Phasors

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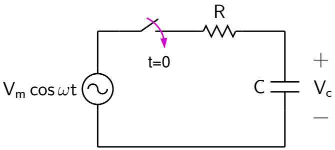
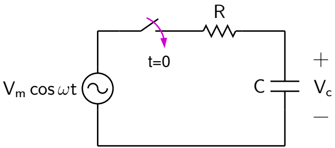
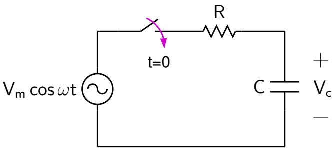

R
t=0
$V_m \cos \omega t$
C
+
V_c
−



$$R\,(C\,V_c') + V_c = V_m\, \cos\, \omega t\,, \qquad t > 0\,. \tag{1}$$



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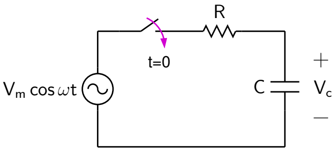
The solution $V_c(t)$ is made up of two components, $V_c(t) = V_c^{(h)}(t) + V_c^{(p)}(t)\,.$



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$V_c^{(h)}(t)$ satisfies the homogeneous differential equation,

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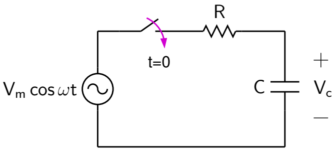
from which, $V_c^{(h)}(t) = A\,\exp(-t/\tau)\,$, with $\tau = RC\,.$

Sinusoidal steady state



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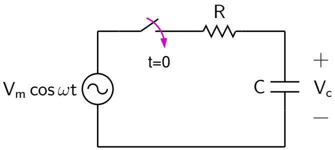

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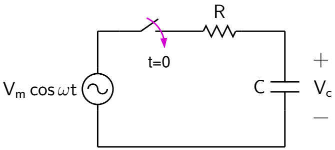
Substituting in (1), we get,

$$\omega R\,C\,(-C_1 \sin \omega t + C_2 \cos \omega t) + C_1 \cos \omega t + C_2 \sin \omega t = V_m \cos \omega t\,.$$



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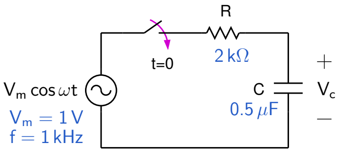
$$\omega R\,C\,(-C_1\,\sin\,\omega t + C_2\,\cos\,\omega t) + C_1\,\cos\,\omega t + C_2\,\sin\,\omega t = V_m\,\cos\,\omega t\,.$$

C_1 and C_2 can be found by equating the coefficients of sin ωt and cos ωt on the left and right sides.

Sinusoidal steady state



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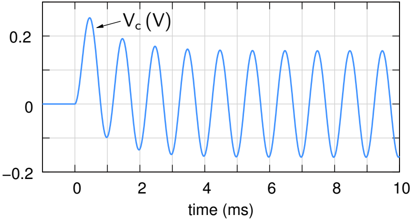

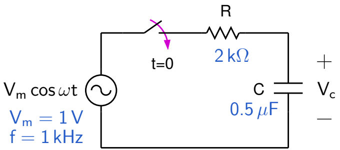

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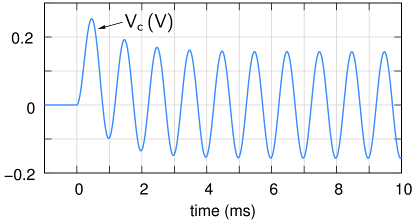

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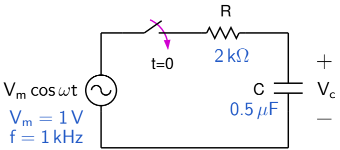

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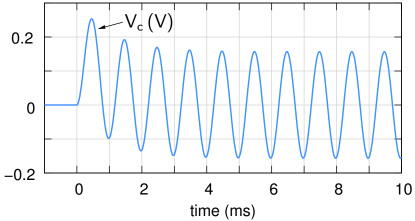

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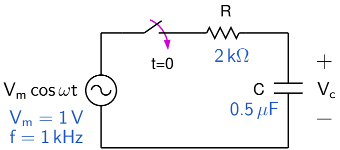

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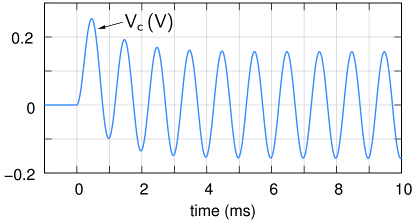

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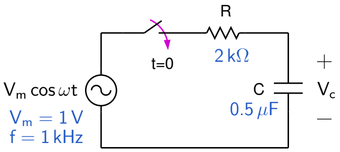

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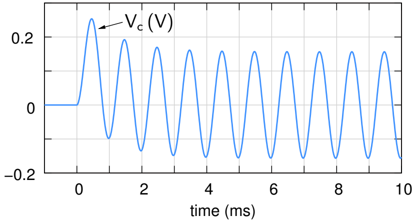

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* This is known as the "sinusoidal steady state" response since all quantities (currents and voltages) in the circuit are sinusoidal in nature.
* Any circuit containing resistors, capacitors, inductors, sinusoidal voltage and current sources (of the same frequency), dependent (linear) sources behaves in a similar manner, viz., each current and voltage in the circuit becomes purely sinusoidal as $t \rightarrow \infty$.

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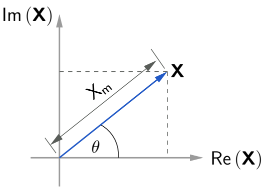
* Use of phasors substantially simplifies analysis of circuits in the sinusoidal steady state.
* Note that a phasor can be written in the polar form or rectangular form,
  $\mathbf{X} = X_m \angle\theta = X_m \, \exp(j\theta) = X_m \cos\theta + j\, X_m \sin\theta$.
  The term ωt is always *implicit*.



Phasors: examples

Time domain	Frequency domain
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$i_3(t) = \sqrt{2}\cos(\omega t + 45^\circ)$ A	$I_3 = 1 + j1$ A $= \sqrt{2}\angle 45^\circ$ A

Consider addition of two sinusoidal quantities:

$$
\begin{aligned}
v(t) &= v_1(t) + v_2(t) \\
&= V_{m1} \cos(\omega t + \theta_1) + V_{m2} \cos(\omega t + \theta_2)
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In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$\tilde{v}(t) = Re\left[\mathbf{V} e^{j\omega t}\right]$$

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In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$\tilde{v}(t) = Re\left[\mathbf{V}e^{j\omega t}\right]$$
$$= Re\left[\left(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}\right)e^{j\omega t}\right]$$

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$$= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)$$

Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$$
$$= V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}$$

In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$\tilde{v}(t) = Re\left[\mathbf{V}e^{j\omega t}\right]$$
$$= Re\left[\left(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}\right)e^{j\omega t}\right]$$
$$= Re\left[V_{m1}e^{j(\omega t + \theta_1)} + V_{m2}e^{j(\omega t + \theta_2)}\right]$$

Consider addition of two sinusoidal quantities:

$$
\begin{aligned}
v(t) &= v_1(t) + v_2(t) \\
&= V_{m1}\,\cos(\omega t + \theta_1) + V_{m2}\,\cos(\omega t + \theta_2)
\end{aligned}
$$

Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$
\begin{aligned}
\mathbf{V} &= \mathbf{V_1} + \mathbf{V_2} \\
&= V_{m1} e^{j\theta_1} + V_{m2} e^{j\theta_2}
\end{aligned}
$$

In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$
\begin{aligned}
\tilde{v}(t) &= Re\left[\mathbf{V} e^{j\omega t}\right] \\
&= Re\left[\left(V_{m1} e^{j\theta_1} + V_{m2} e^{j\theta_2}\right) e^{j\omega t}\right] \\
&= Re\left[V_{m1} e^{j(\omega t+\theta_1)} + V_{m2} e^{j(\omega t+\theta_2)}\right] \\
&= V_{m1}\,\cos(\omega t + \theta_1) + V_{m2}\,\cos(\omega t + \theta_2)
\end{aligned}
$$

Consider addition of two sinusoidal quantities:

$$v(t) = v_1(t) + v_2(t)$$
$$= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)$$

Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

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In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$\tilde{v}(t) = Re\left[\mathbf{V}e^{j\omega t}\right]$$
$$= Re\left[\left(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}\right)e^{j\omega t}\right]$$
$$= Re\left[V_{m1}e^{j(\omega t+\theta_1)} + V_{m2}e^{j(\omega t+\theta_2)}\right]$$
$$= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)$$

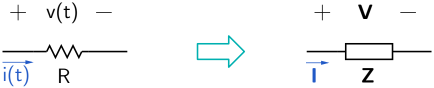
which is the same as $v(t)$.

* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.

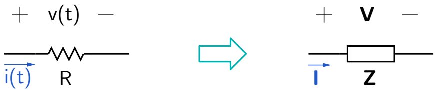
Addition of phasors

* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.
* The KCL and KVL equations,
  $\sum i_k(t) = 0$ at a node, and
  $\sum v_k(t) = 0$ in a loop,
  amount to addition of sinusoidal quantities and can therefore be replaced by the corresponding phasor equations,
  $\sum \mathbf{I}_k = \mathbf{0}$ at a node, and
  $\sum \mathbf{V}_k = \mathbf{0}$ in a loop.

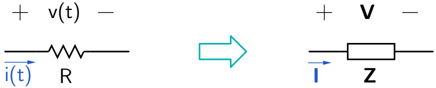
Impedance of a resistor



Impedance of a resistor

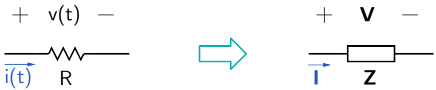


Let $i(t) = I_m \cos(\omega t + \theta)$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

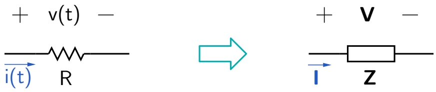
$v(t) = R\, i(t)$



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = R\, i(t)$$
$$= R\, I_m \cos(\omega t + \theta)$$

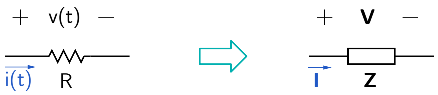
Impedance of a resistor



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$
\begin{aligned}
v(t) &= R\, i(t) \\
&= R\, I_m \cos(\omega t + \theta) \\
&\equiv V_m \cos(\omega t + \theta).
\end{aligned}
$$

Impedance of a resistor

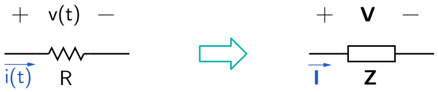


Let $i(t) = I_m \cos(\omega t + \theta)$.

$$
\begin{aligned}
v(t) &= R\, i(t) \\
&= R\, I_m \cos(\omega t + \theta) \\
&\equiv V_m \cos(\omega t + \theta).
\end{aligned}
$$

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

$\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = R \times I_m \angle\theta$.



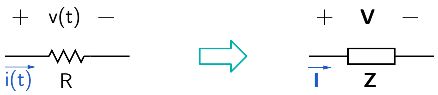
Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = R\, i(t)$$
$$= R\, I_m \cos(\omega t + \theta)$$
$$\equiv V_m \cos(\omega t + \theta).$$

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

$\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = R \times I_m \angle\theta$.

We have therefore the following relationship between **V** and **I**: $\mathbf{V} = R \times \mathbf{I}$.



Let $i(t) = I_m \cos{(\omega t + \theta)}$.

$$\begin{aligned} v(t) &= R\, i(t) \\ &= R\, I_m \cos{(\omega t + \theta)} \\ &\equiv V_m \cos{(\omega t + \theta)}. \end{aligned}$$

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

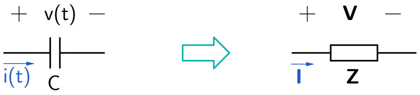
$\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = R \times I_m \angle\theta$.

We have therefore the following relationship between $\mathbf{V}$ and $\mathbf{I}$: $\mathbf{V} = R \times \mathbf{I}$.

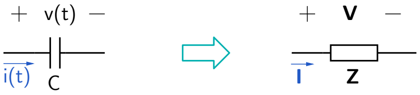
Thus, the *impedance* of a resistor, defined as, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is

$$\boxed{\mathbf{Z} = R + j\,0}$$

Impedance of a capacitor

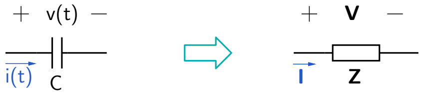


Impedance of a capacitor



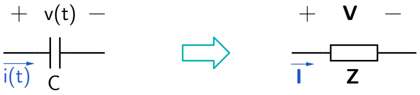
Let $v(t) = V_m \cos(\omega t + \theta)$.

Impedance of a capacitor



Let $v(t) = V_m \cos(\omega t + \theta)$.

$$i(t) = C\,\frac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta).$$



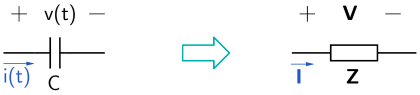
Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$i(t) = C\,\omega\,V_m\ \cos(\omega t + \theta + \pi/2)$.

Impedance of a capacitor



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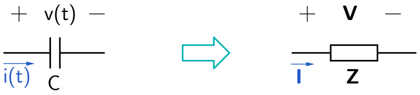
$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$i(t) = C\,\omega\,V_m\,\cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{V} = V_m \angle \theta$, $\mathbf{I} = \omega C V_m \angle (\theta + \pi/2)$.



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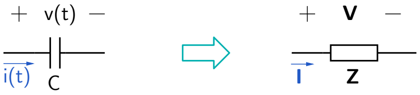
Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$i(t) = C\,\omega\,V_m\,\cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = \omega C V_m \angle(\theta + \pi/2)$.

$\mathbf{I}$ can be rewritten as,

$\mathbf{I} = \omega C V_m\, e^{j(\theta+\pi/2)} = \omega C V_m\, e^{j\theta}\, e^{j\pi/2} = j\omega C\,\left(V_m\, e^{j\theta}\right) = j\omega C\,\mathbf{V}$



Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$i(t) = C\,\omega\,V_m\;\cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = \omega C V_m \angle(\theta+\pi/2)$.

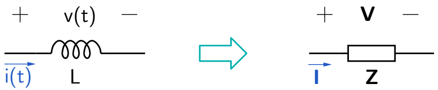
$\mathbf{I}$ can be rewritten as,

$\mathbf{I} = \omega C V_m\, e^{j(\theta+\pi/2)} = \omega C V_m\, e^{j\theta}\, e^{j\pi/2} = j\omega C\;\left(V_m\, e^{j\theta}\right) = j\omega C\,\mathbf{V}$

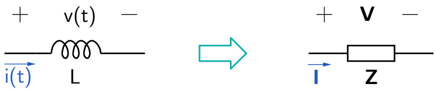
Thus, the *impedance* of a capacitor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = 1/(j\omega C)}$,

and the *admittance* of a capacitor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = j\omega C}$.

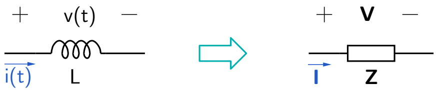
Impedance of an inductor



Impedance of an inductor

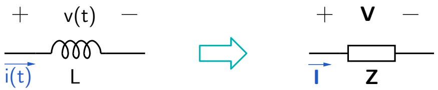


Let $i(t) = I_m \cos(\omega t + \theta)$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = L\,\frac{di}{dt} = -L\,\omega\,I_m \sin(\omega t + \theta).$$



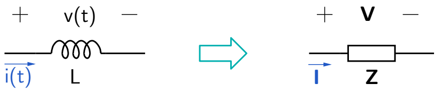
Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = L\,\dfrac{di}{dt} = -L\,\omega\,I_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$v(t) = L\,\omega\,I_m\,\cos(\omega t + \theta + \pi/2)$.

Impedance of an inductor



Let $i(t) = I_m \cos(\omega t + \theta)$.

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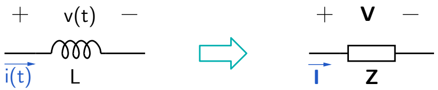
Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$v(t) = L\,\omega\,I_m\ \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{I} = I_m\angle\theta$, $\mathbf{V} = \omega L I_m\ \angle(\theta+\pi/2)$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = L \dfrac{di}{dt} = -L\,\omega\, I_m \sin(\omega t + \theta)$.

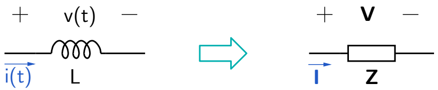
Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$v(t) = L\,\omega\, I_m\ \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = \omega L I_m\ \angle(\theta + \pi/2)$.

$\mathbf{V}$ can be rewritten as,

$\mathbf{V} = \omega L I_m\, e^{j(\theta+\pi/2)} = \omega L I_m\, e^{j\theta}\, e^{j\pi/2} = j\omega L\ \left(I_m\, e^{j\theta}\right) = j\omega L\, \mathbf{I}$



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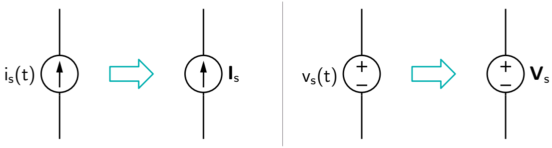
$\mathbf{V}$ can be rewritten as,

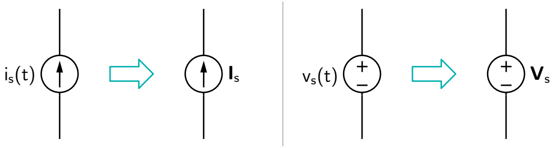
$\mathbf{V} = \omega L I_m\, e^{j(\theta+\pi/2)} = \omega L I_m\, e^{j\theta}\, e^{j\pi/2} = j\omega L\,\left(I_m\, e^{j\theta}\right) = j\omega L\,\mathbf{I}$

Thus, the *impedance* of an indcutor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = j\omega L}$,

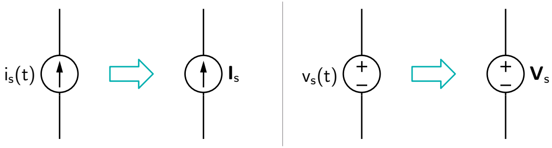
and the *admittance* of an inductor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = 1/(j\omega L)}$.

Sources





* An independent sinusoidal current source, $i_s(t) = I_m \cos(\omega t + \theta)$, can be represented by the phasor $I_m \angle\theta$ (i.e., a *constant* complex number).



* An independent sinusoidal current source, $i_s(t) = I_m \cos(\omega t + \theta)$, can be represented by the phasor $I_m \angle \theta$ (i.e., a *constant* complex number).
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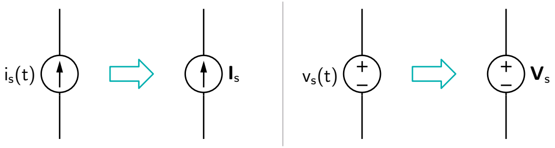

* An independent sinusoidal current source, $i_s(t) = I_m \cos(\omega t + \theta)$, can be represented by the phasor $I_m \angle \theta$ (i.e., a *constant* complex number).
* An independent sinusoidal voltage source, $v_s(t) = V_m \cos(\omega t + \theta)$, can be represented by the phasor $V_m \angle \theta$ (i.e., a *constant* complex number).
* Dependent (linear) sources can be treated in the sinusoidal steady state in the same manner as a resistor, i.e., by the corresponding phasor relationship.
  For example, for a CCVS, we have,
  $v(t) = r\, i_c(t)$ in the time domain.
  $\mathbf{V} = r\, \mathbf{I_c}$ in the frequency domain.

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.

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* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).

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* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = \textit{constant}$ (a complex number).

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* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = \textit{constant}$ (a complex number).
* For dependent sources, a time-domain relationship such as $i(t) = \beta\, i_c(t)$ translates to $\mathbf{I} = \beta\,\mathbf{I_c}$ in the frequency domain.

Use of phasors in circuit analysis

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.
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* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.

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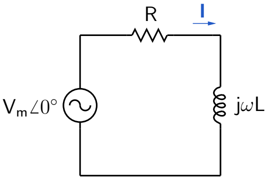
* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.
* Series/parallel formulas for resistors, nodal analysis, mesh analysis, Thevenin's and Norton's theorems can be directly applied to circuits in the sinusoidal steady state.

RL circuit



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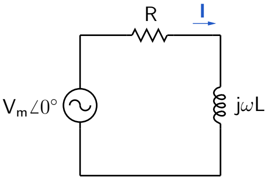
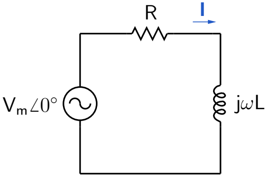


$$\mathbf{I} = \frac{V_m\angle 0}{R + j\omega L} \equiv I_m\angle(-\theta),$$

where $I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$, and $\theta = \tan^{-1}(\omega L/R)$.

RL circuit



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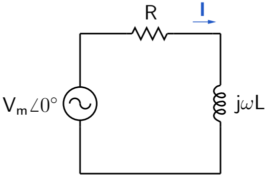
In the time domain, $i(t) = I_m \cos(\omega t - \theta)$, which *lags* the source voltage since the peak (or zero) of $i(t)$ occurs $t = \theta/\omega$ seconds after that of the source voltage.



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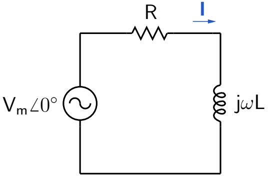
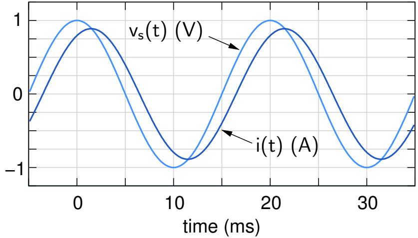
For $R = 1\,\Omega$, $L = 1.6\,\text{mH}$, $f = 50\,\text{Hz}$, $\theta = 26.6^\circ$, $t_{\text{lag}} = 1.48$ ms.

(SEQUEL file: `ee101_rl_ac_1.sqproj`)





R = 1 Ω
L = 1.6 mH

$\mathbf{I} = \dfrac{V_m\angle 0}{R + j\omega L} \equiv I_m\angle(-\theta)$,

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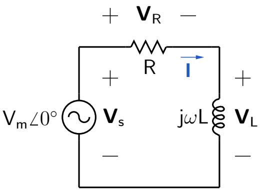
RL circuit



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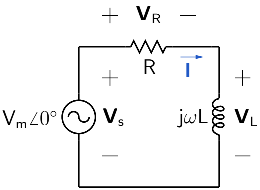

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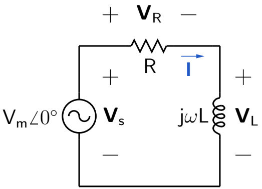
RL circuit



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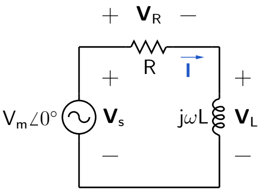
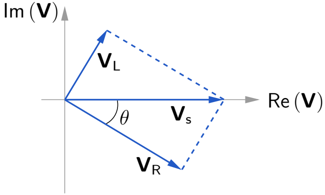
The KVL equation, $\mathbf{V_s} = \mathbf{V_R} + \mathbf{V_L}$, can be represented in the complex plane by a “phasor diagram.”





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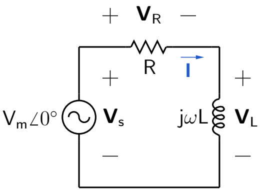

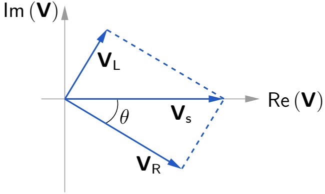

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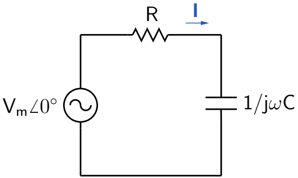
RC circuit



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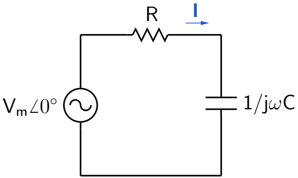

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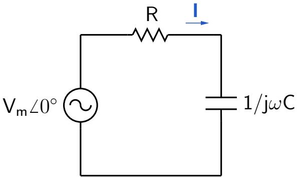

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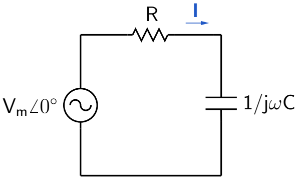

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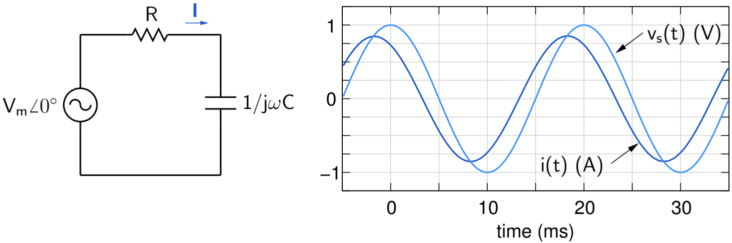
For $R = 1\,\Omega$, $C = 5.3\,\text{mF}$, $f = 50\,\text{Hz}$, $\theta = 31^\circ$, $t_{\text{lead}} = 1.72$ ms.

(SEQUEL file: `ee101_rc_ac_1.sqproj`)



R = 1 Ω
C = 5.3 mF

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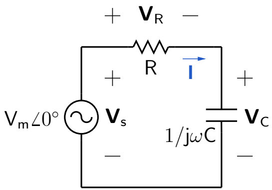

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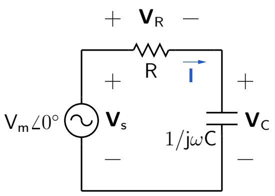

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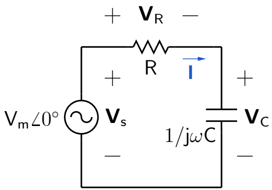
RC circuit



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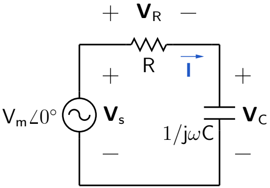

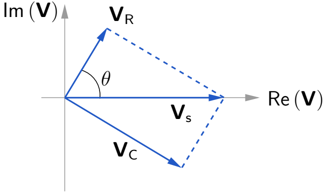

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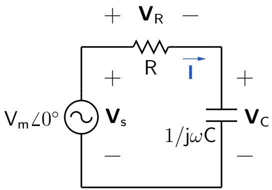

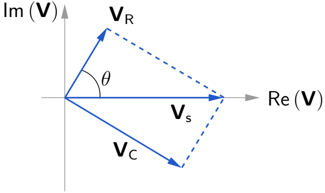

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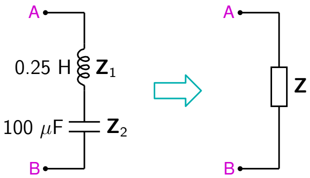
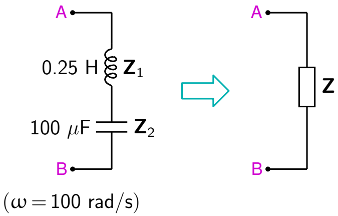
Series/parallel connections



$(\omega = 100 \text{ rad/s})$

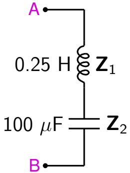
Series/parallel connections



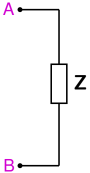
$$\mathbf{Z}_1 = \mathrm{j} \times 100 \times 0.25 = \mathrm{j}\,25\,\Omega$$

$$\mathbf{Z}_2 = -\mathrm{j}/(100 \times 100 \times 10^{-6}) = -\mathrm{j}\,100\,\Omega$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = -\mathrm{j}\,75\,\Omega$$



$(\omega = 100$ rad/s$)$



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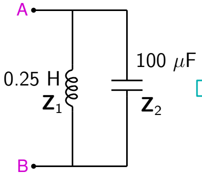

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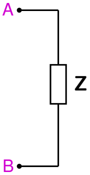

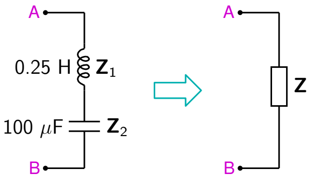

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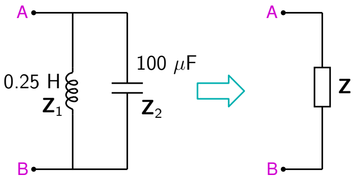


($\omega = 100$ rad/s)

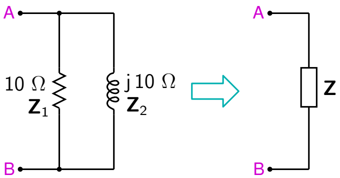
$$\mathbf{Z} = \frac{\mathbf{Z}_1 \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$$= \frac{(\mathrm{j}\,25) \times (-\mathrm{j}\,100)}{\mathrm{j}\,25 - \mathrm{j}\,100}$$

$$= \frac{25 \times 100}{-\mathrm{j}\,75}$$

$$= \mathrm{j}\,33.3\,\Omega$$

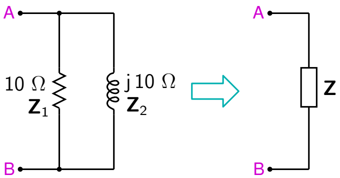
Obtain Z in polar form.



$(\omega = 100 \text{ rad/s})$

Impedance example

Obtain Z in polar form.



$(\omega = 100$ rad/s$)$

Method 1:

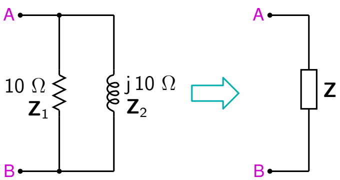
$$\mathbf{Z} = \frac{10 \times \mathrm{j}10}{10 + \mathrm{j}10} = \frac{\mathrm{j}10}{1+\mathrm{j}}$$

$$= \frac{\mathrm{j}10}{1+\mathrm{j}} \times \frac{1-\mathrm{j}}{1-\mathrm{j}}$$

$$= \frac{10+\mathrm{j}10}{2} = 5 + \mathrm{j}5\,\Omega$$

Convert to polar form $\rightarrow \mathbf{Z} = 7.07 \angle 45^\circ\,\Omega$

Obtain Z in polar form.



$(\omega = 100 \text{ rad/s})$

Method 1:

$$\mathbf{Z} = \frac{10 \times j10}{10 + j10} = \frac{j10}{1 + j}$$

$$= \frac{j10}{1 + j} \times \frac{1 - j}{1 - j}$$

$$= \frac{10 + j10}{2} = 5 + j5\,\Omega$$

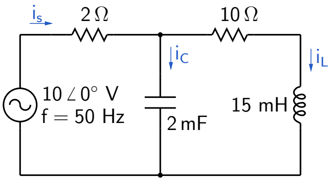
Convert to polar form $\rightarrow \mathbf{Z} = 7.07\,\angle\,45^\circ\,\Omega$

Method 2:

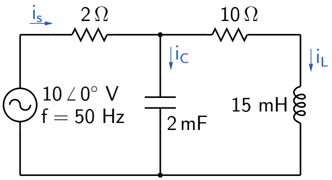
$$\mathbf{Z} = \frac{10 \times j10}{10 + j10} = \frac{100\,\angle\,\pi/2}{10\sqrt{2}\,\angle\,\pi/4}$$

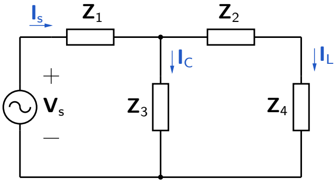
$$= 5\sqrt{2}\,\angle\,(\pi/2 - \pi/4) = 7.07\,\angle\,45^\circ\,\Omega$$

Circuit example

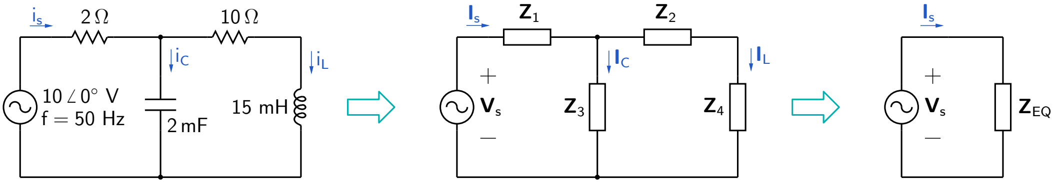


Circuit example

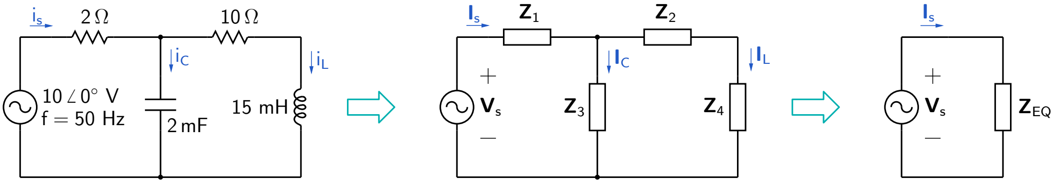




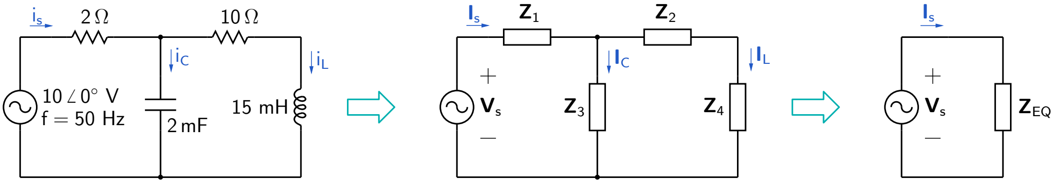
Circuit example



Circuit example

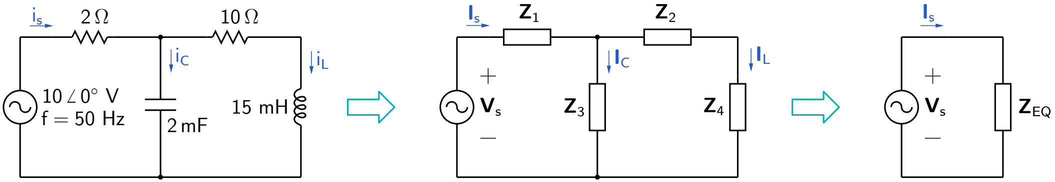


$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = j\,2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$



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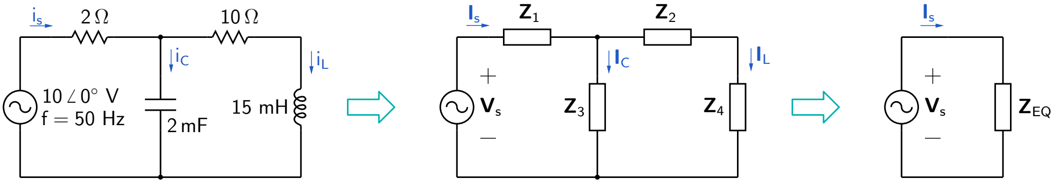
$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

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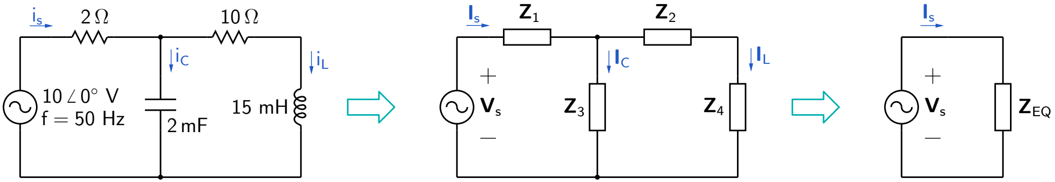
$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

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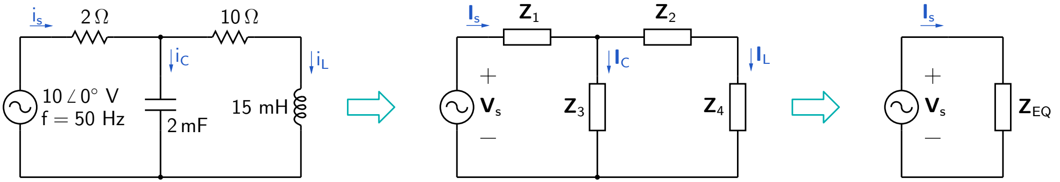
$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

$$= 2 + \frac{1.6\angle\,(-90^\circ) \times 11.05\angle\,(25.2^\circ)}{10.47\angle\,(17.2^\circ)} = 2 + \frac{17.7\angle\,(-64.8^\circ)}{10.47\angle\,(17.2^\circ)}$$

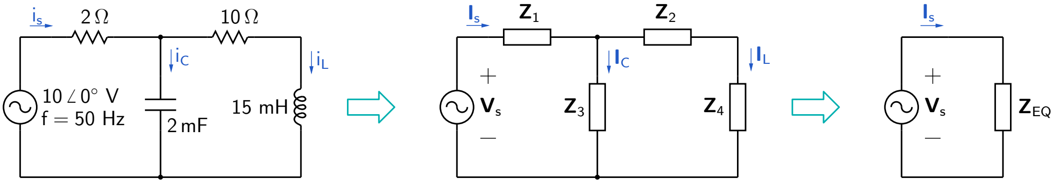


$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = j\,2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\begin{aligned}
\mathbf{Z}_{EQ} &= \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4) \\
&= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7} \\
&= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)} \\
&= 2 + 1.69\angle(-82^\circ) = 2 + (0.235 - j\,1.67)
\end{aligned}$$

Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

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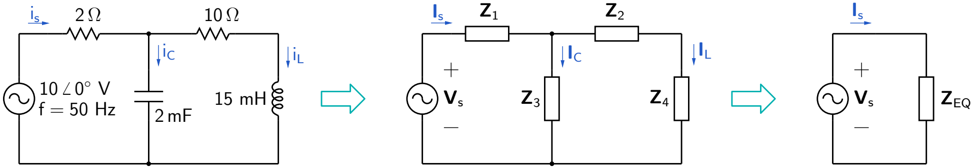
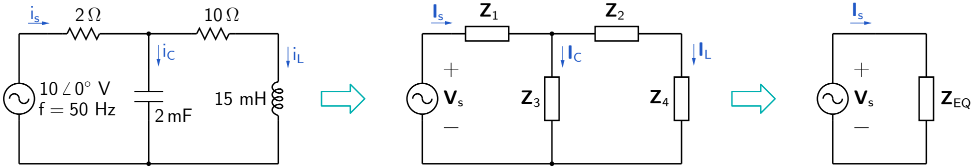
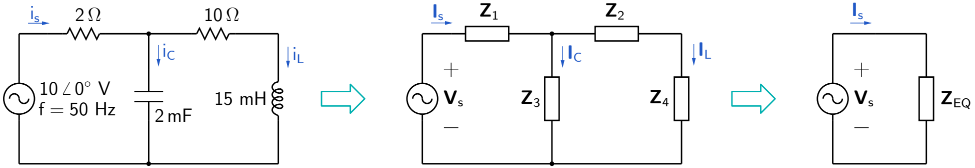
$$\begin{aligned}
\mathbf{Z}_{EQ} &= \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4) \\
&= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7} \\
&= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)} \\
&= 2 + 1.69\angle(-82^\circ) = 2 + (0.235 - j\,1.67) \\
&= 2.235 - j\,1.67 = 2.79\angle(-36.8^\circ)\ \Omega
\end{aligned}$$


i_s
2 Ω
10 Ω
i_C
i_L
10 ∠ 0° V
f = 50 Hz
2 mF
15 mH
I_s
Z_1
Z_2
I_C
I_L
+
V_s
−
Z_3
Z_4
I_s
+
V_s
−
Z_EQ

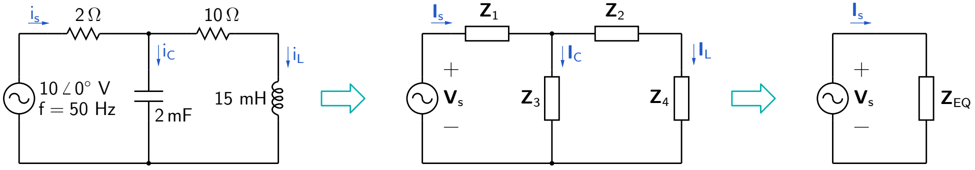


$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

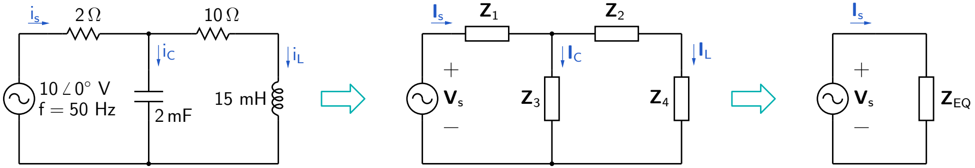


$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_3}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 0.546\angle(-70.6^\circ)\ A$$

Circuit example (continued)



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\,\angle\,(0^\circ)}{2.79\,\angle\,(-36.8^\circ)} = 3.58\,\angle\,(36.8^\circ)\;A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\,\angle\,(44.6^\circ)\;A$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_3}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 0.546\,\angle\,(-70.6^\circ)\;A$$