depending on the sign of V_{OS}).

Eventually, the Op Amp will be driven into saturation.



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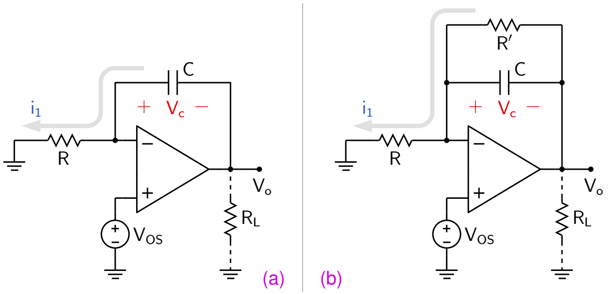
Even with $V_i = 0\,V$, V_c will keep rising or falling (depending on the sign of V_{OS}).

Eventually, the Op Amp will be driven into saturation.

$\rightarrow$ need to address this issue!

Effect of V_{OS}: integrator with $V_i = 0$



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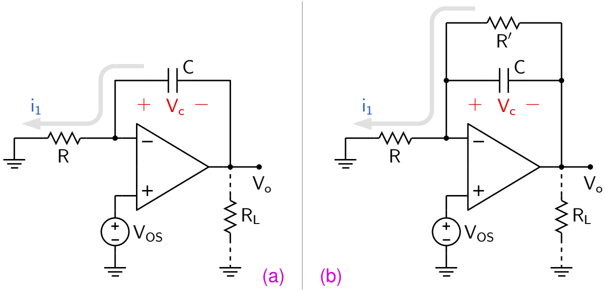

(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

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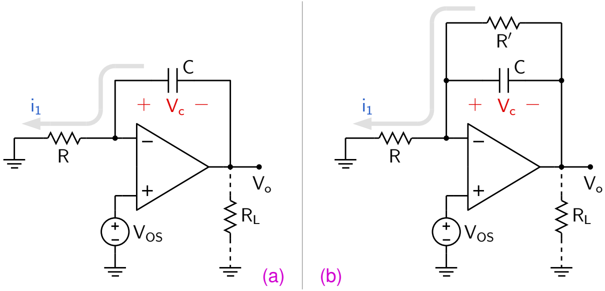

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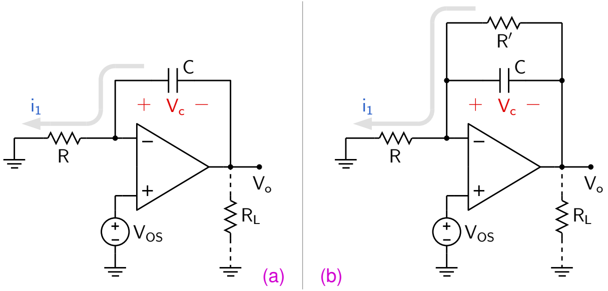

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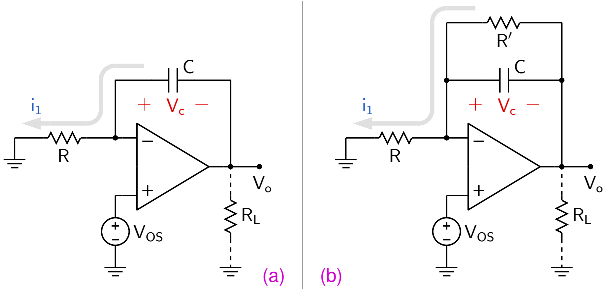

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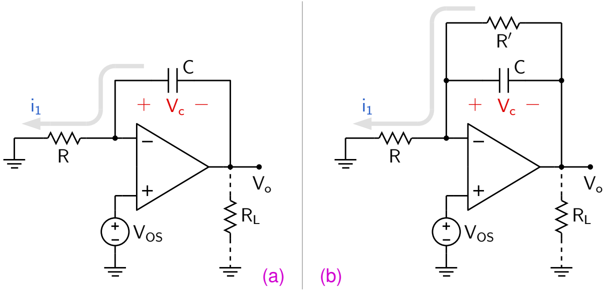

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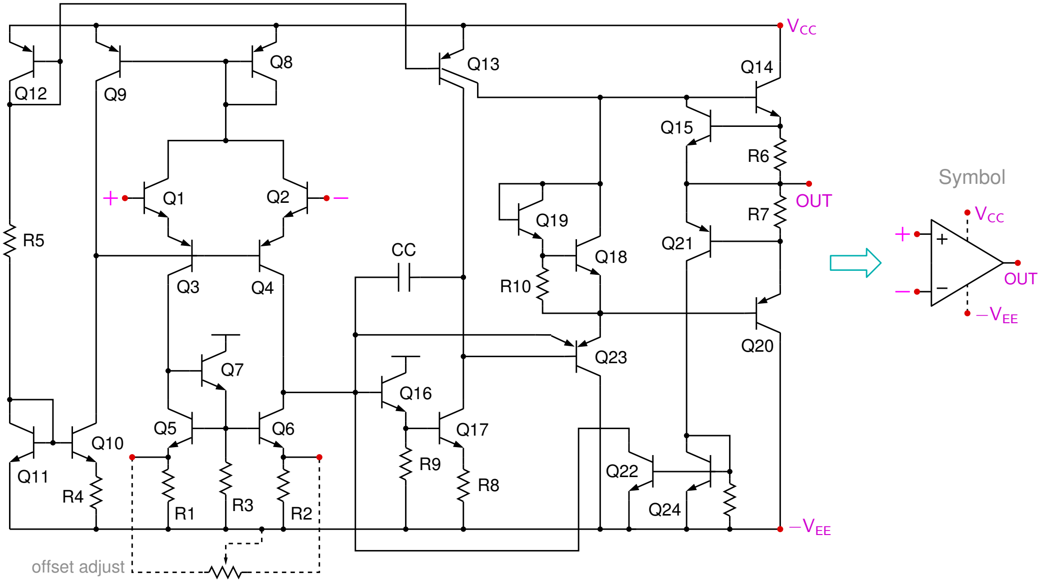
$\rightarrow R' \gg 1/\omega C$ at the frequency of interest.

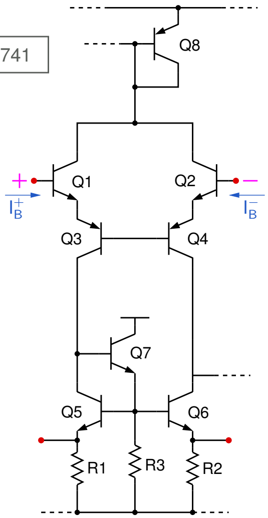
Op-amp 741: offset null

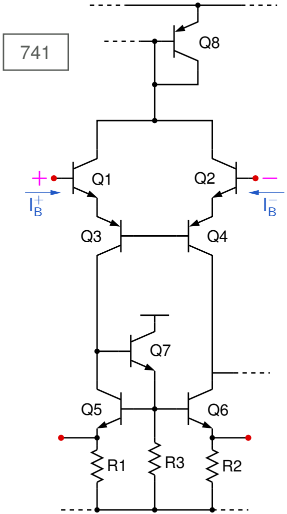


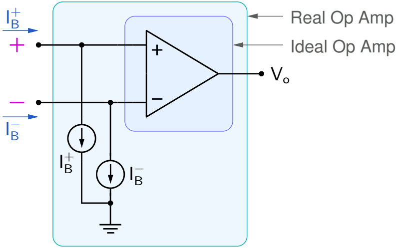
M. B. Patil, IIT Bombay

Input bias currents

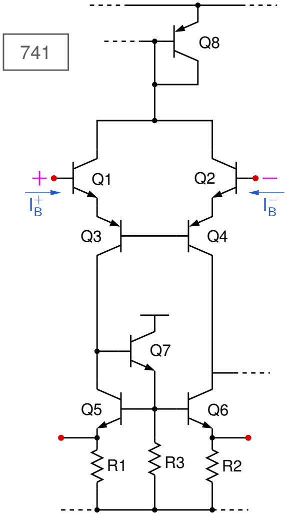
741

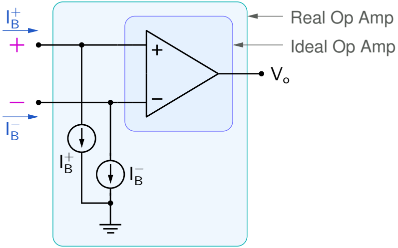



741
Q8
Q1
Q2
Q3
Q4
Q7
Q5
Q6
R1
R3
R2
+
−
I_B^+
I_B^-


I_B^+
+
−
I_B^-
Real Op Amp
Ideal Op Amp
V_o
I_B^+
I_B^-

Input bias currents



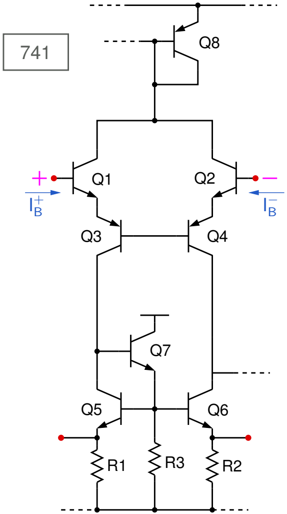


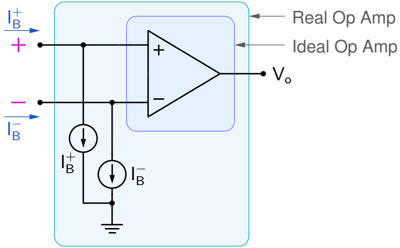
I_B^+ and I_B^- are generally not equal.

$|I_B^+ - I_B^-|$: "offset current" (I_{OS})

$(I_B^+ + I_B^-)/2$: "bias current" (I_B)

Input bias currents





Typical values

Op–Amp	I_B	I_{OS}	V_{OS}	Type
741	80 nA	20 nA	1 mV	BJT input
OP77	1.2 nA	0.3 nA	10 μV	BJT input
411	50 pA	25 pA	0.8 mV	FET input

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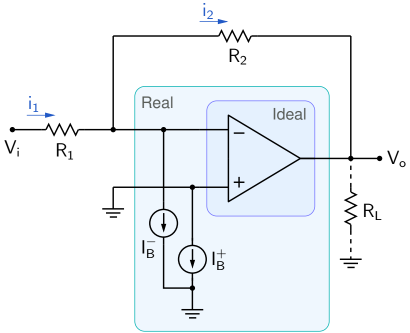
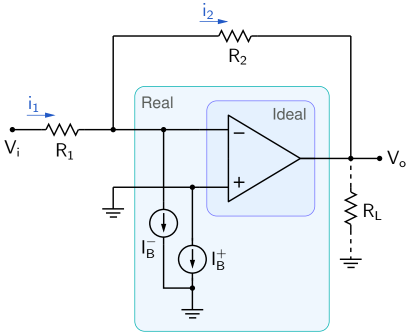


i2
R2
i1
Real
Ideal
Vi
R1
Vo
RL
I−B
I+B

Effect of bias currents: inverting amplifier



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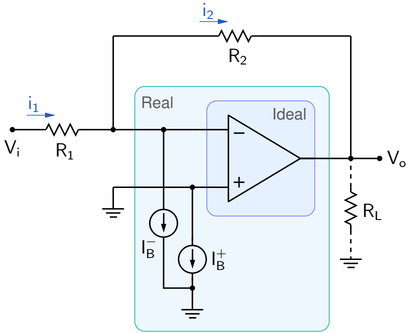

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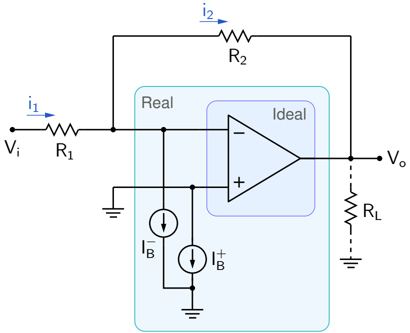

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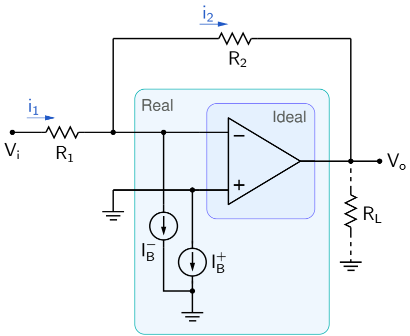

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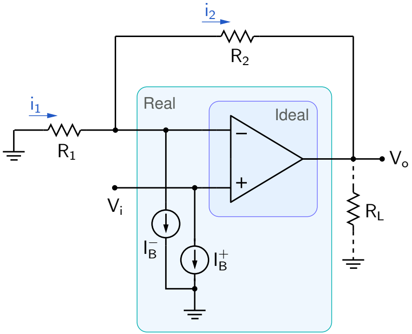
i.e., the bias current causes a DC shift in V_o.

For $I_B^- = 80\,\text{n}A$, $R_2 = 10\,\text{k}$, $\Delta V_o = 0.8\,\text{m}V$.

Effect of bias currents: non-inverting amplifier



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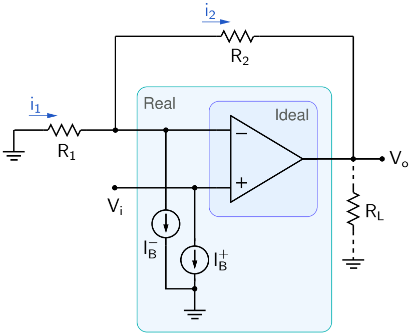

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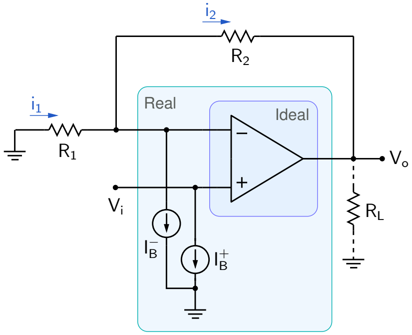

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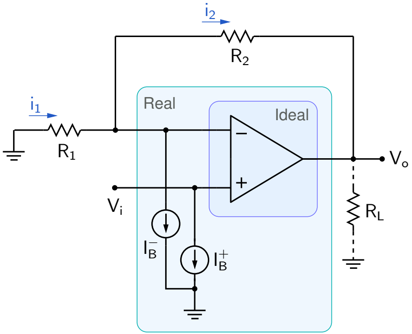

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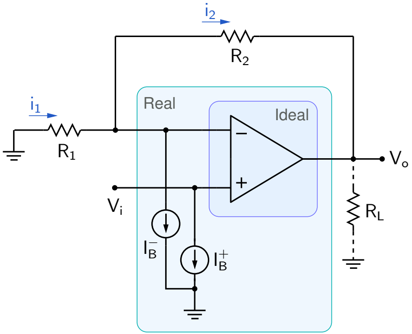

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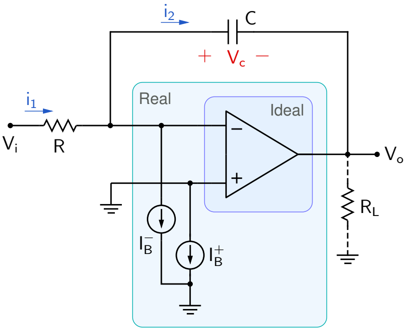
$\rightarrow$ Again, a DC shift ΔV_o.

Effect of bias currents: integrator



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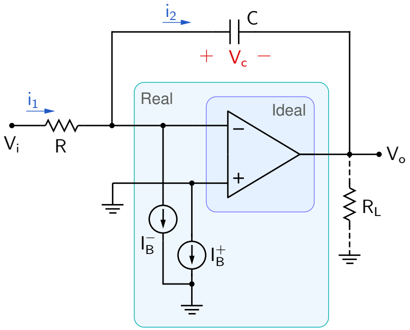

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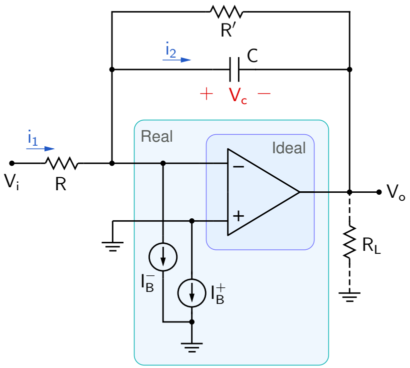

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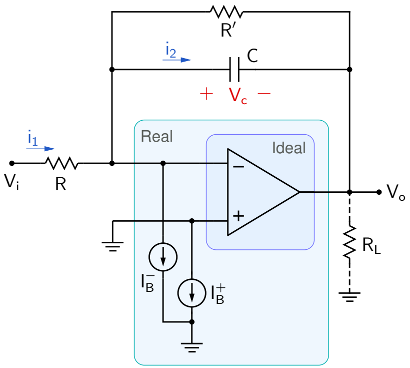

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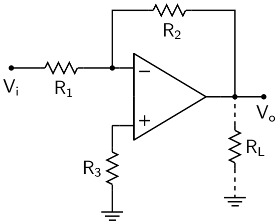

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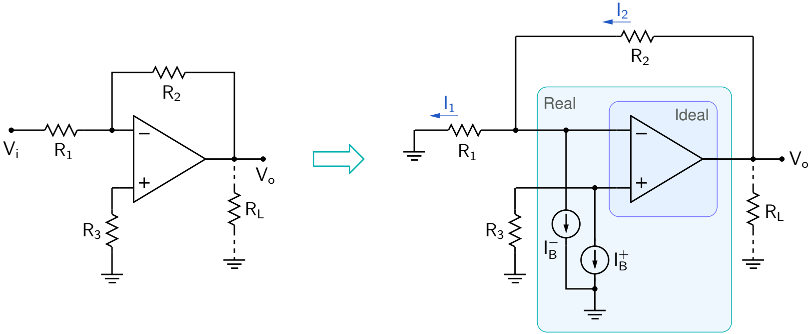
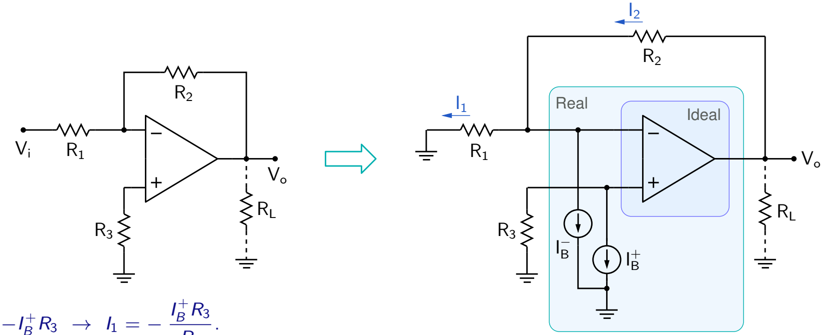
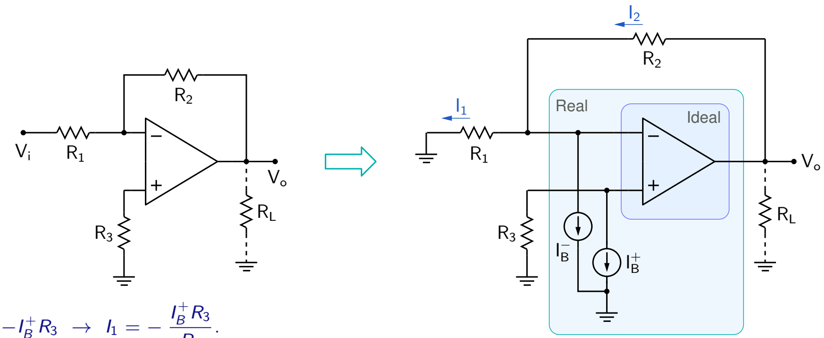




$$V_- \approx V_+ = -I_B^+ R_3 \;\rightarrow\; I_1 = -\frac{I_B^+ R_3}{R_1}.$$



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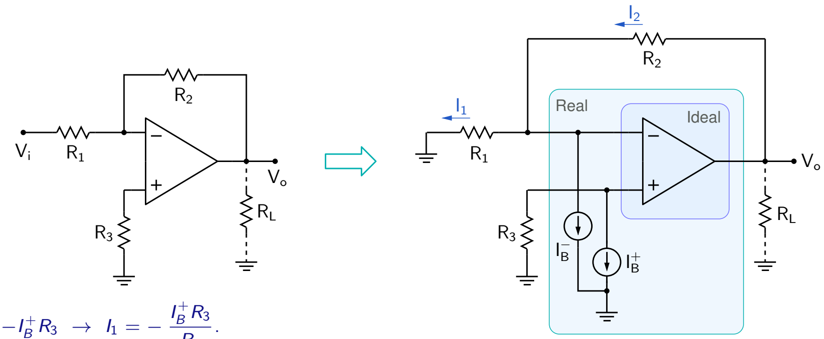

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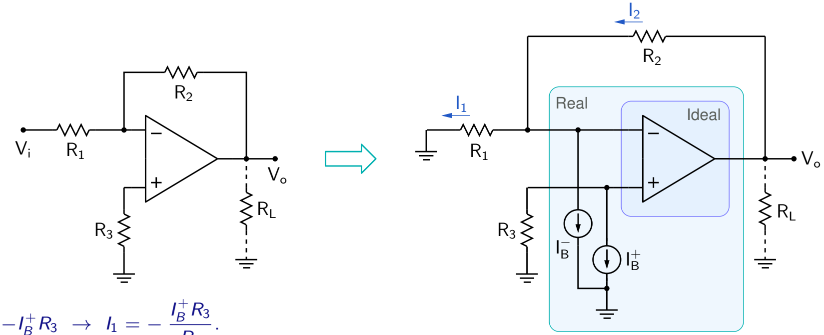

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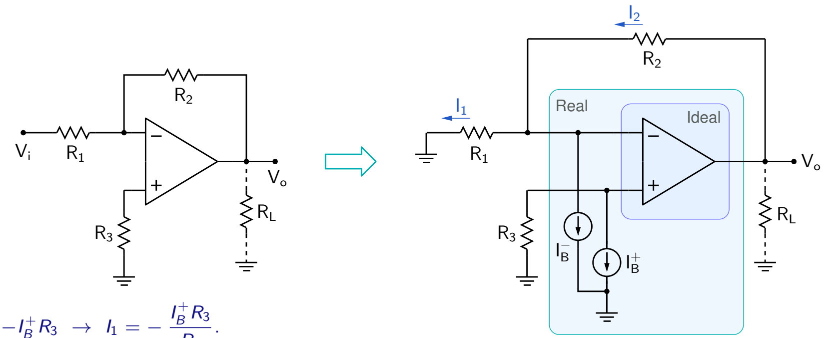
$$V_o = -R_3 \left(1 + \frac{R_2}{R_1}\right) \left(I_B + \frac{I_{OS}}{2}\right) + R_2 \left(I_B - \frac{I_{OS}}{2}\right) = \left(1 + \frac{R_2}{R_1}\right) \left\{ \left[(R_1 \,\|\, R_2) - R_3 \right] I_B - \left[(R_1 \,\|\, R_2) + R_3 \right] \frac{I_{OS}}{2} \right\}$$

Effect of bias currents: inverting amplifier



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The first term can be made zero if we select $R_3 = R_1 \,\|\, R_2$.

$\rightarrow V_o = -R_2 I_{OS}$ (Compare with $V_o = R_2 I_B^-$ when R_3 is not connected.)

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* A DC shift is a matter of concern when the output is expected to be a DC (or slowly varying) quantity, e.g., a temperature sensor or a strain gauge circuit.