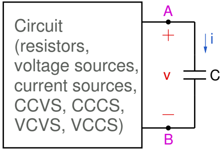
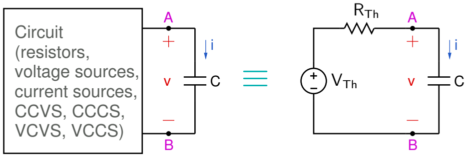
RC and *RL* Circuits with Piecewise Constant Sources

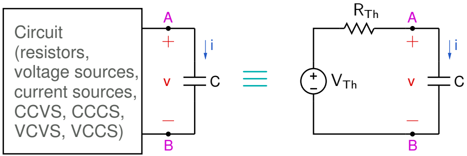
M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay

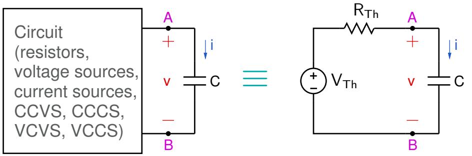
RC circuits with DC sources



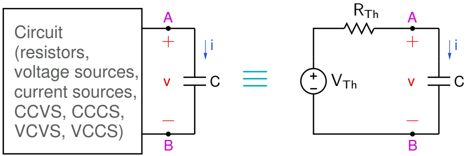

Circuit
(resistors,
voltage sources,
current sources,
CCVS, CCCS,
VCVS, VCCS)
A
B
i
v
C
R_{Th}
V_{Th}
A
B
i
v
C



* If all sources are DC (constant), $V_{Th} = \text{constant}$.

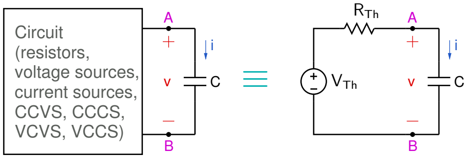


- $*$ If all sources are DC (constant), $V_{Th} = \text{constant}$.
- $*$ KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C \dfrac{dv}{dt} + v \rightarrow \dfrac{dv}{dt} + \dfrac{v}{R_{Th} C} = \dfrac{V_{Th}}{R_{Th} C}$.



* If all sources are DC (constant), V_{Th} = constant .
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th}C\,\dfrac{dv}{dt} + v \rightarrow \dfrac{dv}{dt} + \dfrac{v}{R_{Th}C} = \dfrac{V_{Th}}{R_{Th}C}$.
* Homogeneous solution:

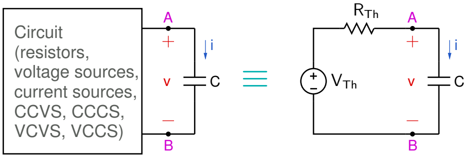
$\dfrac{dv}{dt} + \dfrac{1}{\tau}\,v = 0$, where $\tau = R_{Th}\,C$ is the "time constant." $\left(\dfrac{\text{V}}{\text{Coul/sec}} \times \dfrac{\text{Coul}}{V}\right)$



* If all sources are DC (constant), $V_{Th} =$ constant .
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th}C\,\dfrac{dv}{dt} + v \rightarrow \dfrac{dv}{dt} + \dfrac{v}{R_{Th}C} = \dfrac{V_{Th}}{R_{Th}C}$.
* Homogeneous solution:

$$\frac{dv}{dt} + \frac{1}{\tau}\,v = 0\,,\ \text{where } \tau = R_{Th}\,C \text{ is the “time constant.”} \left(\frac{\text{V}}{\text{Coul/sec}} \times \frac{\text{Coul}}{V}\right)$$

$$\rightarrow\ \frac{dv}{v} = -\,\frac{dt}{\tau}\ \rightarrow\ \log\, v = -\,\frac{t}{\tau} + K_0\ \rightarrow\ v^{(h)} = \exp\left[(-t/\tau) + K_0\right] = K\, \exp(-t/\tau)\,.$$

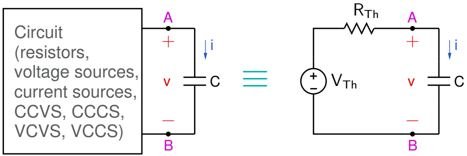


* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C \dfrac{dv}{dt} + v \rightarrow \dfrac{dv}{dt} + \dfrac{v}{R_{Th} C} = \dfrac{V_{Th}}{R_{Th} C}$.
* Homogeneous solution:

$$\frac{dv}{dt} + \frac{1}{\tau}\, v = 0\,, \text{ where } \tau = R_{Th}\, C \text{ is the ``time constant.''} \left(\frac{\text{V}}{\text{Coul/sec}} \times \frac{\text{Coul}}{V}\right)$$

$$\rightarrow \frac{dv}{v} = -\frac{dt}{\tau} \rightarrow \log\, v = -\frac{t}{\tau} + K_0 \rightarrow v^{(h)} = \exp\left[(-t/\tau) + K_0\right] = K\, \exp(-t/\tau)\,.$$

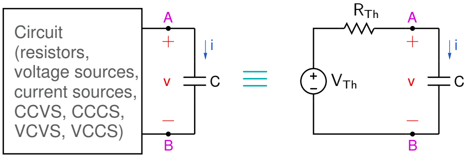
* Particular solution is a specific function that satisfies the differential equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i = 0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).



* If all sources are DC (constant), $V_{Th} =$ constant .
* KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C\,\dfrac{dv}{dt} + v \rightarrow \dfrac{dv}{dt} + \dfrac{v}{R_{Th} C} = \dfrac{V_{Th}}{R_{Th} C}$.
* Homogeneous solution:

  $\dfrac{dv}{dt} + \dfrac{1}{\tau}\, v = 0$, where $\tau = R_{Th}\, C$ is the "time constant." $\left(\dfrac{\text{V}}{\text{Coul/sec}} \times \dfrac{\text{Coul}}{V}\right)$

  $\rightarrow\ \dfrac{dv}{v} = -\dfrac{dt}{\tau}\ \rightarrow\ \log\, v = -\dfrac{t}{\tau} + K_0\ \rightarrow\ v^{(h)} = \exp\left[(-t/\tau) + K_0\right] = K\, \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differential equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i = 0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).
* $v = v^{(h)} + v^{(p)} = K\, \exp(-t/\tau) + V_{Th}$.

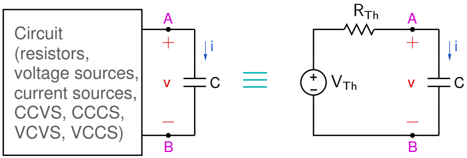


- If all sources are DC (constant), $V_{Th} = \text{constant}$.
- KVL: $V_{Th} = R_{Th}\, i + v \rightarrow V_{Th} = R_{Th} C\,\dfrac{dv}{dt} + v \rightarrow \dfrac{dv}{dt} + \dfrac{v}{R_{Th} C} = \dfrac{V_{Th}}{R_{Th} C}$.
- Homogeneous solution:

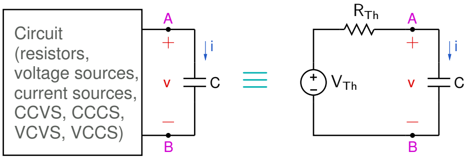
  $\dfrac{dv}{dt} + \dfrac{1}{\tau}\, v = 0$, where $\tau = R_{Th}\, C$ is the "time constant." $\left(\dfrac{\text{V}}{\text{Coul/sec}} \times \dfrac{\text{Coul}}{V}\right)$

  $\rightarrow \dfrac{dv}{v} = -\dfrac{dt}{\tau} \rightarrow \log v = -\dfrac{t}{\tau} + K_0 \rightarrow v^{(h)} = \exp\left[(-t/\tau) + K_0\right] = K\, \exp(-t/\tau)$.
- Particular solution is a specific function that satisfies the differential equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $i = 0$, and we get $v^{(p)} = V_{Th}$ as a particular solution (which happens to be simply a constant).
- $v = v^{(h)} + v^{(p)} = K\, \exp(-t/\tau) + V_{Th}$.
- In general, $v(t) = A\, \exp(-t/\tau) + B$, where A and B can be obtained from known conditions on v.

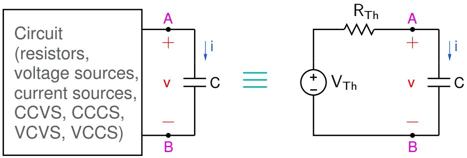
RC circuits with DC sources (continued)



* If all sources are DC (constant), we have
  $v(t) = A\,\exp(-t/\tau) + B\,,\ \tau = R_{Th}C\,.$



* If all sources are DC (constant), we have
  $v(t) = A\,\exp(-t/\tau) + B\,,\ \tau = R_{Th}C\,.$
* $i(t) = C\,\dfrac{dv}{dt} = C \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,\exp(-t/\tau)\,.$

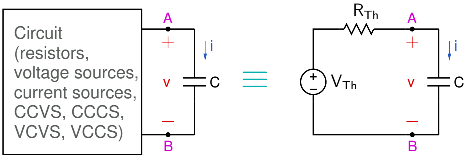


* If all sources are DC (constant), we have
  $v(t) = A\,\exp(-t/\tau) + B\,,\ \tau = R_{Th}C\,.$

* $i(t) = C\,\dfrac{dv}{dt} = C \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,\exp(-t/\tau)\,.$

* As $t \rightarrow \infty$, $i \rightarrow 0$, i.e., the capacitor behaves like an open circuit since all derivatives vanish.

RC circuits with DC sources (continued)



* If all sources are DC (constant), we have
  $v(t) = A\,\exp(-t/\tau) + B\,,\ \tau = R_{Th}C\,.$
* $i(t) = C\,\dfrac{dv}{dt} = C \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,\exp(-t/\tau)\,.$
* As $t \rightarrow \infty$, $i \rightarrow 0$, i.e., the capacitor behaves like an open circuit since all derivatives vanish.
* Since the circuit in the black box is linear, *any* variable (current or voltage) in the circuit can be expressed as
  $x(t) = K_1\,\exp(-t/\tau) + K_2\,,$
  where K_1 and K_2 can be obtained from suitable conditions on $x(t)$.

Plot of $f(t) = e^{-t/\tau}$

t/τ	$e^{-t/\tau}$	$1 - e^{-t/\tau}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933

Plot of $f(t) = e^{-t/\tau}$

t/τ	$e^{-t/\tau}$	$1 - e^{-t/\tau}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933

* For $t/\tau = 5$, $e^{-t/\tau} \simeq 0$, $1 - e^{-t/\tau} \simeq 1$.

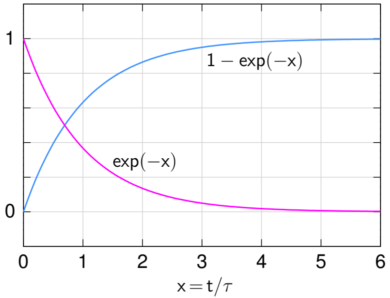
Plot of $f(t) = e^{-t/\tau}$

t/τ	$e^{-t/\tau}$	$1 - e^{-t/\tau}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933

* For $t/\tau = 5$, $e^{-t/\tau} \simeq 0$, $1 - e^{-t/\tau} \simeq 1$.
* We can say that the transient lasts for about 5 time constants.

Plot of $f(t) = e^{-t/\tau}$

t/τ	$e^{-t/\tau}$	$1 - e^{-t/\tau}$
0.0	1.0	0.0
1.0	0.3679	0.6321
2.0	0.1353	0.8647
3.0	4.9787×10^{-2}	0.9502
4.0	1.8315×10^{-2}	0.9817
5.0	6.7379×10^{-3}	0.9933



- For $t/\tau = 5$, $e^{-t/\tau} \simeq 0$, $1 - e^{-t/\tau} \simeq 1$.
- We can say that the transient lasts for about 5 time constants.

Plot of $f(t) = A\,e^{-t/\tau} + B$

Plot of $f(t) = A\,e^{-t/\tau} + B$

* At $t = 0$, $f = A + B$.

Plot of $f(t) = A\, e^{-t/\tau} + B$

* At $t = 0$, $f = A + B$.
* As $t \to \infty$, $f \to B$.

Plot of $f(t) = A\,e^{-t/\tau} + B$

* At $t = 0$, $f = A + B$.
* As $t \rightarrow \infty$, $f \rightarrow B$.
* The graph of $f(t)$ lies between $(A + B)$ and B.
  Note: If $A > 0$, $A + B > B$. If $A < 0$, $A + B < B$.

Plot of $f(t) = A\,e^{-t/\tau} + B$

* At $t = 0$, $f = A + B$.
* As $t \rightarrow \infty$, $f \rightarrow B$.
* The graph of $f(t)$ lies between $(A + B)$ and B.

  Note: If $A > 0$, $A + B > B$. If $A < 0$, $A + B < B$.
* At $t = 0$, $\dfrac{df}{dt} = A\,e^{-t/\tau}\left(-\dfrac{1}{\tau}\right) = -\dfrac{A}{\tau}$.

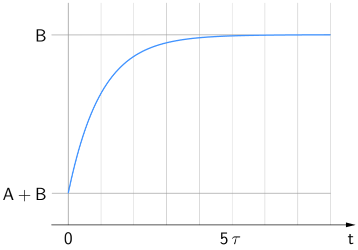
  If $A > 0$, the derivative (slope) at $t = 0$ is negative; else, it is positive.

Plot of $f(t) = A\,e^{-t/\tau} + B$

* At $t = 0$, $f = A + B$.
* As $t \rightarrow \infty$, $f \rightarrow B$.
* The graph of $f(t)$ lies between $(A + B)$ and B.
  Note: If $A > 0$, $A + B > B$. If $A < 0$, $A + B < B$.
* At $t = 0$, $\dfrac{df}{dt} = A\,e^{-t/\tau}\left(-\dfrac{1}{\tau}\right) = -\dfrac{A}{\tau}$.
  If $A > 0$, the derivative (slope) at $t = 0$ is negative; else, it is positive.
* As $t \rightarrow \infty$, $\dfrac{df}{dt} \rightarrow 0$, i.e., f becomes constant (equal to B).

Plot of $f(t) = A\,e^{-t/\tau} + B$

* At $t = 0$, $f = A + B$.
* As $t \rightarrow \infty$, $f \rightarrow B$.
* The graph of $f(t)$ lies between $(A + B)$ and B.
  Note: If $A > 0$, $A + B > B$. If $A < 0$, $A + B < B$.
* At $t = 0$, $\dfrac{df}{dt} = A\,e^{-t/\tau}\left(-\dfrac{1}{\tau}\right) = -\dfrac{A}{\tau}$.
  If $A > 0$, the derivative (slope) at $t = 0$ is negative; else, it is positive.
* As $t \rightarrow \infty$, $\dfrac{df}{dt} \rightarrow 0$, i.e., f becomes constant (equal to B).



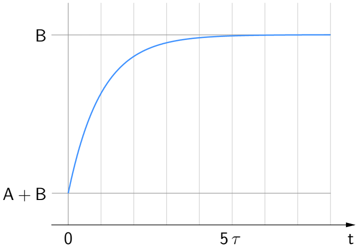
$\boxed{A < 0}$

Plot of $f(t) = A\,e^{-t/\tau} + B$

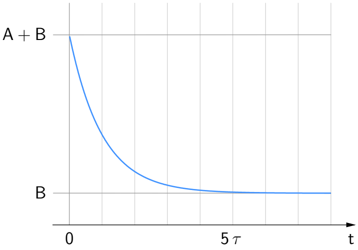
* At $t=0$, $f = A + B$.
* As $t \rightarrow \infty$, $f \rightarrow B$.
* The graph of $f(t)$ lies between $(A+B)$ and B.

  Note: If $A > 0$, $A+B > B$. If $A < 0$, $A+B < B$.
* At $t=0$, $\dfrac{df}{dt} = A\,e^{-t/\tau}\left(-\dfrac{1}{\tau}\right) = -\dfrac{A}{\tau}$.

  If $A > 0$, the derivative (slope) at $t=0$ is negative; else, it is positive.
* As $t \rightarrow \infty$, $\dfrac{df}{dt} \rightarrow 0$, i.e., f becomes constant (equal to B).

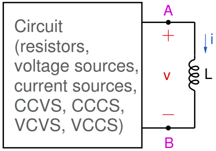


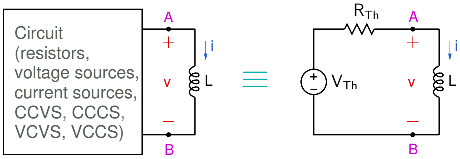
$\boxed{A < 0}$

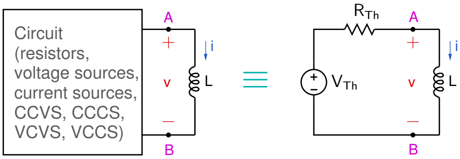


$\boxed{A > 0}$

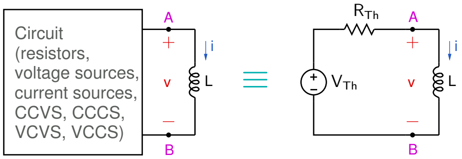
RL circuits with DC sources



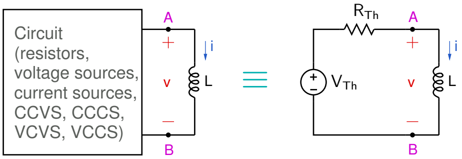

Circuit
(resistors,
voltage sources,
current sources,
CCVS, CCCS,
VCVS, VCCS)
A
B
+
v
−
i
L
R_{Th}
V_{Th}
A
B
+
v
−
i
L



* If all sources are DC (constant), $V_{Th} =$ constant .



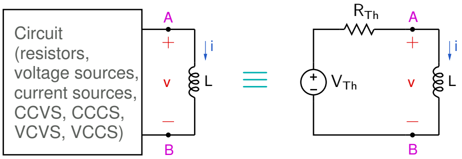
* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.
* Homogeneous solution:

  $\dfrac{di}{dt} + \dfrac{1}{\tau}\, i = 0$, where $\tau = L/R_{Th}$

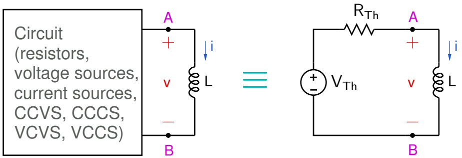
  $\rightarrow i^{(h)} = K\, \exp(-t/\tau)$.



* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.
* Homogeneous solution:

  $\dfrac{di}{dt} + \dfrac{1}{\tau}\, i = 0$, where $\tau = L/R_{Th}$

  $\rightarrow i^{(h)} = K\, \exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differential equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).

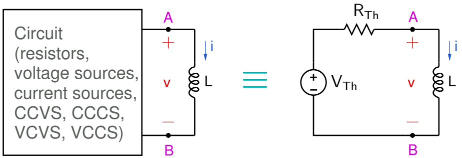


* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.
* Homogeneous solution:

$$\frac{di}{dt} + \frac{1}{\tau}\, i = 0\,,\ \text{where}\ \tau = L/R_{Th}$$

$$\rightarrow i^{(h)} = K\ \exp(-t/\tau)\,.$$

* Particular solution is a specific function that satisfies the differential equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).
* $i = i^{(h)} + i^{(p)} = K\ \exp(-t/\tau) + V_{Th}/R_{Th}$.

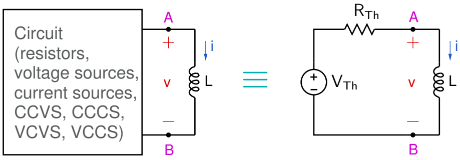


* If all sources are DC (constant), $V_{Th} = \text{constant}$.
* KVL: $V_{Th} = R_{Th}\, i + L\,\dfrac{di}{dt}$.
* Homogeneous solution:

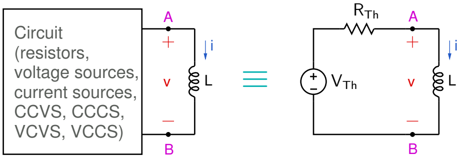
  $\dfrac{di}{dt} + \dfrac{1}{\tau}\, i = 0$, where $\tau = L/R_{Th}$

  $\rightarrow i^{(h)} = K\,\exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differential equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).
* $i = i^{(h)} + i^{(p)} = K\,\exp(-t/\tau) + V_{Th}/R_{Th}$.
* In general, $i(t) = A\,\exp(-t/\tau) + B$, where A and B can be obtained from known conditions on i.

RL circuits with DC sources (continued)

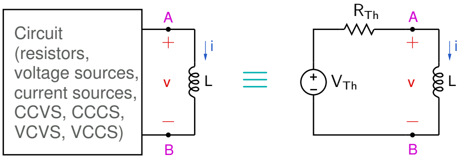


* If all sources are DC (constant), we have
  $i(t) = A\,\exp(-t/\tau) + B$, $\tau = L/R_{Th}$.



* If all sources are DC (constant), we have
  $i(t) = A\,\exp(-t/\tau) + B\,,\ \tau = L/R_{Th}\,.$
* $v(t) = L\,\dfrac{di}{dt} = L \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,\exp(-t/\tau)\,.$

RL circuits with DC sources (continued)

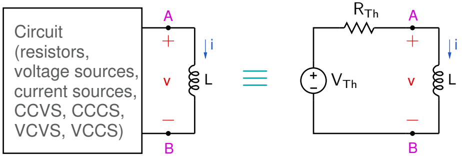


* If all sources are DC (constant), we have
  $i(t) = A\, \exp(-t/\tau) + B\,,\ \tau = L/R_{Th}\,.$

* $v(t) = L\,\dfrac{di}{dt} = L \times A\, \exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\, \exp(-t/\tau)\,.$

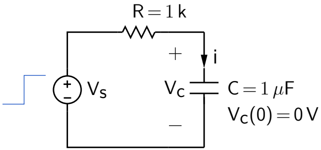
* As $t \rightarrow \infty$, $v \rightarrow 0$, i.e., the inductor behaves like a short circuit since all derivatives vanish.

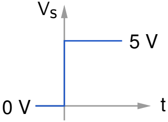
RL circuits with DC sources (continued)



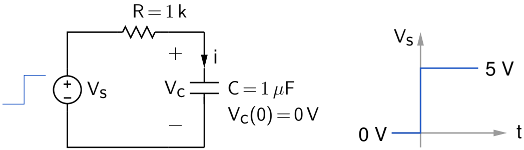
* If all sources are DC (constant), we have
  $i(t) = A\,\exp(-t/\tau) + B\,,\ \tau = L/R_{Th}\,.$
* $v(t) = L\,\dfrac{di}{dt} = L \times A\,\exp(-t/\tau)\left(-\dfrac{1}{\tau}\right) \equiv A'\,\exp(-t/\tau)\,.$
* As $t \rightarrow \infty$, $v \rightarrow 0$, i.e., the inductor behaves like a short circuit since all derivatives vanish.
* Since the circuit in the black box is linear, *any* variable (current or voltage) in the circuit can be expressed as
  $x(t) = K_1\,\exp(-t/\tau) + K_2\,,$
  where K_1 and K_2 can be obtained from suitable conditions on $x(t)$.

RC circuits: Can V_c change "suddenly?"

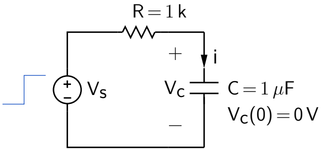


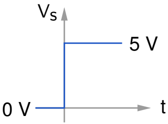


RC circuits: Can V_c change "suddenly?"

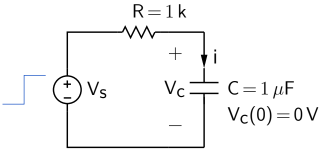


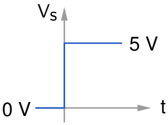
* V_s changes from 0 V (at $t = 0^-$), to 5 V (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?





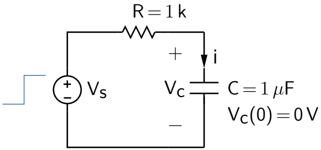
* V_s changes from $0\ V$ (at $t = 0^-$), to $5\ V$ (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by $1\ V$ in $1\,\mu$s at a constant rate of $1\ V/1\,\mu\text{s} = 10^6\ V/\text{s}$?

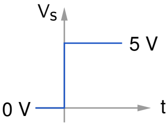




* V_s changes from $0\ V$ (at $t = 0^-$), to $5\ V$ (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by $1\ V$ in $1\ \mu$s at a constant rate of $1\ V/1\ \mu s = 10^6\ V/s$?
* $i = C\,\dfrac{dV_c}{dt} = 1\,\mu F \times 10^6\,\dfrac{V}{s} = 1\,A\,.$

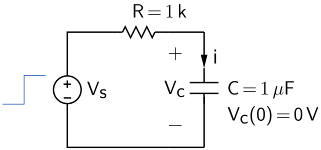
RC circuits: Can V_c change "suddenly?"

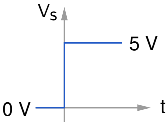




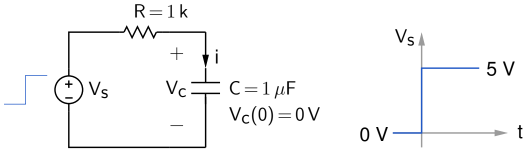
* V_s changes from $0\;V$ (at $t=0^-$), to $5\;V$ (at $t=0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by $1\;V$ in $1\,\mu s$ at a constant rate of $1\;V/1\,\mu s = 10^6\;V/s$?
* $i = C\,\dfrac{dV_c}{dt} = 1\,\mu F \times 10^6\,\dfrac{V}{s} = 1\,A\,.$
* With $i = 1\,A$, the voltage drop across R would be $1000\;V$! Not allowed by KVL.

RC circuits: Can V_c change "suddenly?"



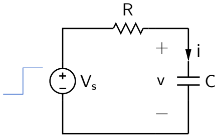


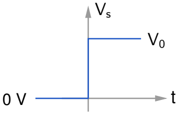
* V_s changes from $0\,V$ (at $t=0^-$), to $5\,V$ (at $t=0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by $1\,V$ in $1\,\mu$s at a constant rate of $1\,V/1\,\mu s = 10^6\,V/s$?
* $i = C\,\dfrac{dV_c}{dt} = 1\,\mu F \times 10^6\,\dfrac{V}{s} = 1\,A\,.$
* With $i = 1\,A$, the voltage drop across R would be $1000\,V$! Not allowed by KVL.
* We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.



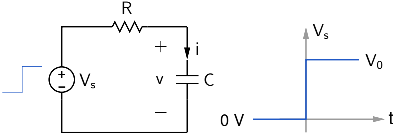
* V_s changes from $0\ V$ (at $t = 0^-$), to $5\ V$ (at $t = 0^+$). As a result of this change, V_c will rise. How fast can V_c change?
* For example, what would happen if V_c changes by $1\ V$ in $1\ \mu s$ at a constant rate of $1\ V/1\ \mu s = 10^6\ V/s$?
* $i = C\,\dfrac{dV_c}{dt} = 1\,\mu F \times 10^6\,\dfrac{V}{s} = 1\,A\,.$
* With $i = 1\,A$, the voltage drop across R would be $1000\ V$! Not allowed by KVL.
* We conclude that $V_c(0^+) = V_c(0^-) \Rightarrow$ A capacitor does not allow abrupt changes in V_c if there is a finite resistance in the circuit.
* Similarly, an inductor does not allow abrupt changes in i_L.

RC circuits: charging and discharging transients

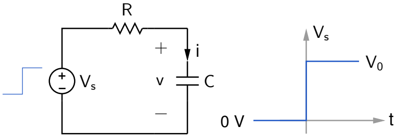




RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

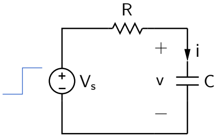
$v(0^+) \simeq v(0^-) = 0$ V

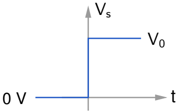
Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty\,, i \to 0\ \to v(\infty) = V_s(\infty) = V_0$

RC circuits: charging and discharging transients





Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

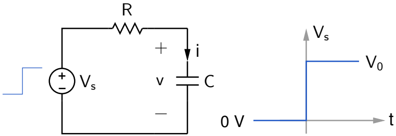
(2) As $t \rightarrow \infty$, $i \rightarrow 0 \rightarrow v(\infty) = V_s(\infty) = V_0$

Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B$,

$t \rightarrow \infty$: $V_0 = B$.

i.e., $B = V_0$, $A = -V_0$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on v(t):

(1) $v(0^-) = V_S(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty$, $i \rightarrow 0 \rightarrow v(\infty) = V_S(\infty) = V_0$

Imposing (1) and (2) on Eq. (A), we get

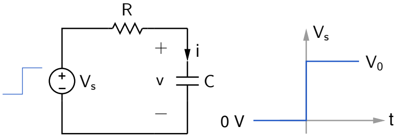
$t = 0^+$: $0 = A + B$,

$t \rightarrow \infty$: $V_0 = B$.

i.e., $B = V_0$, $A = -V_0$

$$\boxed{v(t) = V_0\left[1 - \exp(-t/\tau)\right]}$$

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0 \qquad \text{(A)}$

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty\,, i \rightarrow 0\ \rightarrow v(\infty) = V_s(\infty) = V_0$

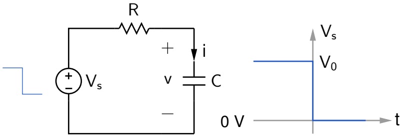
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B\,,$

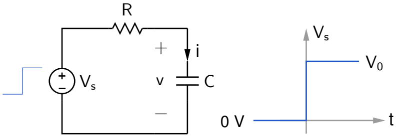
$t \rightarrow \infty$: $V_0 = B\,.$

i.e., $B = V_0\,, A = -V_0$

$$\boxed{v(t) = V_0\left[1 - \exp(-t/\tau)\right]}$$



RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \rightarrow \infty$, $i \rightarrow 0 \rightarrow v(\infty) = V_s(\infty) = V_0$

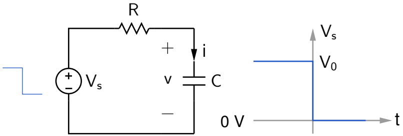
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B$,

$t \rightarrow \infty$: $V_0 = B$.

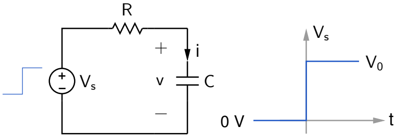
i.e., $B = V_0$, $A = -V_0$

$$\boxed{v(t) = V_0\left[1 - \exp(-t/\tau)\right]}$$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

RC circuits: charging and discharging transients



Let $v(t) = A \exp(-t/\tau) + B, \quad t > 0 \qquad$ (A)

Conditions on v(t):

(1) $v(0^-) = V_s(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty\,, i \to 0\ \to v(\infty) = V_s(\infty) = V_0$

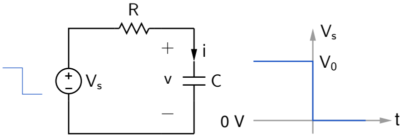
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B\,,$

$t \to \infty$: $V_0 = B\,.$

i.e., $B = V_0\,, A = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$



Let $v(t) = A \exp(-t/\tau) + B, \quad t > 0 \qquad$ (A)

Conditions on v(t):

(1) $v(0^-) = V_s(0^-) = V_0$

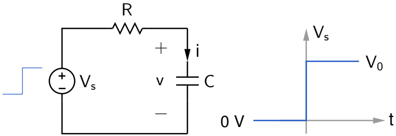
$v(0^+) \simeq v(0^-) = V_0$

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty\,, i \to 0\ \to v(\infty) = V_s(\infty) = 0$ V

RC circuits: charging and discharging transients



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-) because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty$, $i \to 0 \to v(\infty) = V_S(\infty) = V_0$

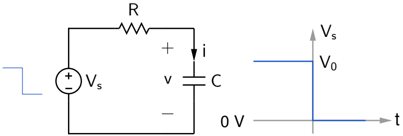
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B$,

$t \to \infty$: $V_0 = B$.

i.e., $B = V_0$, $A = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = V_0$

$v(0^+) \simeq v(0^-) = V_0$

Note that we need the condition at 0^+ (and not at 0^-) because Eq. (A) applies only for $t > 0$.

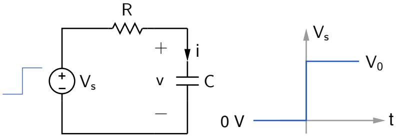
(2) As $t \to \infty$, $i \to 0 \to v(\infty) = V_S(\infty) = 0$ V

Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $V_0 = A + B$,

$t \to \infty$: $0 = B$.

i.e., $A = V_0$, $B = 0$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = 0$ V

$v(0^+) \simeq v(0^-) = 0$ V

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty$, $i \to 0 \;\to v(\infty) = V_S(\infty) = V_0$

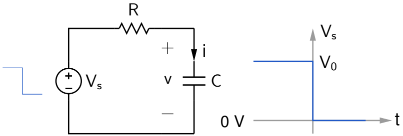
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $0 = A + B$,

$t \to \infty$: $V_0 = B$.

i.e., $B = V_0$, $A = -V_0$

$$\boxed{v(t) = V_0\,[1 - \exp(-t/\tau)]}$$



Let $v(t) = A\exp(-t/\tau) + B, \quad t > 0$ (A)

Conditions on $v(t)$:

(1) $v(0^-) = V_S(0^-) = V_0$

$v(0^+) \simeq v(0^-) = V_0$

Note that we need the condition at 0^+ (and not at 0^-)

because Eq. (A) applies only for $t > 0$.

(2) As $t \to \infty$, $i \to 0 \;\to v(\infty) = V_S(\infty) = 0$ V

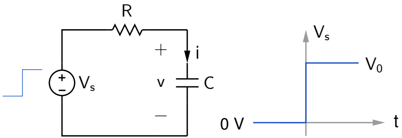
Imposing (1) and (2) on Eq. (A), we get

$t = 0^+$: $V_0 = A + B$,

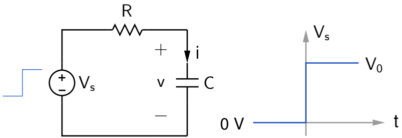
$t \to \infty$: $0 = B$.

i.e., $A = V_0$, $B = 0$

$$\boxed{v(t) = V_0\exp(-t/\tau)}$$



Compute $\mathrm{i}(\mathrm{t})$, $\mathrm{t} > 0$.

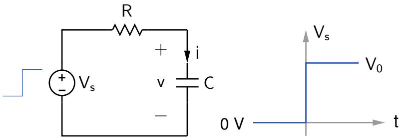


Compute $i(t)$, $t > 0$.

(A) $$i(t) = C\frac{d}{dt}V_0\left[1 - \exp(-t/\tau)\right]$$

$$= \frac{CV_0}{\tau}\exp(-t/\tau) = \frac{V_0}{R}\exp(-t/\tau)$$

RC circuits: charging and discharging transients



Compute $i(t)$, $t > 0$.

(A) $$i(t) = C\,\frac{d}{dt}\,V_0\,[1 - \exp(-t/\tau)]$$

$$= \frac{CV_0}{\tau}\,\exp(-t/\tau) = \frac{V_0}{R}\,\exp(-t/\tau)$$

(B) Let $i(t) = A'\exp(-t/\tau) + B'\,,\quad t > 0\,.$

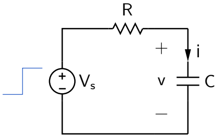
$t = 0^+$: $v = 0\,,\ V_S = V_0\ \Rightarrow i(0^+) = V_0/R\,.$

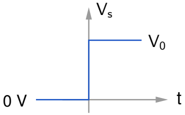
$t \rightarrow \infty$: $i(t) = 0\,.$

Using these conditions, we obtain

$$A' = \frac{V_0}{R}\,,\ \ B' = 0\ \Rightarrow i(t) = \frac{V_0}{R}\,\exp(-t/\tau)$$

RC circuits: charging and discharging transients





Compute $i(t)$, $t > 0$.

(A) $i(t) = C \dfrac{d}{dt} V_0 \left[1 - \exp(-t/\tau)\right]$

$$= \frac{CV_0}{\tau} \exp(-t/\tau) = \frac{V_0}{R} \exp(-t/\tau)$$

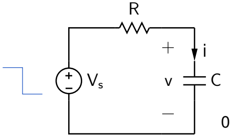
(B) Let $i(t) = A' \exp(-t/\tau) + B'\,, \quad t > 0\,.$

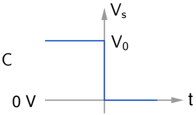
$t = 0^+$: $v = 0\,,\ V_S = V_0 \ \Rightarrow i(0^+) = V_0/R\,.$

$t \rightarrow \infty$: $i(t) = 0\,.$

Using these conditions, we obtain

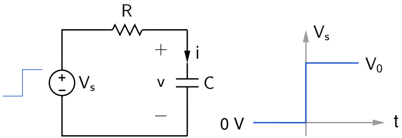
$$A' = \frac{V_0}{R}\,,\ \ B' = 0 \ \Rightarrow i(t) = \frac{V_0}{R} \exp(-t/\tau)$$





Compute $i(t)$, $t > 0$.

RC circuits: charging and discharging transients



Compute $i(t)$, $t > 0$.

(A) $$i(t) = C\,\frac{d}{dt}\,V_0\,[1 - \exp(-t/\tau)]$$

$$= \frac{CV_0}{\tau}\exp(-t/\tau) = \frac{V_0}{R}\exp(-t/\tau)$$

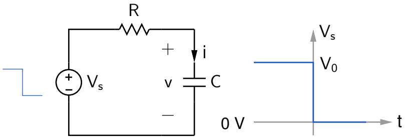
(B) Let $i(t) = A'\exp(-t/\tau) + B'\,,\quad t > 0\,.$

$t = 0^+$: $v = 0\,,\ V_S = V_0\ \Rightarrow i(0^+) = V_0/R\,.$

$t \rightarrow \infty$: $i(t) = 0\,.$

Using these conditions, we obtain

$$A' = \frac{V_0}{R}\,,\ \ B' = 0\ \Rightarrow i(t) = \frac{V_0}{R}\exp(-t/\tau)$$

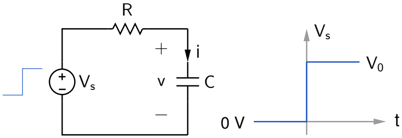


Compute $i(t)$, $t > 0$.

(A) $$i(t) = C\,\frac{d}{dt}\,V_0\,[\exp(-t/\tau)]$$

$$= -\frac{CV_0}{\tau}\exp(-t/\tau) = -\frac{V_0}{R}\exp(-t/\tau)$$

RC circuits: charging and discharging transients



Compute $i(t)$, $t > 0$.

(A) $i(t) = C\dfrac{d}{dt}V_0\left[1 - \exp(-t/\tau)\right]$

$$= \frac{CV_0}{\tau}\exp(-t/\tau) = \frac{V_0}{R}\exp(-t/\tau)$$

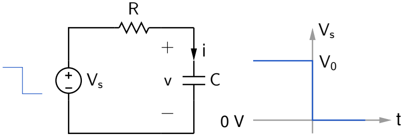
(B) Let $i(t) = A'\exp(-t/\tau) + B'$, $t > 0$.

$t = 0^+$: $v = 0$, $V_S = V_0 \Rightarrow i(0^+) = V_0/R$.

$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = \frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



Compute $i(t)$, $t > 0$.

(A) $i(t) = C\dfrac{d}{dt}V_0\left[\exp(-t/\tau)\right]$

$$= -\frac{CV_0}{\tau}\exp(-t/\tau) = -\frac{V_0}{R}\exp(-t/\tau)$$

(B) Let $i(t) = A'\exp(-t/\tau) + B'$, $t > 0$.

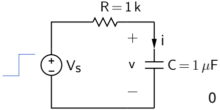
$t = 0^+$: $v = V_0$, $V_S = 0 \Rightarrow i(0^+) = -V_0/R$.

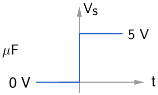
$t \rightarrow \infty$: $i(t) = 0$.

Using these conditions, we obtain

$$A' = -\frac{V_0}{R},\ B' = 0 \Rightarrow i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

RC circuits: charging and discharging transients

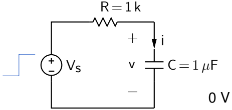


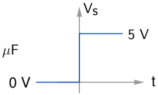


$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$

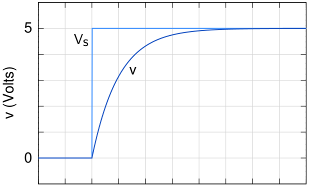
RC circuits: charging and discharging transients



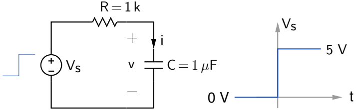


$$v(t) = V_0 \left[1 - \exp(-t/\tau)\right]$$

$$i(t) = \frac{V_0}{R} \exp(-t/\tau)$$

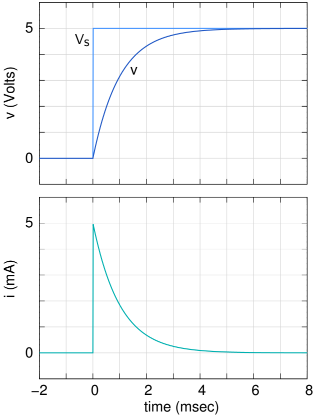


RC circuits: charging and discharging transients

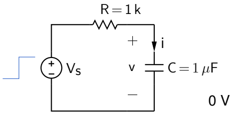


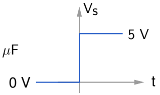
$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



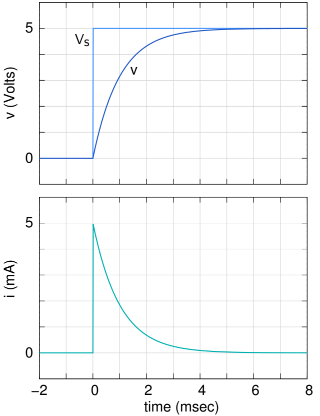
RC circuits: charging and discharging transients

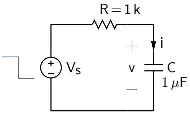


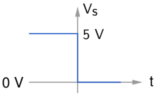


$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



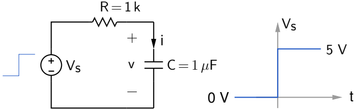




$$v(t) = V_0\exp(-t/\tau)$$

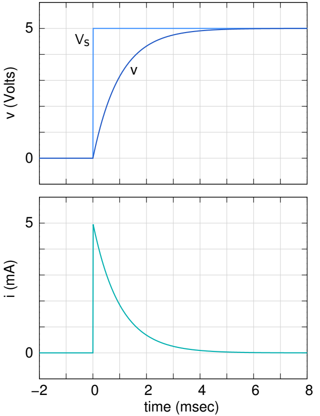
$$i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

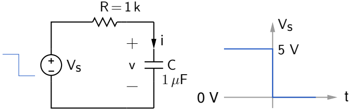
RC circuits: charging and discharging transients



$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

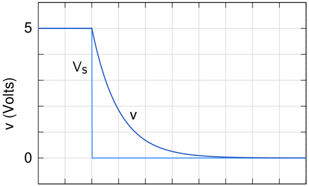
$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



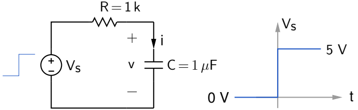


$$v(t) = V_0\exp(-t/\tau)$$

$$i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

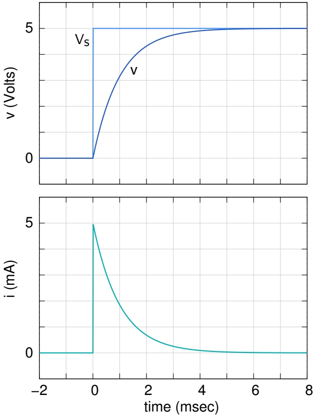


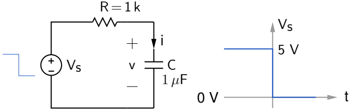
RC circuits: charging and discharging transients



$$v(t) = V_0\,[1 - \exp(-t/\tau)]$$

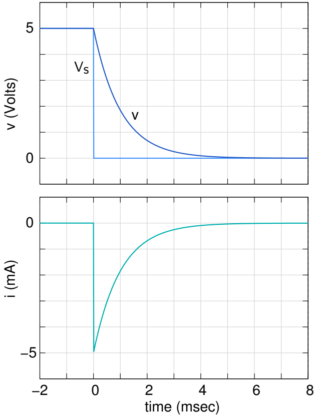
$$i(t) = \frac{V_0}{R}\exp(-t/\tau)$$



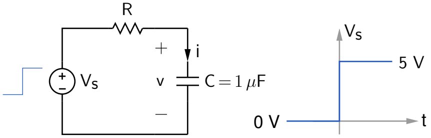


$$v(t) = V_0\exp(-t/\tau)$$

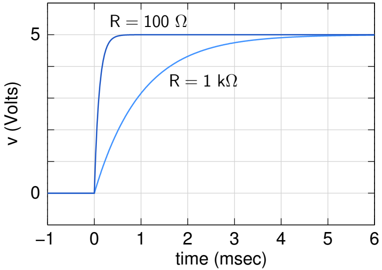
$$i(t) = -\frac{V_0}{R}\exp(-t/\tau)$$

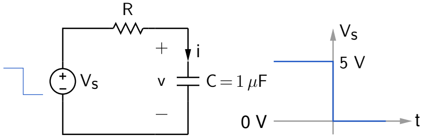


RC circuits: charging and discharging transients

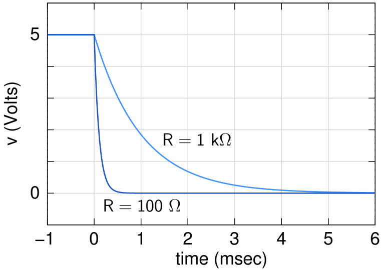


$$v(t) = V_0 \left[1 - \exp(-t/\tau)\right]$$

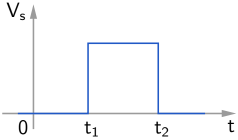




$$v(t) = V_0 \exp(-t/\tau)$$



* Identify intervals in which the source voltages/currents are constant.
  For example,

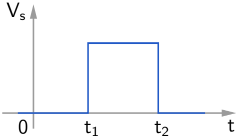


(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

* Identify intervals in which the source voltages/currents are constant.
  For example,



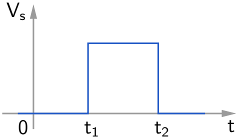
(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
  $x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
  $x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
  $x(t) = A_3 \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Identify intervals in which the source voltages/currents are constant.
  For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

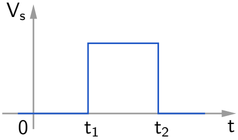
(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
  $x(t) = A_1\ \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
  $x(t) = A_2\ \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
  $x(t) = A_3\ \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Work out suitable conditions on $x(t)$ at specific time points using

Analysis of RC/RL circuits with a piece-wise constant source

* Identify intervals in which the source voltages/currents are constant.
  For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
  $x(t) = A_1 \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
  $x(t) = A_2 \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
  $x(t) = A_3 \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

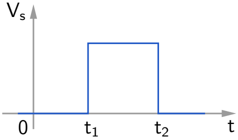
* Work out suitable conditions on $x(t)$ at specific time points using

  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.

  $$\Rightarrow i_C = C\,\frac{dV_c}{dt} = 0\,, \text{ and } V_L = L\,\frac{di_L}{dt} = 0\,.$$

Analysis of RC/RL circuits with a piece-wise constant source

* Identify intervals in which the source voltages/currents are constant.
  For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
  $x(t) = A_1\ \exp(-t/\tau) + B_1\,, \quad t < t_1\,,$
  $x(t) = A_2\ \exp(-t/\tau) + B_2\,, \quad t_1 < t < t_2\,,$
  $x(t) = A_3\ \exp(-t/\tau) + B_3\,, \quad t > t_2\,.$

* Work out suitable conditions on $x(t)$ at specific time points using

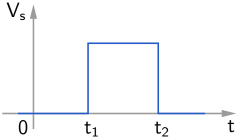
  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.

  $$\Rightarrow i_C = C\,\frac{dV_c}{dt} = 0\,,\ \text{and}\ V_L = L\,\frac{di_L}{dt} = 0\,.$$

  (b) When a source voltage (or current) changes, say, at $t = t_0$, $V_c(t)$ or $i_L(t)$ cannot change abruptly, i.e.,
  $V_c(t_0^+) = V_c(t_0^-)$, and $i_L(t_0^+) = i_L(t_0^-)$.

Analysis of RC/RL circuits with a piece-wise constant source

* Identify intervals in which the source voltages/currents are constant.
  For example,



(1) $t < t_1$

(2) $t_1 < t < t_2$

(3) $t > t_2$

* For *any* current or voltage $x(t)$, write general expressions such as,
  $x(t) = A_1 \exp(-t/\tau) + B_1 , \quad t < t_1 ,$
  $x(t) = A_2 \exp(-t/\tau) + B_2 , \quad t_1 < t < t_2 ,$
  $x(t) = A_3 \exp(-t/\tau) + B_3 , \quad t > t_2 .$

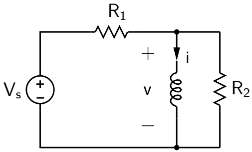
* Work out suitable conditions on $x(t)$ at specific time points using

  (a) If the source voltage/current has not changed for a "long" time (long compared to τ), all derivatives are zero.

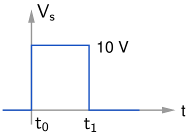
  $$\Rightarrow i_C = C\,\frac{dV_c}{dt} = 0 \,, \text{ and } V_L = L\,\frac{di_L}{dt} = 0 \,.$$

  (b) When a source voltage (or current) changes, say, at $t = t_0$, $V_c(t)$ or $i_L(t)$ cannot change abruptly, i.e.,
  $V_c(t_0^+) = V_c(t_0^-)$, and $i_L(t_0^+) = i_L(t_0^-)$.

* Compute A_1, B_1, $\cdots$ using the conditions on $x(t)$.



Find $i(t)$.



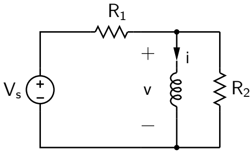
$R_1 = 10\,\Omega$

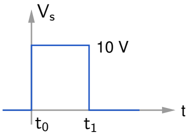
$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$

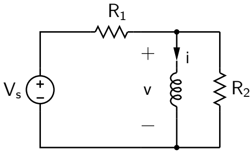
Find $i(t)$.

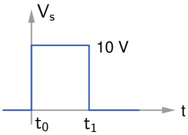
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$

Find i(t).

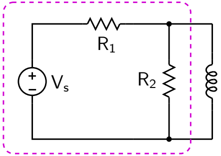
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:

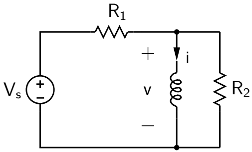


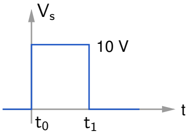
$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

$= 0.8\ \text{H}/8\,\Omega$

$= 0.1\ \text{s}$





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$

Find $i(t)$.

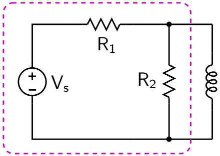
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

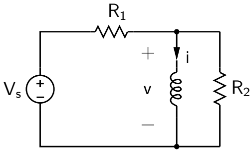
$= 0.8\ \mathrm{H}/8\,\Omega$

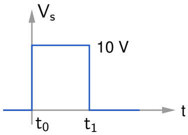
$= 0.1\ \mathrm{s}$

At $t = t_0^-$, $v = 0\ \mathrm{V}$, $V_s = 0\ \mathrm{V}$.

$\Rightarrow i(t_0^-) = 0\ \mathrm{A} \Rightarrow i(t_0^+) = 0\ \mathrm{A}$.

RL circuit: example





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$

Find $i(t)$.

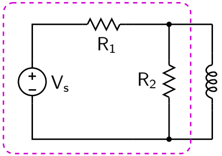
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

$= 0.8\ \text{H}/8\,\Omega$

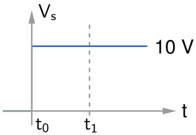
$= 0.1\ \text{s}$

At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V .

$\Rightarrow i(t_0^-) = 0$ A $\Rightarrow i(t_0^+) = 0$ A .

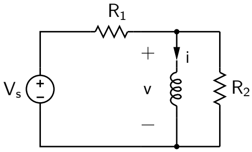
If V_s did not change at $t = t_1$,

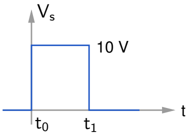
we would have



$v(\infty) = 0$ V, $i(\infty) = 10\ \text{V}/10\ \Omega = 1$ A .

RL circuit: example





$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$

Find $i(t)$.

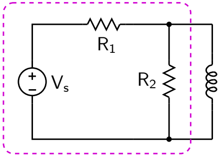
There are three intervals of constant V_s:

(1) $t < t_0$

(2) $t_0 < t < t_1$

(3) $t > t_1$

R_{Th} seen by L is the same in all intervals:



$R_{Th} = R_1 \parallel R_2 = 8\,\Omega$

$\tau = L/R_{Th}$

$= 0.8\ \text{H}/8\,\Omega$

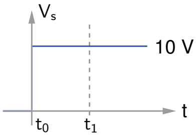
$= 0.1\ \text{s}$

At $t = t_0^-$, $v = 0$ V, $V_s = 0$ V.

$\Rightarrow i(t_0^-) = 0\ \text{A} \Rightarrow i(t_0^+) = 0\ \text{A}$.

If V_s did not change at $t = t_1$,

we would have

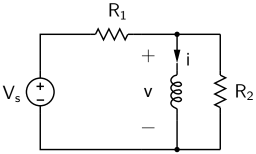


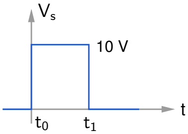
$v(\infty) = 0$ V, $i(\infty) = 10\ \text{V}/10\ \Omega = 1$ A.

Using $i(t_0^+)$ and $i(\infty)$, we can obtain

$i(t),\ t > 0$ (See next slide).

RL circuit: example





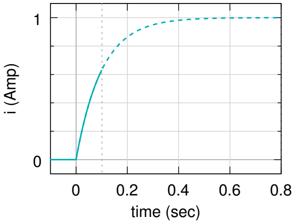
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

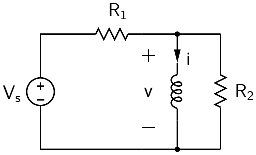
$L = 0.8\,H$

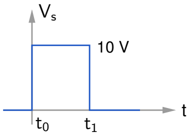
$t_0 = 0$

$t_1 = 0.1\,s$



RL circuit: example





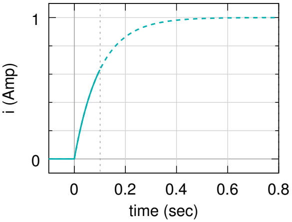
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

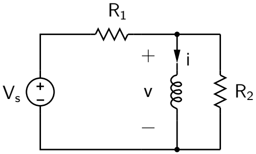
$t_0 = 0$

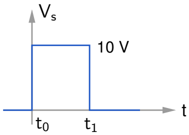
$t_1 = 0.1\,\text{s}$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

RL circuit: example





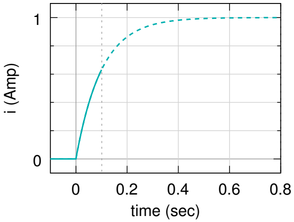
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$

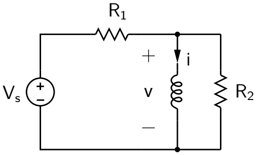


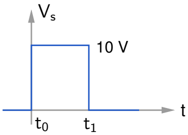
In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

RL circuit: example





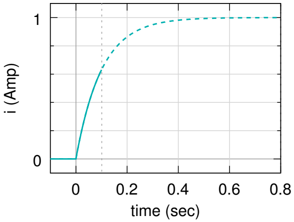
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$



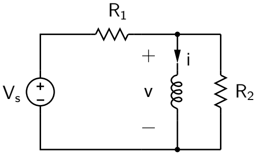
In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

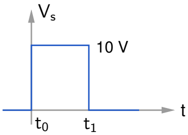
For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

$i(\infty) = 0$ A.





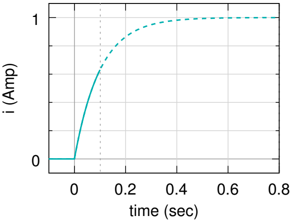
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\text{H}$

$t_0 = 0$

$t_1 = 0.1\,\text{s}$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

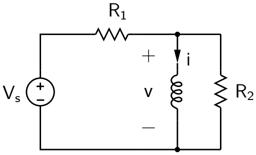
Consider $t > t_1$.

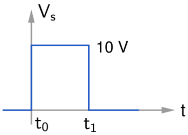
$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

$i(\infty) = 0$ A.

Let $i(t) = A\exp(-t/\tau) + B$.

RL circuit: example





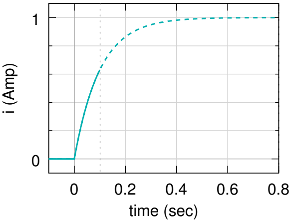
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

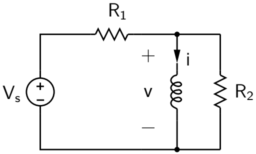
$i(\infty) = 0$ A.

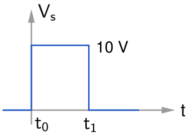
Let $i(t) = A\exp(-t/\tau) + B$.

It is convenient to rewrite $i(t)$ as

$i(t) = A'\exp[-(t - t_1)/\tau] + B$.

RL circuit: example





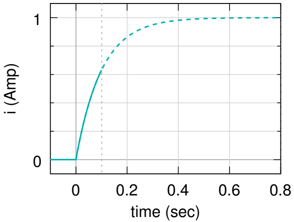
$R_1 = 10\,\Omega$

$R_2 = 40\,\Omega$

$L = 0.8\,\mathrm{H}$

$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$



In reality, V_s changes at $t = t_1$, and we need to work out the solution for $t > t_1$ separately.

For $t_0 < t < t_1$, $i(t) = 1 - \exp(-t/\tau)$ Amp.

Consider $t > t_1$.

$i(t_1^+) = i(t_1^-) = 1 - e^{-1} = 0.632$ A (Note: $t_1/\tau = 1$).

$i(\infty) = 0$ A.

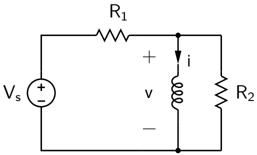
Let $i(t) = A\exp(-t/\tau) + B$.

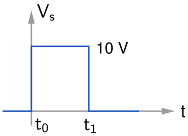
It is convenient to rewrite $i(t)$ as

$i(t) = A'\exp[-(t - t_1)/\tau] + B$.

Using $i(t_1^+)$ and $i(\infty)$, we get

$i(t) = 0.632\exp[-(t - t_1)/\tau]$ A.





$R_1 = 10\,\Omega$

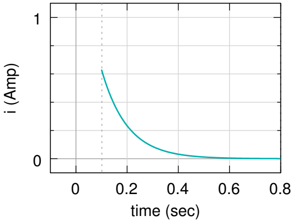
$R_2 = 40\,\Omega$

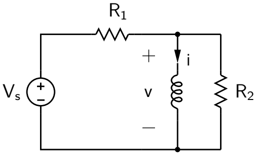
$L = 0.8\,\mathrm{H}$

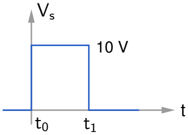
$t_0 = 0$

$t_1 = 0.1\,\mathrm{s}$

$$i(t) = 0.632\,\exp[-(t - t_1)/\tau]\ \text{A}.$$







$\mathsf{R_1} = 10\,\Omega$

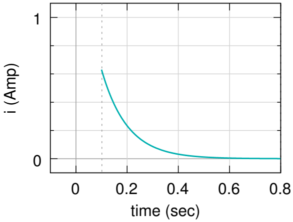
$\mathsf{R_2} = 40\,\Omega$

$\mathsf{L} = 0.8\,\mathsf{H}$

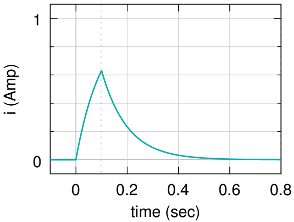
$\mathsf{t_0} = 0$

$\mathsf{t_1} = 0.1\,\mathsf{s}$

$$i(t) = 0.632\,\exp[-(t - t_1)/\tau]\ \text{A}.$$

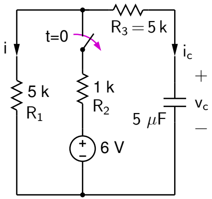


Combining the solutions for $t_0 < t < t_1$ and $t > t_1$, we get

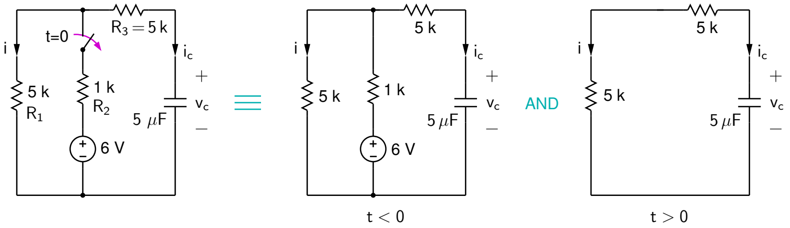


(SEQUEL file: ee101_rl1.sqproj)

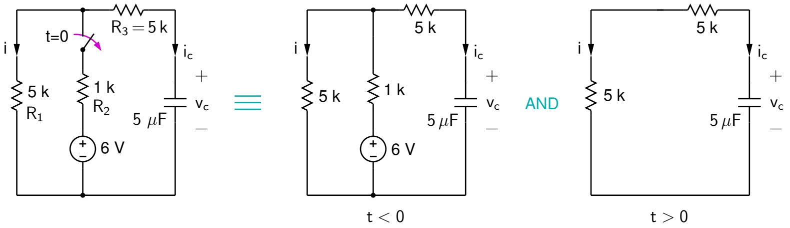
RC circuit: The switch has been closed for a long time and opens at $t = 0$.



RC circuit: The switch has been closed for a long time and opens at $t = 0$.

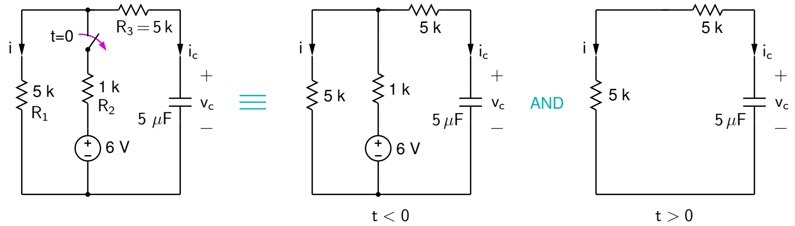


RC circuit: The switch has been closed for a long time and opens at $t = 0$.



t = 0^-: capacitor is an open circuit ⇒ $i(0^-) = 6\ \mathrm{V}/(5\ \mathrm{k} + 1\ \mathrm{k}) = 1$ mA.

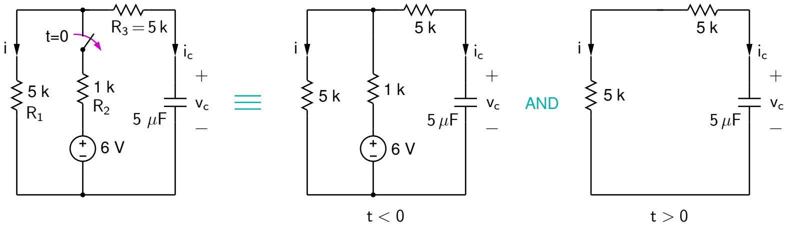
RC circuit: The switch has been closed for a long time and opens at $t = 0$.



$t = 0^-$: capacitor is an open circuit $\Rightarrow$ $i(0^-) = 6\ \text{V}/(5\ \text{k} + 1\ \text{k}) = 1\ \text{mA}$.

$v_c(0^-) = i(0^-)\, R_1 = 5\,\text{V} \Rightarrow v_c(0^+) = 5\,\text{V}$

RC circuit: The switch has been closed for a long time and opens at $t = 0$.

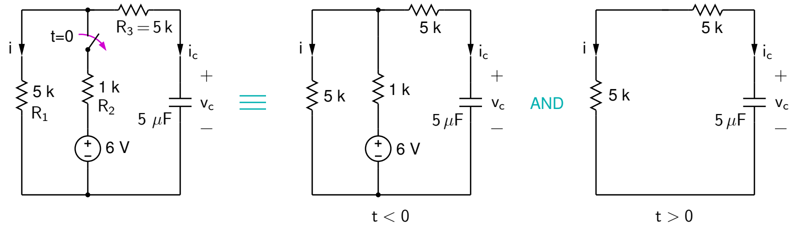


$t = 0^-$: capacitor is an open circuit $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_c(0^-) = i(0^-)\,R_1 = 5\,\text{V} \Rightarrow v_c(0^+) = 5\,\text{V}$

$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

RC circuit: The switch has been closed for a long time and opens at $t = 0$.



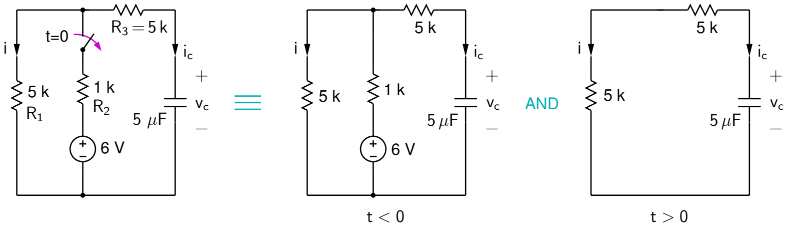
$t = 0^-$: capacitor is an open circuit $\Rightarrow$ $i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_c(0^-) = i(0^-)\,R_1 = 5\,\text{V} \Rightarrow v_c(0^+) = 5\,\text{V}$

$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50\text{ ms}$.

RC circuit: The switch has been closed for a long time and opens at $t = 0$.



$t = 0^-$: capacitor is an open circuit $\Rightarrow$ $i(0^-) = 6\ \mathrm{V}/(5\ \mathrm{k} + 1\ \mathrm{k}) = 1\ \mathrm{mA}$.

$v_c(0^-) = i(0^-)\, R_1 = 5\,\mathrm{V} \Rightarrow v_c(0^+) = 5\,\mathrm{V}$

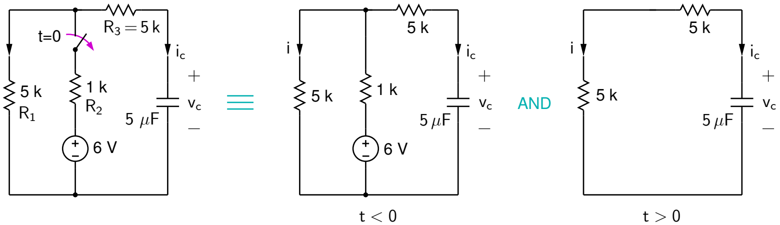
$\Rightarrow i(0^+) = 5\ \mathrm{V}/(5\ \mathrm{k} + 5\ \mathrm{k}) = 0.5\ \mathrm{mA}$.

Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\ \mathrm{k} \times 5\,\mu\mathrm{F} = 50\ \mathrm{ms}$.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get

$i(t) = 0.5\exp(-t/\tau)$ mA.

RC circuit: The switch has been closed for a long time and opens at $t = 0$.



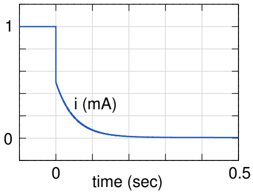
$t = 0^-$: capacitor is an open circuit $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_c(0^-) = i(0^-)\,R_1 = 5\,\text{V} \Rightarrow v_c(0^+) = 5\,\text{V}$

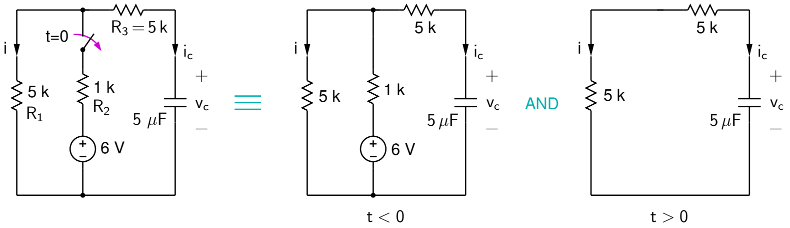
$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

Let $i(t) = A\exp(\text{-}t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50\text{ ms}$.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get
$i(t) = 0.5\exp(\text{-}t/\tau)$ mA.



RC circuit: The switch has been closed for a long time and opens at $t = 0$.



t = 0^-: capacitor is an open circuit $\Rightarrow$ i(0^-) = 6 V/(5 k + 1 k) = 1 mA.

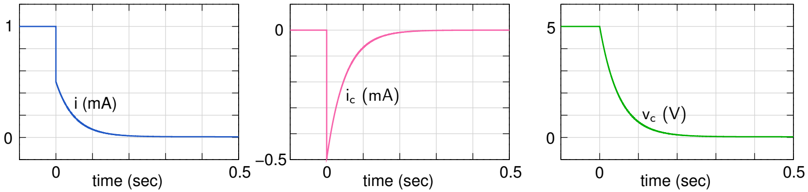
$v_c(0^-) = i(0^-)\,R_1 = 5\,V \Rightarrow v_c(0^+) = 5\,V$

$\Rightarrow$ i(0^+) = 5 V/(5 k + 5 k) = 0.5 mA.

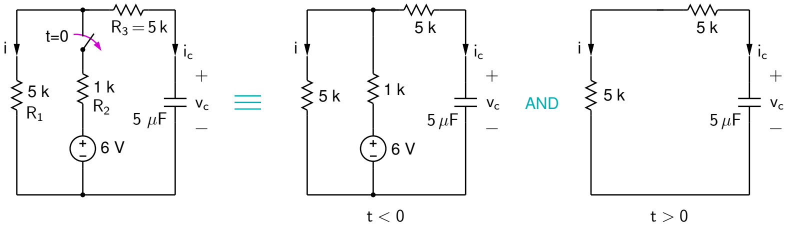
Let i(t) = A exp(-t/τ) + B for t > 0, with τ = 10 k × 5 μF = 50 ms.

Using i(0^+) and i(∞) = 0 A, we get

i(t) = 0.5 exp(-t/τ) mA.



RC circuit: The switch has been closed for a long time and opens at $t = 0$.



$t = 0^-$: capacitor is an open circuit $\Rightarrow i(0^-) = 6\text{ V}/(5\text{ k} + 1\text{ k}) = 1\text{ mA}$.

$v_c(0^-) = i(0^-)\,R_1 = 5\,\text{V} \Rightarrow v_c(0^+) = 5\,\text{V}$

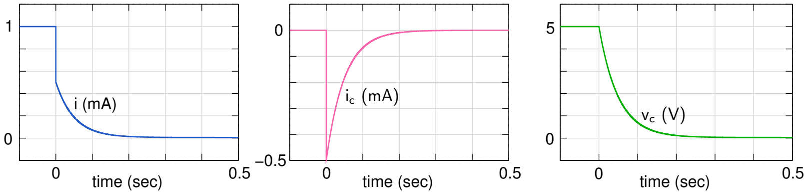
$\Rightarrow i(0^+) = 5\text{ V}/(5\text{ k} + 5\text{ k}) = 0.5\text{ mA}$.

Let $i(t) = A\exp(-t/\tau) + B$ for $t > 0$, with $\tau = 10\text{ k} \times 5\,\mu\text{F} = 50\text{ ms}$.

Using $i(0^+)$ and $i(\infty) = 0$ A, we get

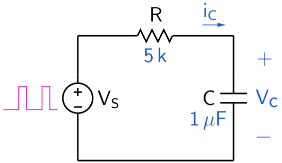
$i(t) = 0.5\exp(-t/\tau)$ mA.

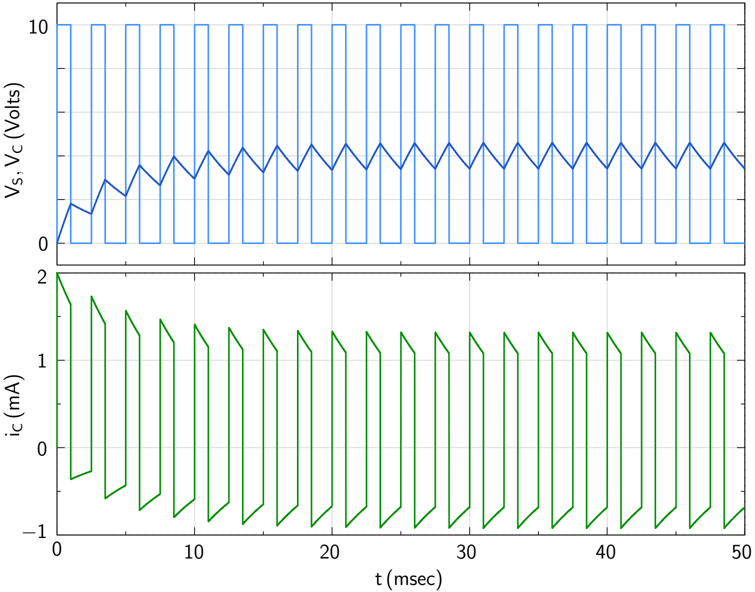
(SEQUEL file: ee101_rc2.sqproj)



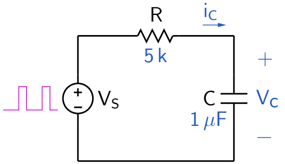
M. B. Patil, IIT Bombay

RC circuit: example

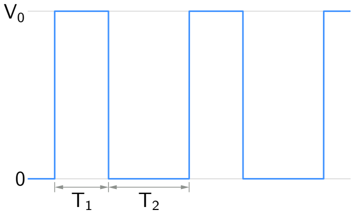


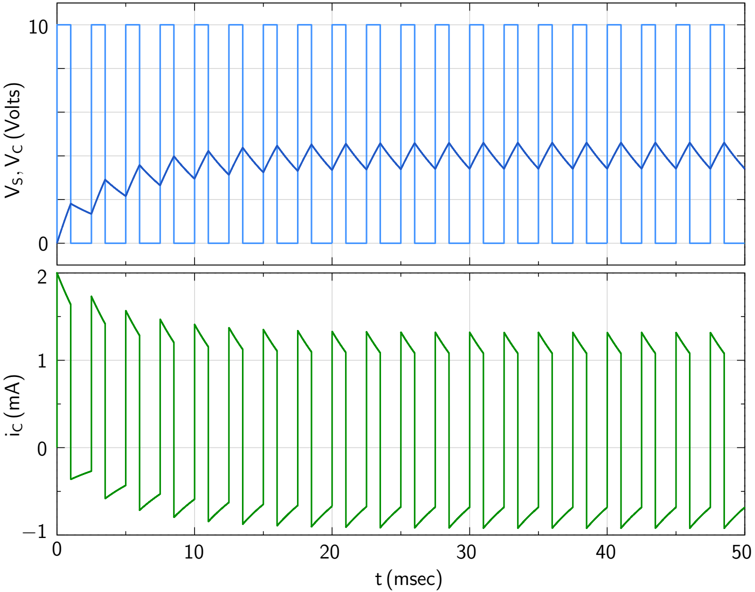


RC circuit: example

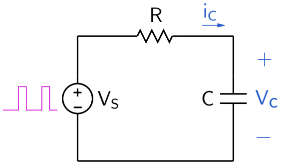


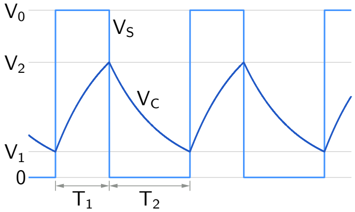
Find expressions for $V_C(t)$ and $i_C(t)$ in steady state (in terms of R, C, V_0, T_1, T_2).

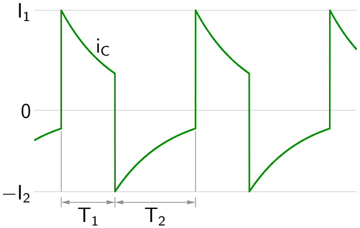




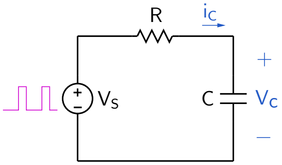
RC circuit: example

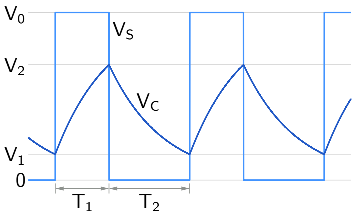


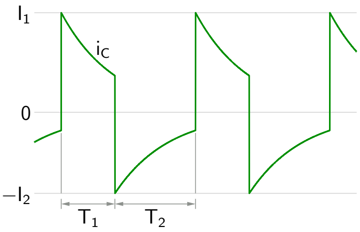




RC circuit: example

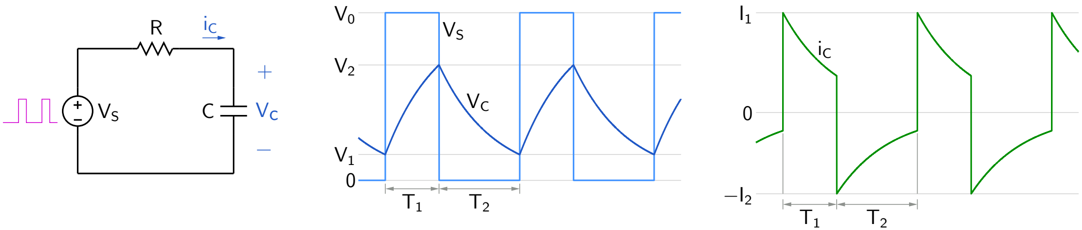






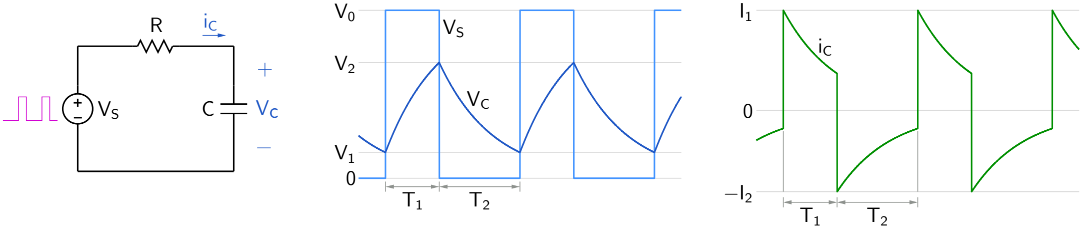
$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

RC circuit: example



$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

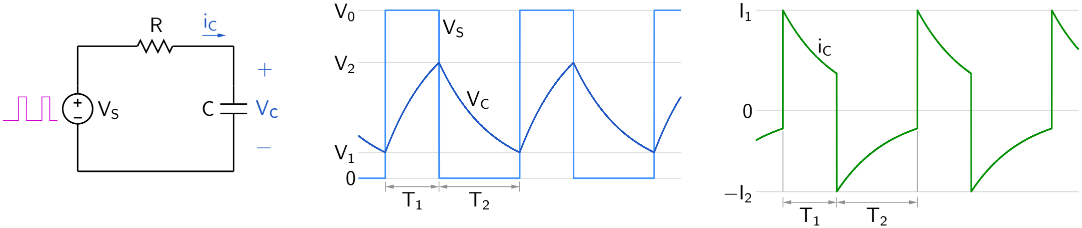
$V_C^{(1)}(0) = V_1,\; V_C^{(1)}(\infty) = V_0$



$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

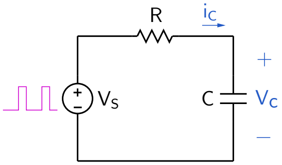


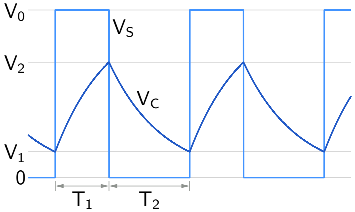
$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\, e^{-t/\tau} + B$

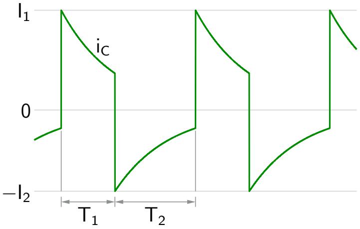
$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

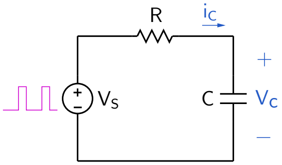
$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

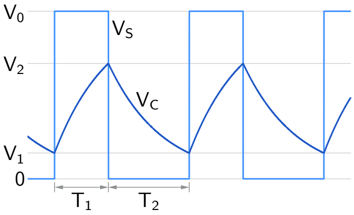
$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

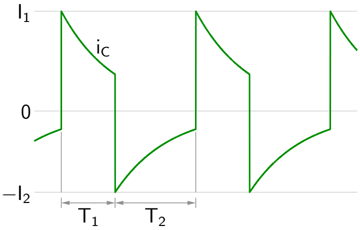
$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

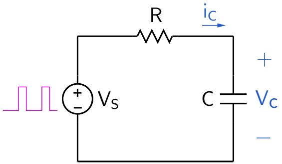
$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

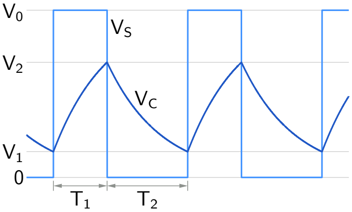
$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

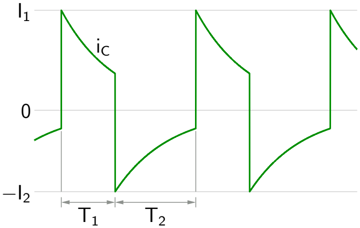
$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1$, $V_C^{(1)}(\infty) = V_0$

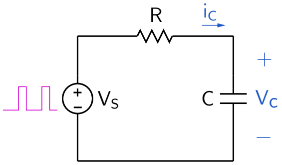
$\rightarrow$ $B = V_0$, $A = V_1 - V_0$.

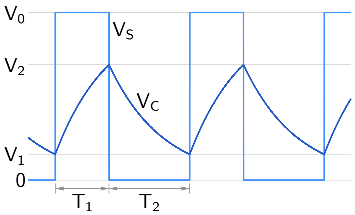
$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

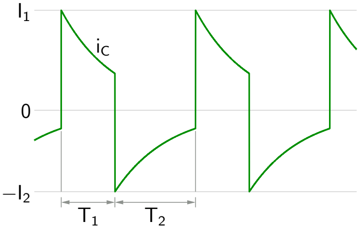
$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2$, $V_C^{(2)}(\infty) = 0$

$\rightarrow$ $B' = 0$, $A' = V_2\,e^{T_1/\tau}$.







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

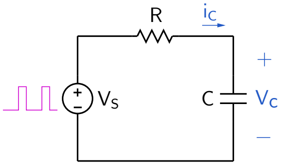
$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

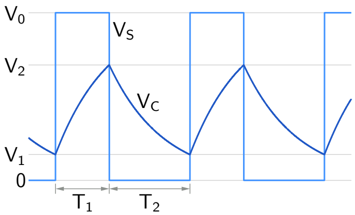
$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

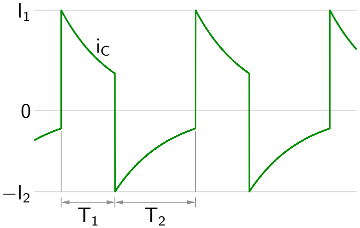
$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = V_2\,e^{T_1/\tau}.$

$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

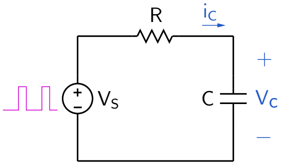
$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$

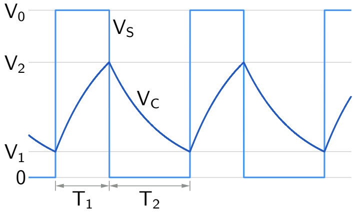
$\rightarrow\ B' = 0,\ A' = V_2\,e^{T_1/\tau}.$

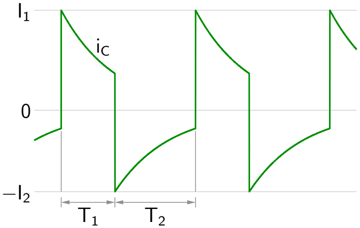
$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

$V_C^{(1)}(T_1) = V_2,\ V_C^{(2)}(T_1 + T_2) = V_1.$







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$

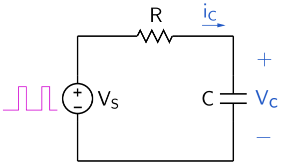
$\rightarrow\ B' = 0,\ A' = V_2\,e^{T_1/\tau}.$

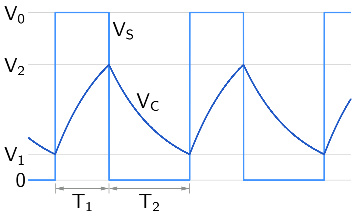
$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

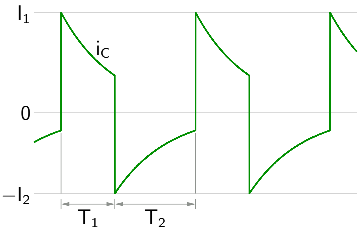
Now use the conditions:

$V_C^{(1)}(T_1) = V_2,\ V_C^{(2)}(T_1 + T_2) = V_1.$

$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = V_2\,e^{T_1/\tau}.$

$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

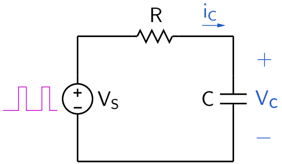
Now use the conditions:

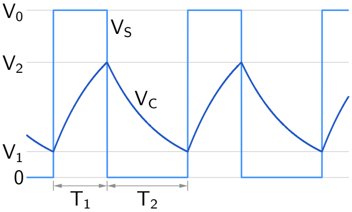
$V_C^{(1)}(T_1) = V_2,\ V_C^{(2)}(T_1 + T_2) = V_1.$

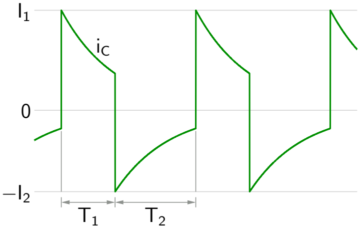
$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$

$$V_1 = V_2 e^{-(T_1+T_2-T_1)/\tau} = V_2 e^{-T_2/\tau} \qquad (4)$$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = V_2\,e^{T_1/\tau}.$

$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

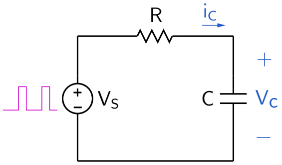
Now use the conditions:

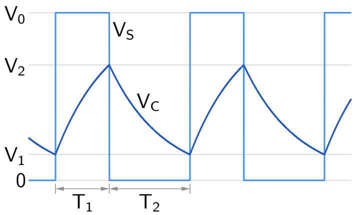
$V_C^{(1)}(T_1) = V_2,\ V_C^{(2)}(T_1 + T_2) = V_1.$

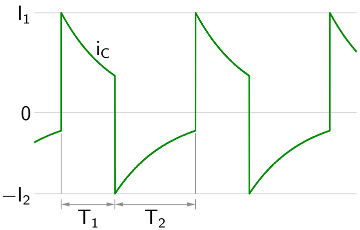
$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$

$$V_1 = V_2 e^{-(T_1+T_2-T_1)/\tau} = V_2 e^{-T_2/\tau} \qquad (4)$$

Rewrite with $a \equiv e^{-T_1/\tau}$, $b \equiv e^{-T_2/\tau}$.







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1$, $V_C^{(1)}(\infty) = V_0$

$\rightarrow$ $B = V_0$, $A = V_1 - V_0$.

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2$, $V_C^{(2)}(\infty) = 0$

$\rightarrow$ $B' = 0$, $A' = V_2\,e^{T_1/\tau}$.

$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

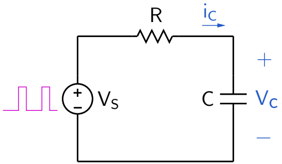
$V_C^{(1)}(T_1) = V_2$, $V_C^{(2)}(T_1 + T_2) = V_1$.

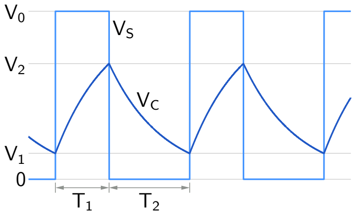
$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$

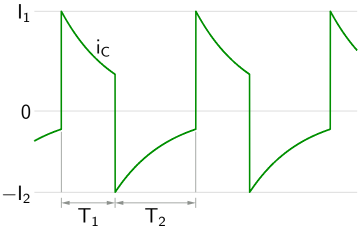
$$V_1 = V_2 e^{-(T_1+T_2-T_1)/\tau} = V_2 e^{-T_2/\tau} \qquad (4)$$

Rewrite with $a \equiv e^{-T_1/\tau}$, $b \equiv e^{-T_2/\tau}$.

$$V_2 = -(V_0 - V_1)a + V_0 \qquad (5)$$







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\ V_C^{(1)}(\infty) = V_0$

$\rightarrow\ B = V_0,\ A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2,\ V_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = V_2\,e^{T_1/\tau}.$

$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

$V_C^{(1)}(T_1) = V_2,\ V_C^{(2)}(T_1 + T_2) = V_1.$

$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$

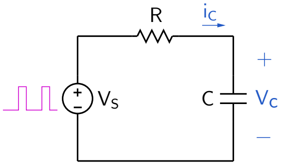
$$V_1 = V_2 e^{-(T_1+T_2-T_1)/\tau} = V_2 e^{-T_2/\tau} \qquad (4)$$

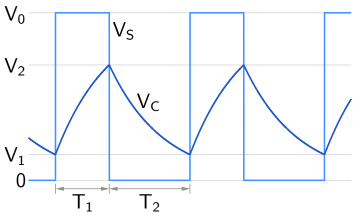
Rewrite with $a \equiv e^{-T_1/\tau},\ b \equiv e^{-T_2/\tau}$.

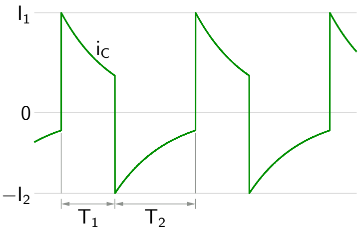
$$V_2 = -(V_0 - V_1)a + V_0 \qquad (5)$$

$$V_1 = b\,V_2 \qquad (6)$$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $V_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$V_C^{(1)}(0) = V_1,\; V_C^{(1)}(\infty) = V_0$

$\rightarrow\; B = V_0,\; A = V_1 - V_0.$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0 \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $V_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$V_C^{(2)}(T_1) = V_2,\; V_C^{(2)}(\infty) = 0$

$\rightarrow\; B' = 0,\; A' = V_2\,e^{T_1/\tau}.$

$$V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

$V_C^{(1)}(T_1) = V_2,\; V_C^{(2)}(T_1 + T_2) = V_1.$

$$V_2 = -(V_0 - V_1)e^{-T_1/\tau} + V_0 \qquad (3)$$

$$V_1 = V_2 e^{-(T_1+T_2-T_1)/\tau} = V_2 e^{-T_2/\tau} \qquad (4)$$

Rewrite with $a \equiv e^{-T_1/\tau},\; b \equiv e^{-T_2/\tau}.$

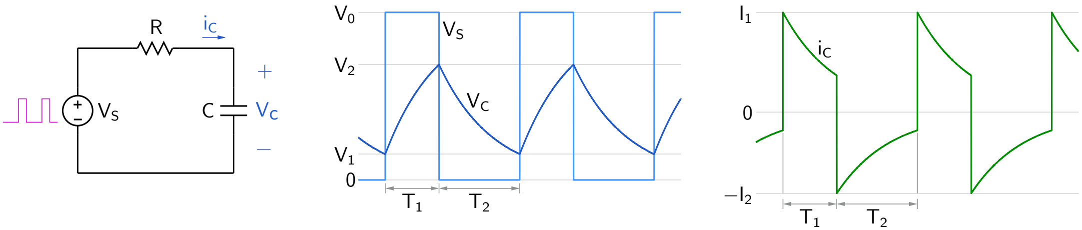
$$V_2 = -(V_0 - V_1)a + V_0 \qquad (5)$$

$$V_1 = b\,V_2 \qquad (6)$$

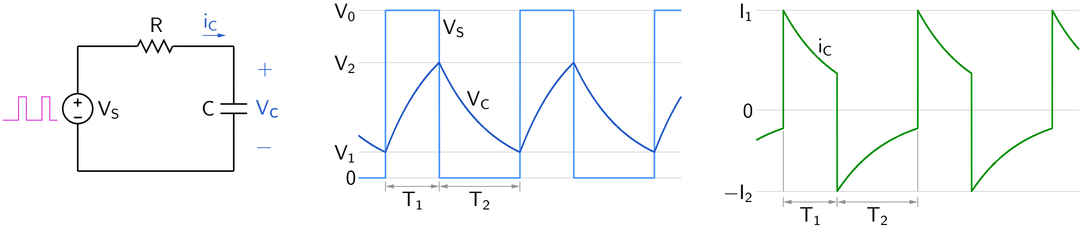
Solve to get

$$\boxed{V_1 = b\,V_0\,\frac{1-a}{1-ab},\quad V_2 = V_0\,\frac{1-a}{1-ab}}$$

RC circuit: example

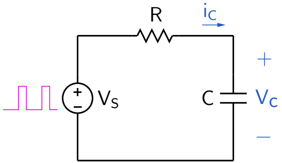


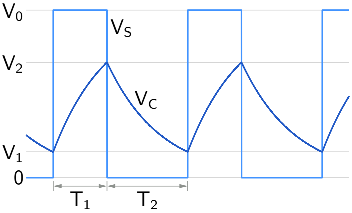
$$V_1 = b\,V_0\,\frac{1-a}{1-ab}, \quad V_2 = V_0\,\frac{1-a}{1-ab}, \quad \text{with} \quad a = e^{-T_1/\tau}, \quad b = e^{-T_2/\tau}.$$

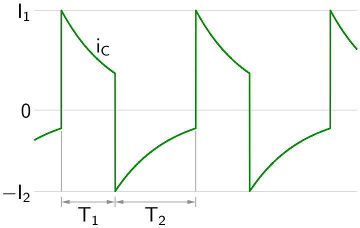


$$V_1 = b\,V_0\,\frac{1-a}{1-ab},\quad V_2 = V_0\,\frac{1-a}{1-ab},\quad \text{with}\quad a = e^{-T_1/\tau},\quad b = e^{-T_2/\tau}.$$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0,\quad V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau}.$$



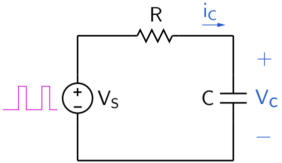


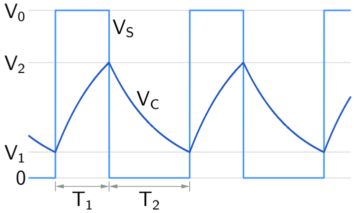


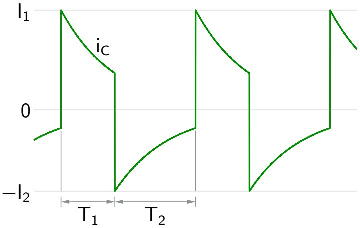
$$V_1 = b\,V_0\,\frac{1-a}{1-ab}, \quad V_2 = V_0\,\frac{1-a}{1-ab}, \quad \text{with} \quad a = e^{-T_1/\tau}, \quad b = e^{-T_2/\tau}.$$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0, \quad V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau}.$$

Current calculation:







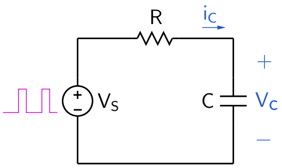
$$V_1 = b\, V_0 \,\frac{1-a}{1-ab}, \quad V_2 = V_0 \,\frac{1-a}{1-ab}, \quad \text{with} \quad a = e^{-T_1/\tau}, \quad b = e^{-T_2/\tau}.$$

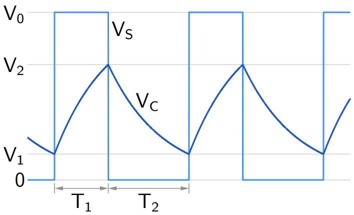
$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0, \quad V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau}.$$

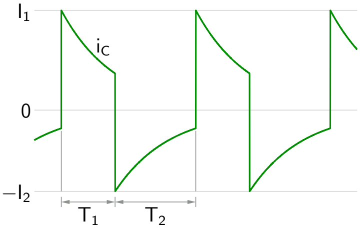
Current calculation:

Method 1:

$$i_C(t) = C\,\frac{dV_C}{dt} \quad \text{(home work)}$$







$$V_1 = b\,V_0\,\frac{1-a}{1-ab}, \quad V_2 = V_0\,\frac{1-a}{1-ab}, \quad \text{with} \quad a = e^{-T_1/\tau}, \quad b = e^{-T_2/\tau}.$$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0, \quad V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau}.$$

Current calculation:

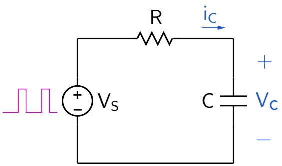
Method 1:

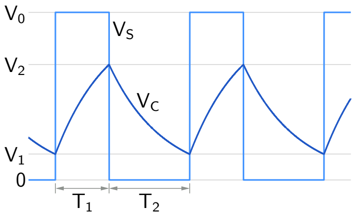
$$i_C(t) = C\,\frac{dV_C}{dt} \quad \text{(home work)}$$

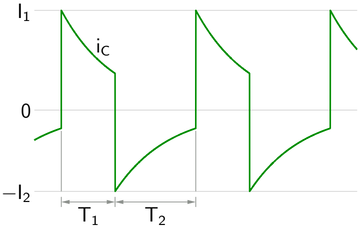
Method 2:

Start from scratch!

RC circuit: example







$$V_1 = b\,V_0\,\frac{1-a}{1-ab},\quad V_2 = V_0\,\frac{1-a}{1-ab},\quad \text{with}\quad a = e^{-T_1/\tau},\ \ b = e^{-T_2/\tau}.$$

$$V_C^{(1)}(t) = -(V_0 - V_1)e^{-t/\tau} + V_0,\quad V_C^{(2)}(t) = V_2 e^{-(t-T_1)/\tau}.$$

Current calculation:

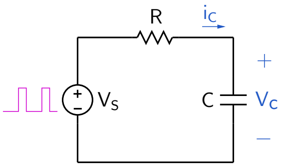
Method 1:

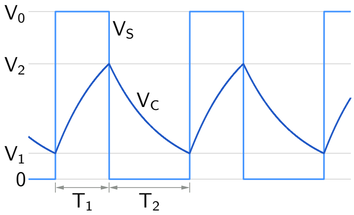
$$i_C(t) = C\,\frac{dV_C}{dt}\ \text{(home work)}$$

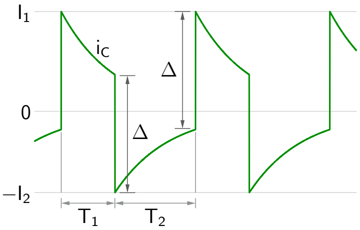
Method 2:

Start from scratch!

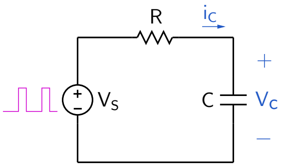
RC circuit: example

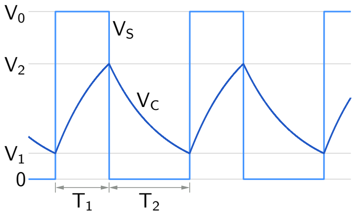


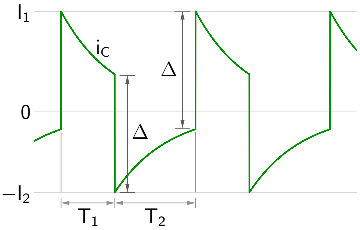




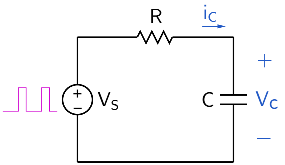
RC circuit: example

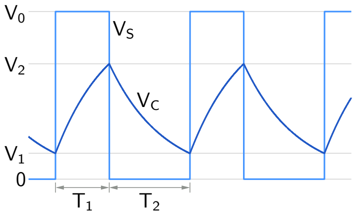


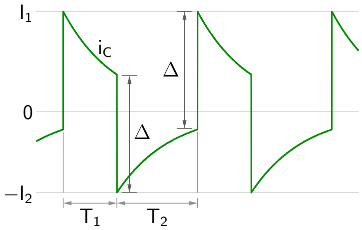




$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\,e^{-t/\tau} + B$



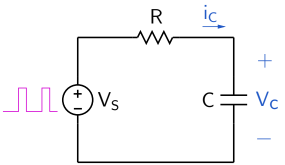


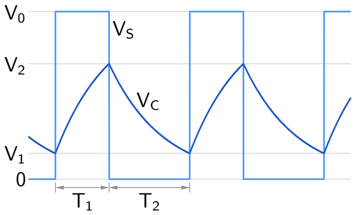


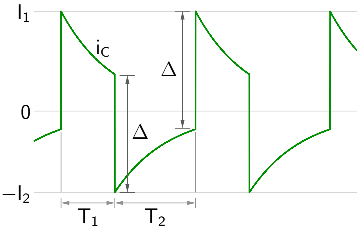
$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

RC circuit: example





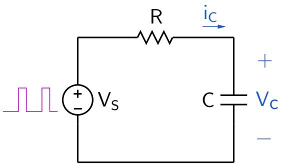


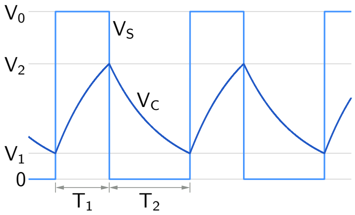
$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

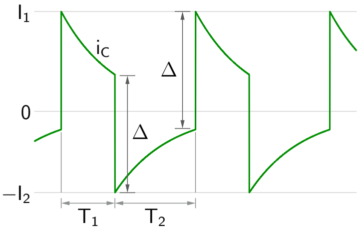
$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

RC circuit: example







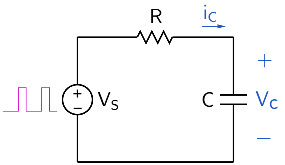
$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

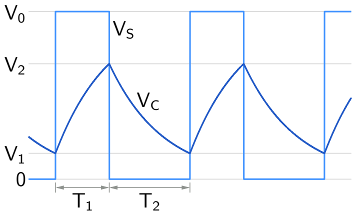
$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

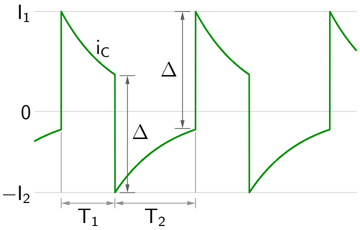
$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

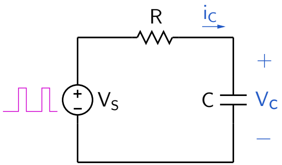
$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

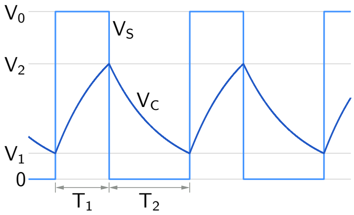
$\rightarrow\ B = 0,\ A = I_1.$

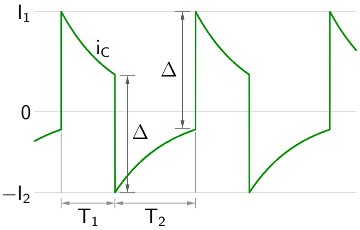
$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

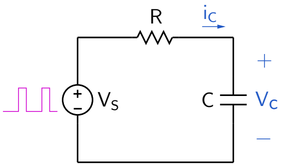
$\rightarrow\ B = 0,\ A = I_1.$

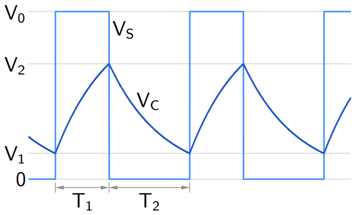
$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

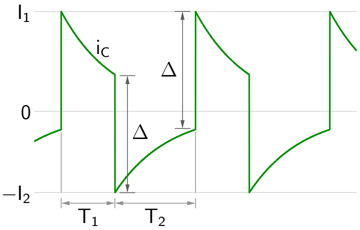
$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

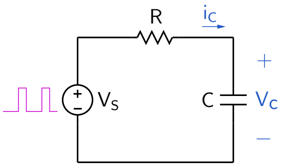
$\rightarrow\ B = 0,\ A = I_1.$

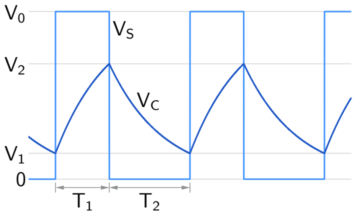
$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

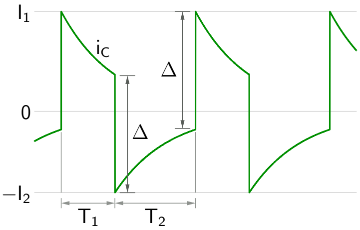
$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = -I_2\, e^{T_1/\tau}.$







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

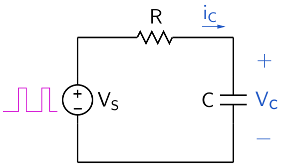
$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

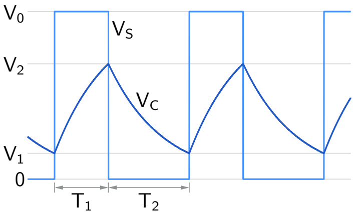
$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

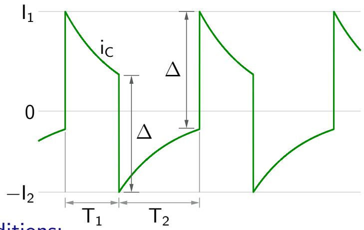
$\rightarrow\ B' = 0,\ A' = -I_2\, e^{T_1/\tau}.$

$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

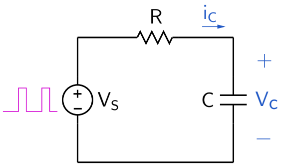
$\rightarrow\ B' = 0,\ A' = -I_2\, e^{T_1/\tau}.$

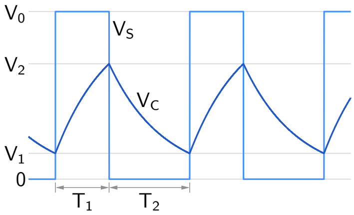
$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

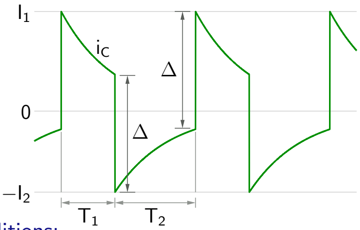
Now use the conditions:

$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

RC circuit: example







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1$, $i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2$, $i_C^{(2)}(\infty) = 0$

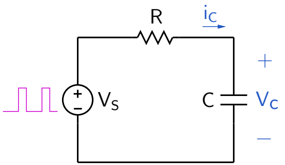
$\rightarrow\ B' = 0,\ A' = -I_2\,e^{T_1/\tau}.$

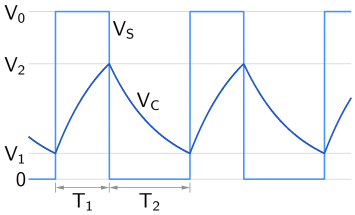
$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

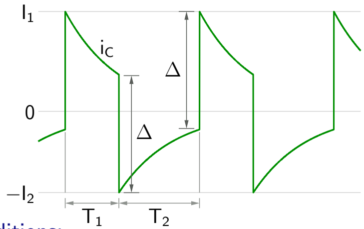
Now use the conditions:

$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

$i_C^{(1)}(0) - i_C^{(2)}(T_1 + T_2) = \Delta = V_0/R.$







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = -I_2\, e^{T_1/\tau}.$

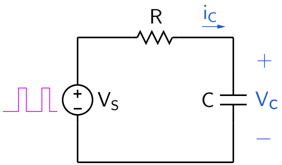
$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

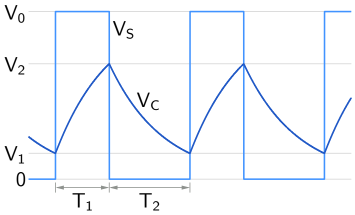
Now use the conditions:

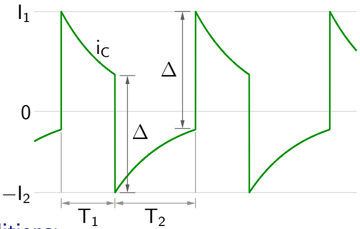
$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

$i_C^{(1)}(0) - i_C^{(2)}(T_1 + T_2) = \Delta = V_0/R.$

$$I_1 e^{-T_1/\tau} - (-I_2) = \Delta \qquad (3)$$







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1$, $i_C^{(1)}(\infty) = 0$

$\rightarrow$ $B = 0$, $A = I_1$.

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2$, $i_C^{(2)}(\infty) = 0$

$\rightarrow$ $B' = 0$, $A' = -I_2\, e^{T_1/\tau}$.

$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

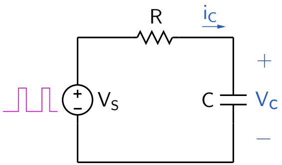
Now use the conditions:

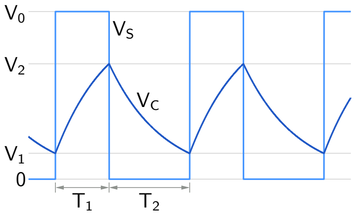
$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R$,

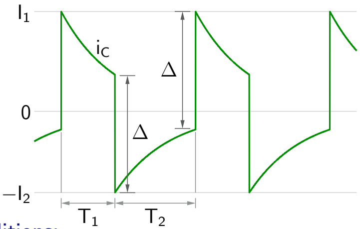
$i_C^{(1)}(0) - i_C^{(2)}(T_1 + T_2) = \Delta = V_0/R$.

$$I_1 e^{-T_1/\tau} - (-I_2) = \Delta \qquad (3)$$

$$I_1 - (-I_2 e^{-(T_1+T_2-T_1)/\tau}) = \Delta \qquad (4)$$







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\, e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\, e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = -I_2\, e^{T_1/\tau}.$

$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

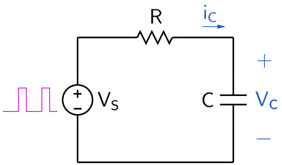
$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

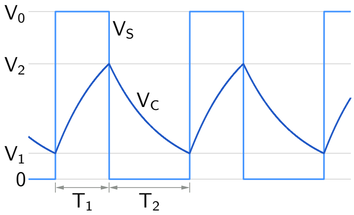
$i_C^{(1)}(0) - i_C^{(2)}(T_1 + T_2) = \Delta = V_0/R.$

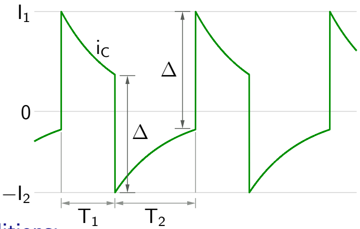
$$I_1 e^{-T_1/\tau} - (-I_2) = \Delta \qquad (3)$$

$$I_1 - (-I_2 e^{-(T_1+T_2-T_1)/\tau}) = \Delta \qquad (4)$$

$$a\, I_1 + I_2 = \Delta \qquad (5)$$







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = -I_2\,e^{T_1/\tau}.$

$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

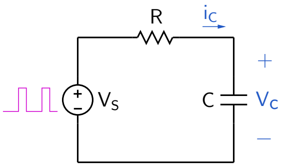
$i_C^{(1)}(0) - i_C^{(2)}(T_1 + T_2) = \Delta = V_0/R.$

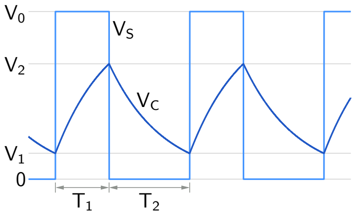
$$I_1 e^{-T_1/\tau} - (-I_2) = \Delta \qquad (3)$$

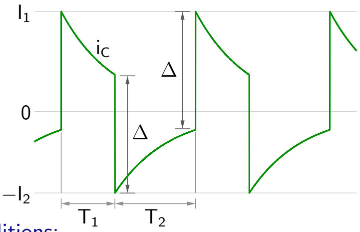
$$I_1 - (-I_2 e^{-(T_1+T_2-T_1)/\tau}) = \Delta \qquad (4)$$

$$a\,I_1 + I_2 = \Delta \qquad (5)$$

$$I_1 + b\,I_2 = \Delta \qquad (6)$$







$\boxed{0 < t < T_1}$ Let $i_C^{(1)}(t) = A\,e^{-t/\tau} + B$

$i_C^{(1)}(0) = I_1,\ i_C^{(1)}(\infty) = 0$

$\rightarrow\ B = 0,\ A = I_1.$

$$i_C^{(1)}(t) = I_1 e^{-t/\tau} \qquad (1)$$

$\boxed{T_1 < t < T_2}$ Let $i_C^{(2)}(t) = A'\,e^{-t/\tau} + B'$

$i_C^{(2)}(T_1) = -I_2,\ i_C^{(2)}(\infty) = 0$

$\rightarrow\ B' = 0,\ A' = -I_2\,e^{T_1/\tau}.$

$$i_C^{(2)}(t) = -I_2 e^{-(t-T_1)/\tau} \qquad (2)$$

Now use the conditions:

$i_C^{(1)}(T_1) - i_C^{(2)}(T_1) = \Delta = V_0/R,$

$i_C^{(1)}(0) - i_C^{(2)}(T_1+T_2) = \Delta = V_0/R.$

$$I_1 e^{-T_1/\tau} - (-I_2) = \Delta \qquad (3)$$

$$I_1 - (-I_2 e^{-(T_1+T_2-T_1)/\tau}) = \Delta \qquad (4)$$

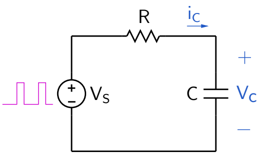
$$a\,I_1 + I_2 = \Delta \qquad (5)$$

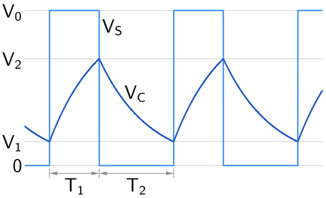
$$I_1 + b\,I_2 = \Delta \qquad (6)$$

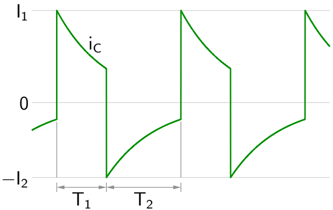
Solve to get

$$\boxed{I_1 = \Delta\,\frac{1-b}{1-ab},\ I_2 = \Delta\,\frac{1-a}{1-ab}}$$

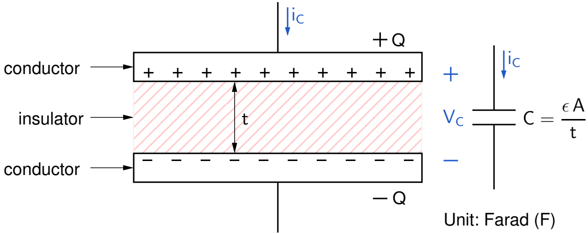
$(a = e^{-T_1/\tau},\ \ b = e^{-T_2/\tau})$



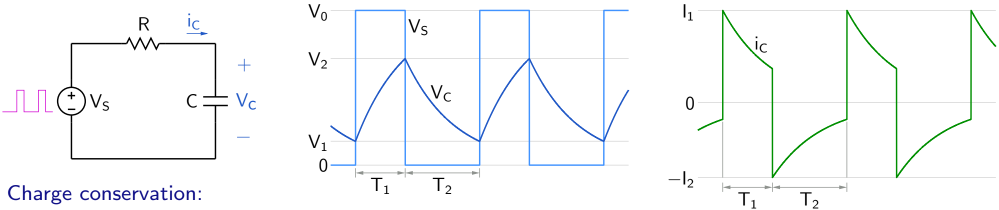




Charge conservation:

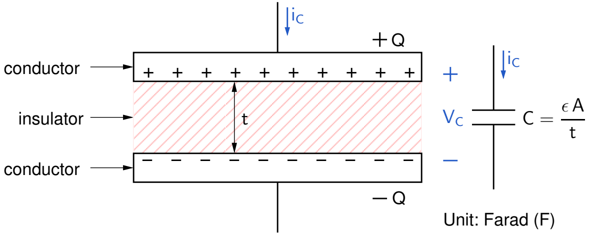


$$i_C = \frac{dQ}{dt} = C\,\frac{dV_C}{dt}$$

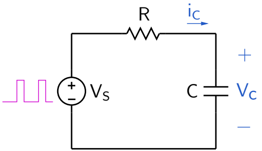


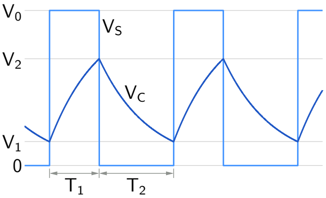
Charge conservation:

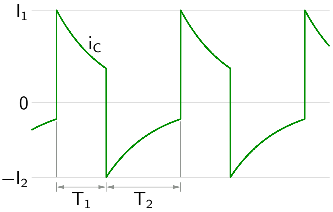
Periodic steady state: All quantities are periodic, i.e., $x(t_0 + T) = x(t_0)$



$$i_C = \frac{dQ}{dt} = C\,\frac{dV_C}{dt}$$



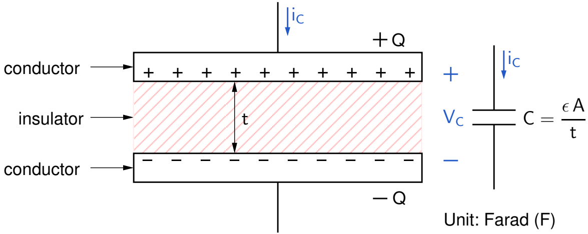




Charge conservation:

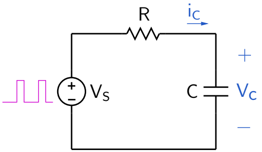
Periodic steady state: All quantities are periodic, i.e., $x(t_0 + T) = x(t_0)$

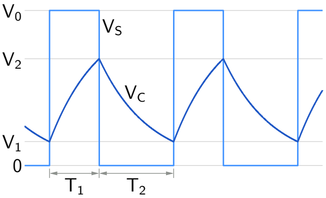
Capacitor charge: $Q(t_0 + T) = Q(t_0)$

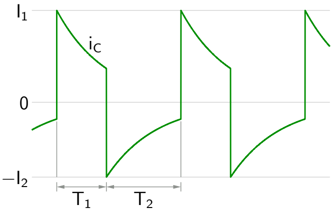


$$i_C = \frac{dQ}{dt} = C\,\frac{dV_C}{dt}$$

RC circuit: example





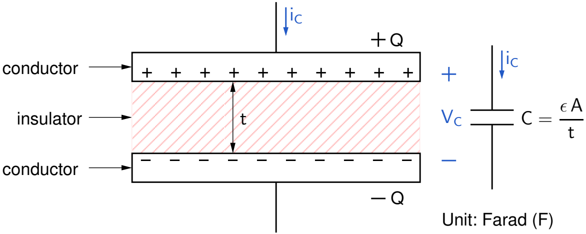


Charge conservation:

Periodic steady state: All quantities are periodic, i.e., $x(t_0 + T) = x(t_0)$

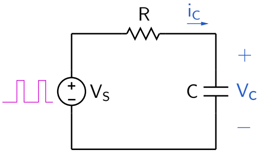
Capacitor charge: $Q(t_0 + T) = Q(t_0)$

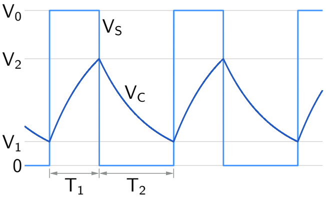
$$i_C = \frac{dQ}{dt} \rightarrow Q = \int i_C \, dt.$$

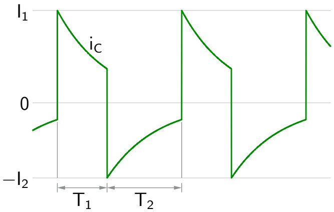


$$i_C = \frac{dQ}{dt} = C\,\frac{dV_C}{dt}$$

RC circuit: example







Charge conservation:

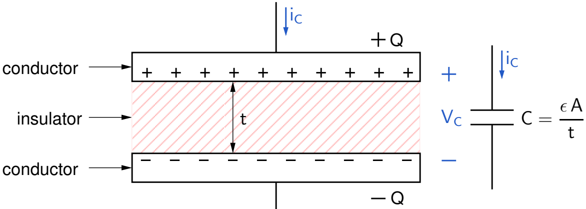
Periodic steady state: All quantities are periodic, i.e.,
$x(t_0 + T) = x(t_0)$

Capacitor charge: $Q(t_0 + T) = Q(t_0)$

$$i_C = \frac{dQ}{dt} \rightarrow Q = \int i_C \, dt.$$

$$Q(t_0 + T) = Q(t_0) \rightarrow Q(t_0 + T) - Q(t_0) = 0$$

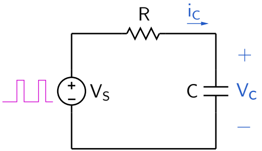
$$\rightarrow \int_{t_0}^{t_0+T} i_C \, dt = 0.$$

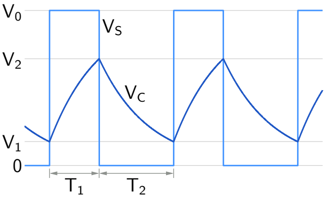


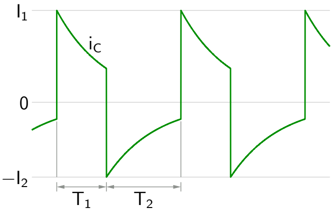
Unit: Farad (F)

$$i_C = \frac{dQ}{dt} = C \frac{dV_C}{dt}$$

RC circuit: example







Charge conservation:

Periodic steady state: All quantities are periodic, i.e., $x(t_0 + T) = x(t_0)$

Capacitor charge: $Q(t_0 + T) = Q(t_0)$

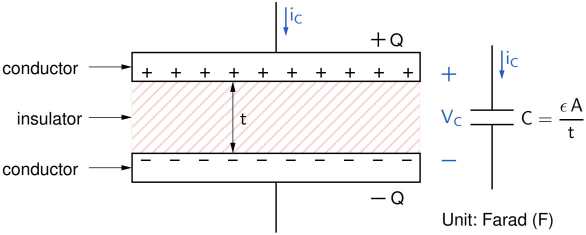
$$i_C = \frac{dQ}{dt} \rightarrow Q = \int i_C \, dt.$$

$$Q(t_0 + T) = Q(t_0) \rightarrow Q(t_0 + T) - Q(t_0) = 0$$

$$\rightarrow \int_{t_0}^{t_0+T} i_C \, dt = 0.$$

$$\int_0^T i_C \, dt = 0 \rightarrow \int_0^{T_1} i_C \, dt + \int_{T_1}^{T_1+T_2} i_C \, dt = 0$$

$$\rightarrow \boxed{\int_{T_1}^{T_1+T_2} i_C \, dt = -\int_0^{T_1} i_C \, dt.}$$



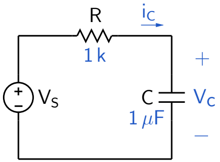
Unit: Farad (F)

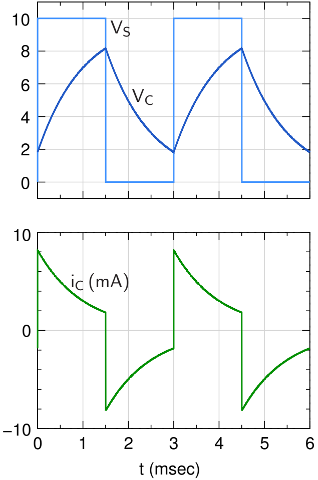
$$i_C = \frac{dQ}{dt} = C \frac{dV_C}{dt}$$

$T_1 = 1.5\,\text{msec},\ T_2 = 1.5\,\text{msec}.$

$V_1 = 1.8\,\text{V},\ V_2 = 8.2\,\text{V}.$

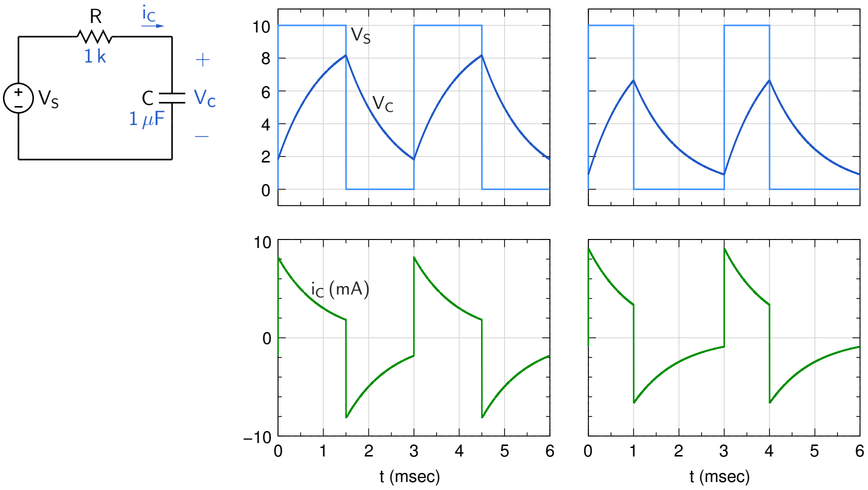
$I_1 = 8.2\,\text{mA},\ I_2 = 8.2\,\text{mA}.$

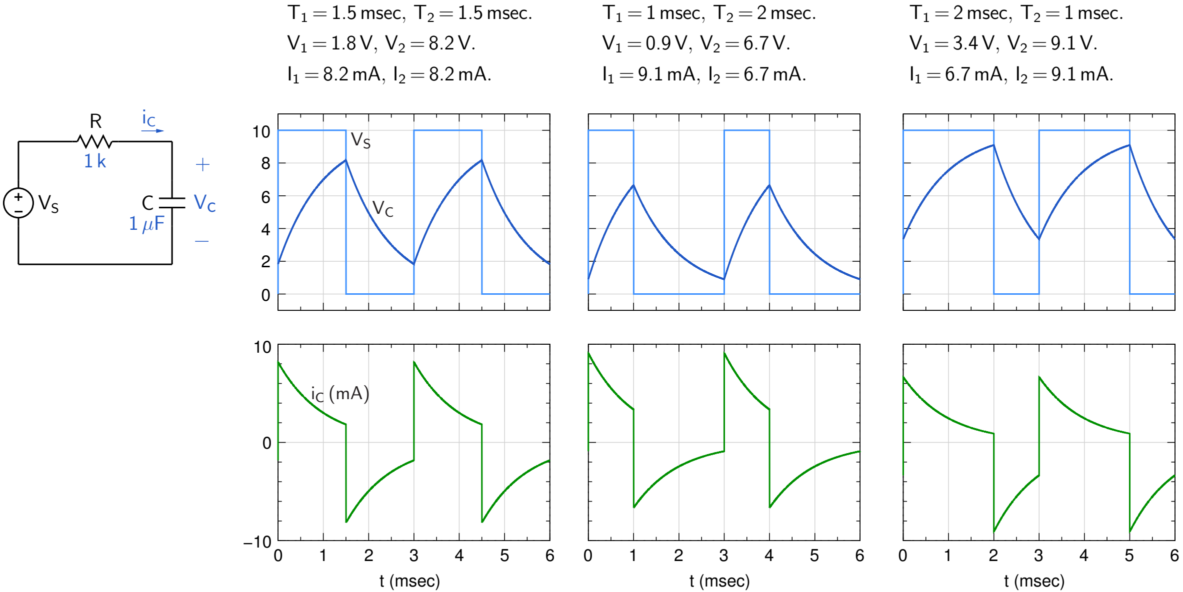




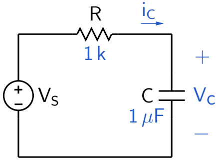
$T_1 = 1.5$ msec, $T_2 = 1.5$ msec.
$V_1 = 1.8$ V, $V_2 = 8.2$ V.
$I_1 = 8.2$ mA, $I_2 = 8.2$ mA.

$T_1 = 1$ msec, $T_2 = 2$ msec.
$V_1 = 0.9$ V, $V_2 = 6.7$ V.
$I_1 = 9.1$ mA, $I_2 = 6.7$ mA.





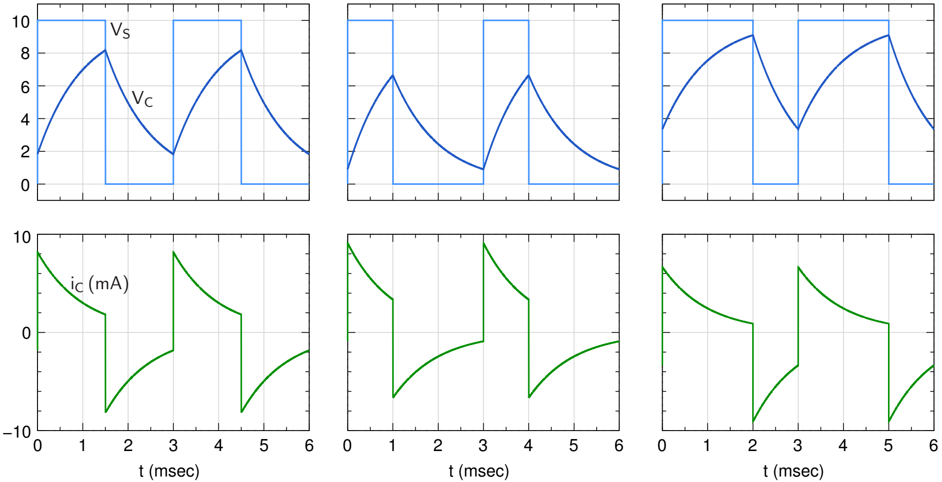
R
i_C
1 k
V_S
C
$1\,\mu F$
+
V_C
−
$T_1 = 1.5$ msec, $T_2 = 1.5$ msec.
$V_1 = 1.8$ V, $V_2 = 8.2$ V.
$I_1 = 8.2$ mA, $I_2 = 8.2$ mA.
$T_1 = 1$ msec, $T_2 = 2$ msec.
$V_1 = 0.9$ V, $V_2 = 6.7$ V.
$I_1 = 9.1$ mA, $I_2 = 6.7$ mA.
$T_1 = 2$ msec, $T_2 = 1$ msec.
$V_1 = 3.4$ V, $V_2 = 9.1$ V.
$I_1 = 6.7$ mA, $I_2 = 9.1$ mA.
V_S
V_C
i_C (mA)
t (msec)
t (msec)
t (msec)



$T_1 = 1.5$ msec, $T_2 = 1.5$ msec.
$V_1 = 1.8$ V, $V_2 = 8.2$ V.
$I_1 = 8.2$ mA, $I_2 = 8.2$ mA.

$T_1 = 1$ msec, $T_2 = 2$ msec.
$V_1 = 0.9$ V, $V_2 = 6.7$ V.
$I_1 = 9.1$ mA, $I_2 = 6.7$ mA.

$T_1 = 2$ msec, $T_2 = 1$ msec.
$V_1 = 3.4$ V, $V_2 = 9.1$ V.
$I_1 = 6.7$ mA, $I_2 = 9.1$ mA.



SEQUEL file: `ee101_rc1b.sqproj`

$T_1 = 1\,\text{msec},\ T_2 = 2\,\text{msec}.$

$V_1 \approx 0\,\text{V},\ V_2 = 10\,\text{V}.$

$I_1 = 100\,\text{mA},\ I_2 = 100\,\text{mA}.$