Circuit Analysis Using Superposition

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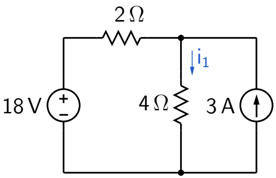
  - Deactivating an independent current source $\Rightarrow$ $I_0 = 0$, i.e., replace the current source with an open circuit.
  - Deactivating an independent voltage source $\Rightarrow$ $V_0 = 0$, i.e., replace the voltage source with a short circuit.

Example 1



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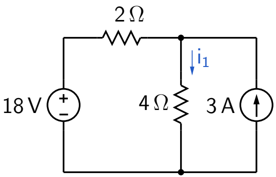
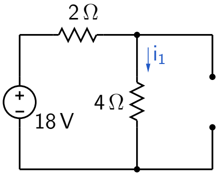


Case 1: Keep V_s, deactivate I_s.



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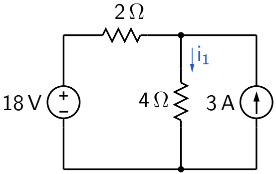

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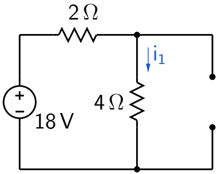

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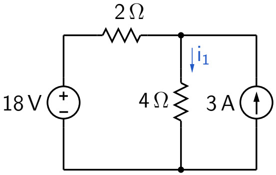

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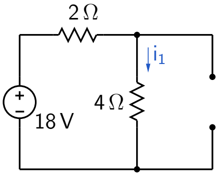

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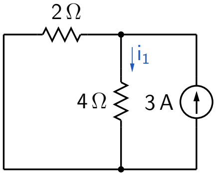

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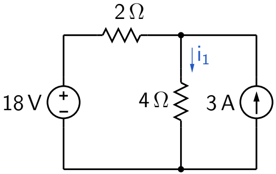

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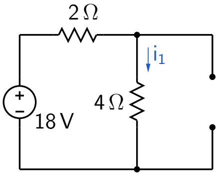

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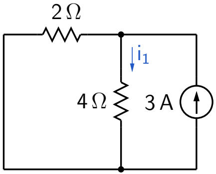

$$i_1^{(2)} = 3\,\mathrm{A} \times \frac{2\,\Omega}{2\,\Omega + 4\,\Omega} = 1\,\mathrm{A}$$

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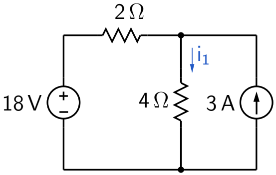

$$i_1^{\text{net}} = i_1^{(1)} + i_1^{(2)} = 3 + 1 = 4\,\text{A}$$

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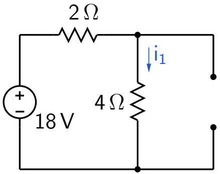

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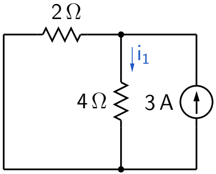

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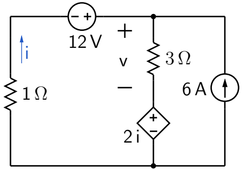

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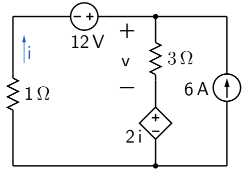

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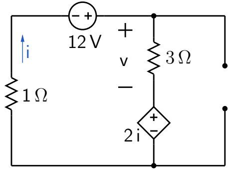

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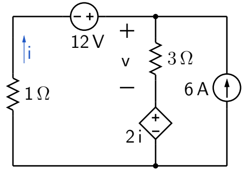

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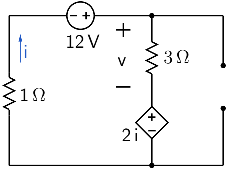

KVL: $-12 + 3\,i + 2\,i + i = 0$

$\Rightarrow i = 2\,A\,, v^{(1)} = 6\,V\,.$

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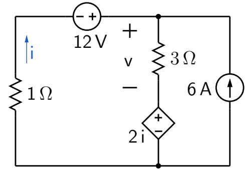

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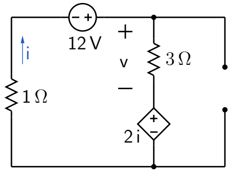

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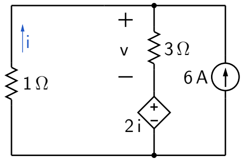

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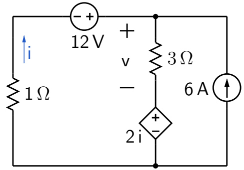

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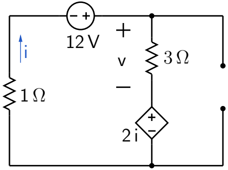

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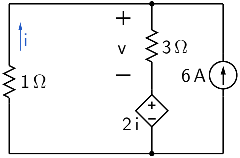

KVL: $i + (6 + i)\,3 + 2\,i = 0$

$\Rightarrow i = -3\,A\,, v^{(2)} = (-3 + 6) \times 3 = 9\,V\,.$

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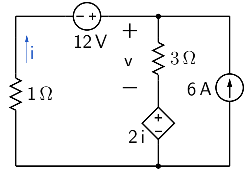

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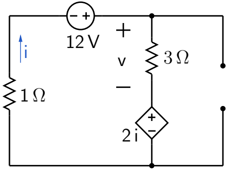

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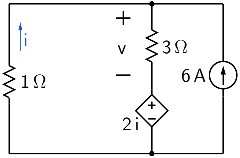

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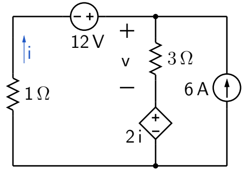

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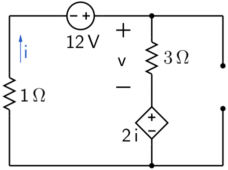

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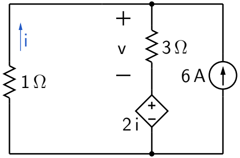

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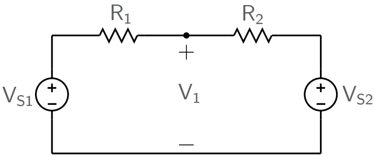
(SEQUEL file: `ee101_superposition_2.sqproj`)

Example 3

Find V_1 using superposition.



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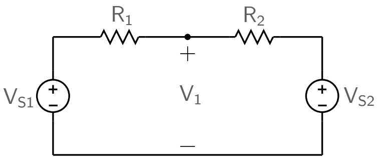
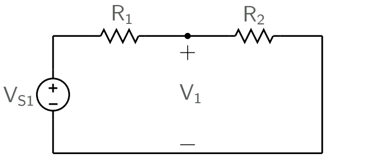


V_{S1} alone:



$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

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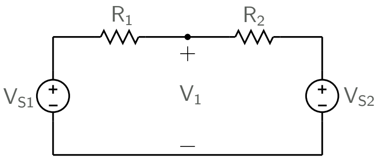

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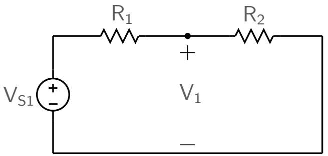

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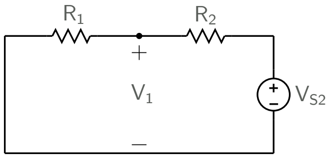

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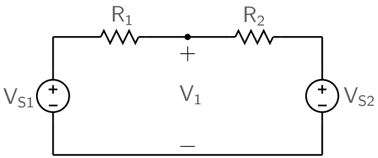

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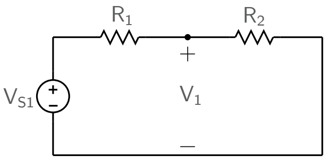

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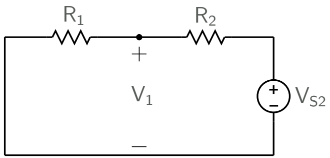
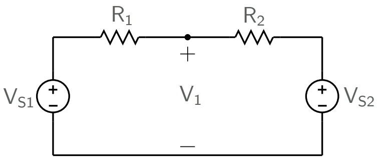


$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

$$V_1^{(\mathrm{net})} = V_1^{(1)} + V_1^{(2)} = \frac{R_2}{R_1 + R_2} V_{S1} + \frac{R_1}{R_1 + R_2} V_{S2}$$

Example 3 (again)

Find V_1 using superposition.



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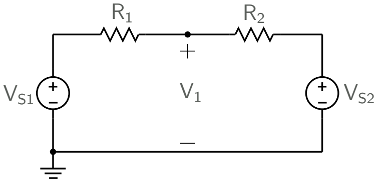

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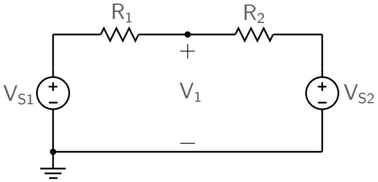

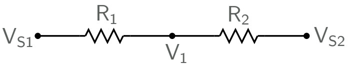

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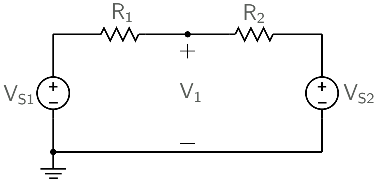
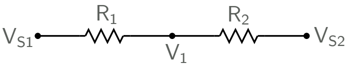
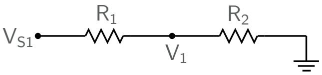




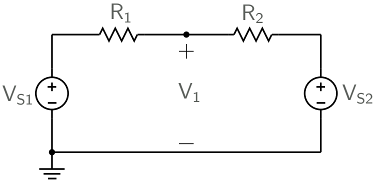
V_{S1} alone:

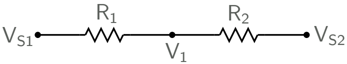


$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

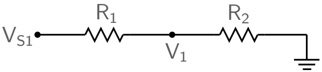
Example 3 (again)

Find V_1 using superposition.



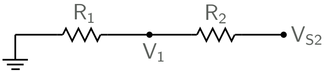


V_{S1} alone:



$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

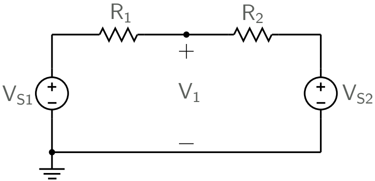
V_{S2} alone:

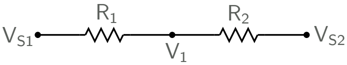


$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

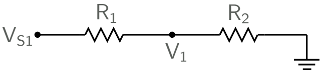
Example 3 (again)

Find V_1 using superposition.



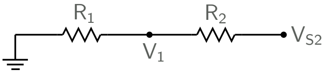


V_{S1} alone:



$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

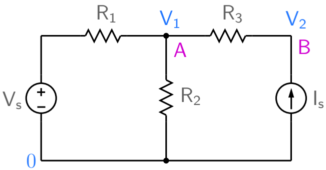
V_{S2} alone:

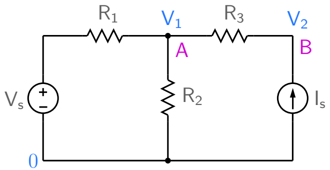


$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

$$V_1^{(\text{net})} = V_1^{(1)} + V_1^{(2)} = \frac{R_2}{R_1 + R_2} V_{S1} + \frac{R_1}{R_1 + R_2} V_{S2}$$

Superposition: Why does it work?



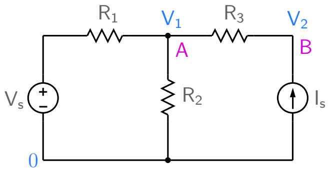


KCL at nodes A and B (taking current leaving a node as positive):

$$\frac{1}{R_1}(V_1 - V_s) + \frac{1}{R_2} V_1 + \frac{1}{R_3}(V_1 - V_2) = 0\,,$$

$$-I_s + \frac{1}{R_3}(V_2 - V_1) = 0\,.$$

Superposition: Why does it work?



KCL at nodes A and B (taking current leaving a node as positive):

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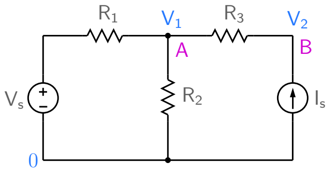
$$-I_s + \frac{1}{R_3}(V_2 - V_1) = 0\,.$$

Writing in a matrix form, we get (using $G_1 = 1/R_1$, etc.),

$$\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}$$

Superposition: Why does it work?



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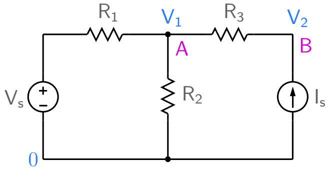
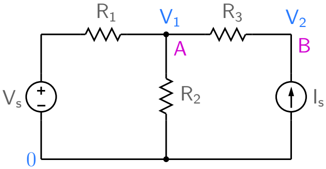
$$\text{i.e.,}\quad \mathbf{A} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$$

Superposition: Why does it work?



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix} .$$

Superposition: Why does it work?

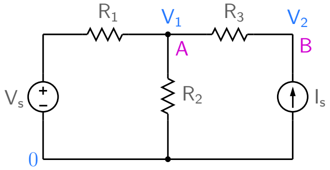


$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix} .$$

We are now in a position to see why superposition works.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ 0 \end{bmatrix} + \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ I_s \end{bmatrix} \equiv \begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \end{bmatrix} + \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \end{bmatrix} .$$

Superposition: Why does it work?



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}.$$

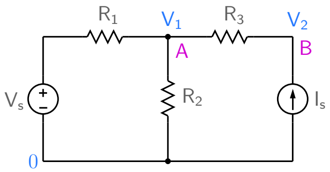
We are now in a position to see why superposition works.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ 0 \end{bmatrix} + \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ I_s \end{bmatrix} \equiv \begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \end{bmatrix} + \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \end{bmatrix}.$$

The first vector is the response due to V_s alone (and I_s deactivated).

The second vector is the response due to I_s alone (and V_s deactivated).

Superposition: Why does it work?



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}.$$

We are now in a position to see why superposition works.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ 0 \end{bmatrix} + \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ I_s \end{bmatrix} \equiv \begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \end{bmatrix} + \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \end{bmatrix}.$$

The first vector is the response due to V_s alone (and I_s deactivated).

The second vector is the response due to I_s alone (and V_s deactivated).

All other currents and voltages are linearly related to V_1 and V_2

$\Rightarrow$ Any voltage (node voltage or branch voltage) or current can also be computed using superposition.