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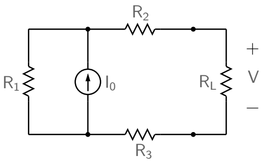
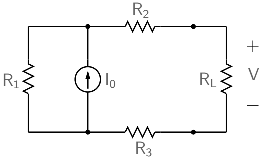
Thevenin's theorem



How is V related to the circuit parameters?



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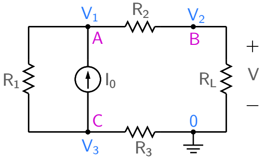

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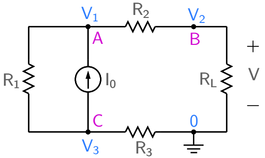

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Let $G_1 \equiv 1/R_1$, etc. Write KCL equation at each node, taking current leaving the node as positive.

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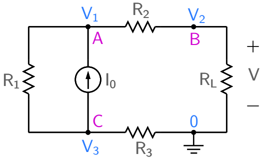

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$$
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\text{KCL at A}: \quad & G_1\,(V_1 - V_3) + G_2\,(V_1 - V_2) - I_0 &= 0\,, \\
\text{KCL at B}: \quad & G_2\,(V_2 - V_1) + G_L\,(V_2 - 0) &= 0\,, \\
\text{KCL at C}: \quad & G_1\,(V_3 - V_1) + G_3\,V_3 + I_0 &= 0\,.
\end{aligned}
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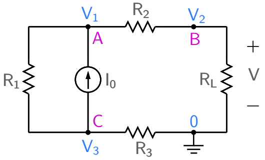

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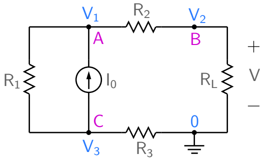
\end{aligned}$$

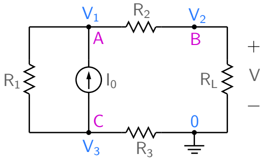
Write in a matrix form:

$$\begin{bmatrix} G_1 + G_2 & -G_2 & -G_1 \\ -G_2 & G_2 + G_L & 0 \\ -G_1 & 0 & G_1 + G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_0 \\ 0 \\ -I_0 \end{bmatrix},$$

i.e., $\mathbf{G}\,\mathbf{V} = \mathbf{I}_s$. We can solve this matrix equation to get V_2, i.e., the voltage across R_L.



V_2 can be found using Cramer's rule: $V_2 = \dfrac{\det \begin{bmatrix} G_1+G_2 & I_0 & -G_1 \\ -G_2 & 0 & 0 \\ -G_1 & -I_0 & G_1+G_3 \end{bmatrix}}{\det(\mathbf{G})} \equiv \dfrac{\Delta_1}{\det(\mathbf{G})}$



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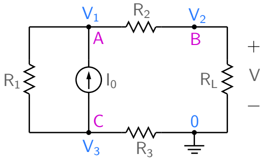
$$\det(\mathbf{G}) = \det \begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2 & G_2+G_L & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix}$$



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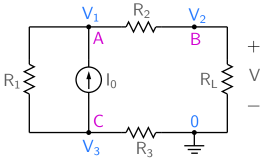

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&= \det \begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2 & G_2 & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix} + \det \begin{bmatrix} G_1+G_2 & 0 & -G_1 \\ -G_2 & G_L & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix} \\
&= \Delta + G_L\Delta_2 \ \ \text{where} \ \ \Delta_2 = \det \begin{bmatrix} G_1+G_2 & 0 & -G_1 \\ -G_2 & 1 & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix}.
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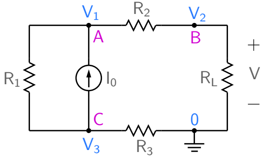

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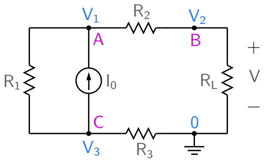
\end{aligned}$$

i.e., $V_2 = \dfrac{\Delta_1}{\det(\mathbf{G})} = \dfrac{\Delta_1}{\Delta + G_L\Delta_2}$ (Note: Δ, Δ_1, and Δ_2 are independent of G_L).



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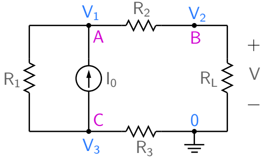

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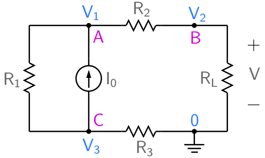

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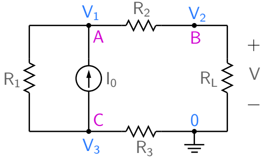

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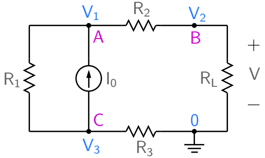
$$V_2 = \frac{R_L}{R_L + R_{\mathrm{Th}}} \, V_2^{OC}.$$

Thevenin's theorem



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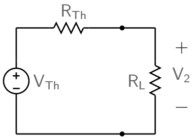

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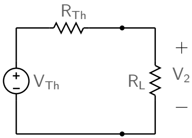
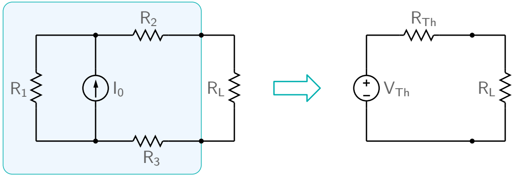


This allows us to replace the original circuit with an equivalent, simpler circuit.



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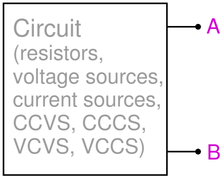
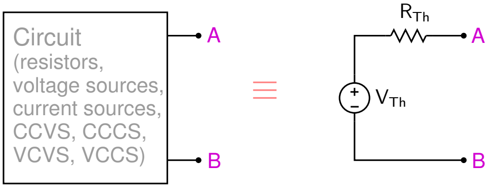
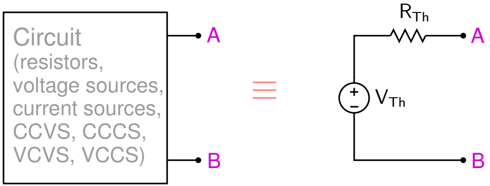
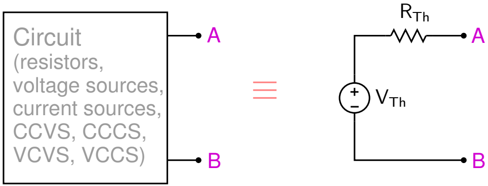



Circuit
(resistors,
voltage sources,
current sources,
CCVS, CCCS,
VCVS, VCCS)
A
B
≡
R_{Th}
V_{Th}
+
−
A
B



* Since the two circuits are equivalent, the open-circuit voltage must be the same in both cases. Let V_{oc} be the open-circuit voltage for the left circuit. For the Thevenin equivalent circuit, the open-circuit voltage is simply V_{Th} since there is no voltage drop across R_{Th} in this case.
  $\rightarrow V_{Th} = V_{oc}$



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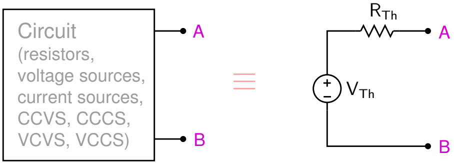
* R_{Th} can be found by different methods.

Thevenin's theorem: R_{Th}

Method 1:



* Deactivate all *independent* sources. This amounts to making $V_{\mathrm{Th}} = 0$ in the Thevenin equivalent circuit.

Thevenin's theorem: R_{Th}

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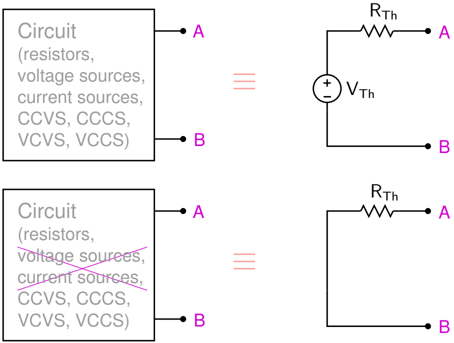

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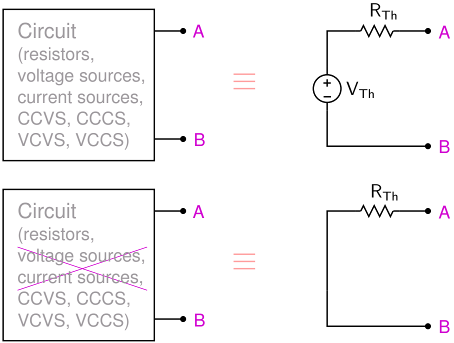

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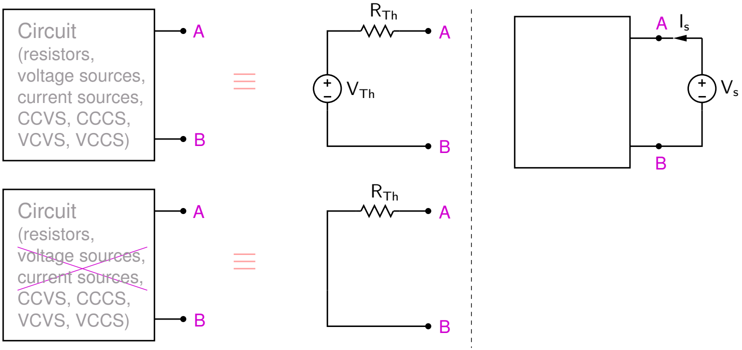

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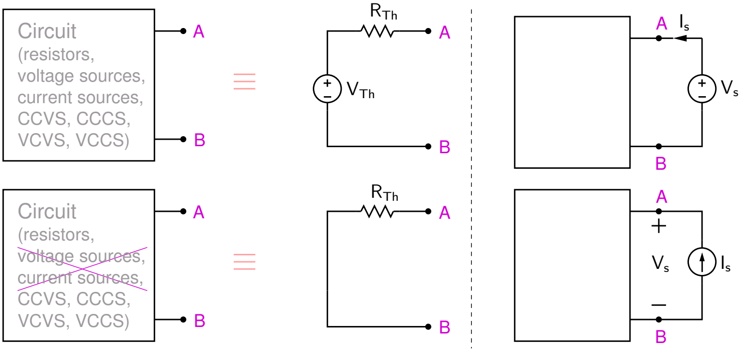

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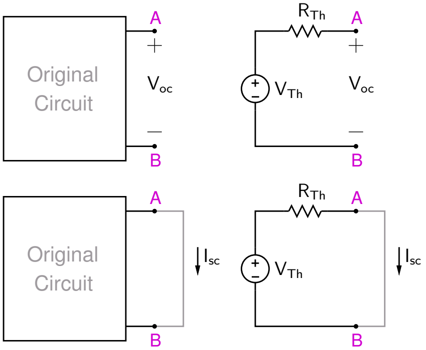

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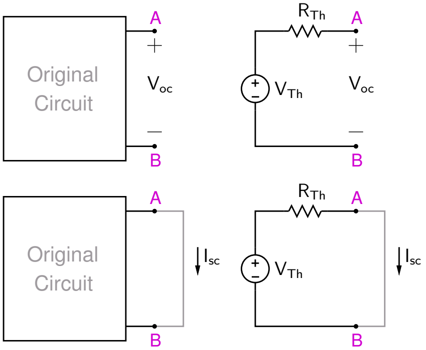

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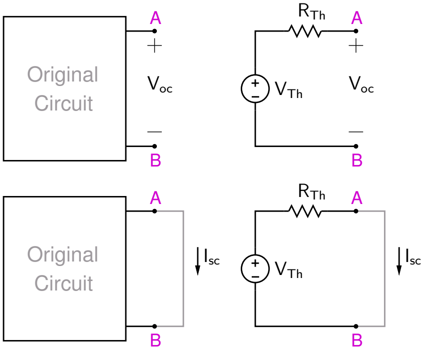
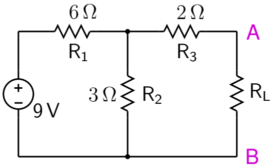


* For the Thevenin equivalent circuit, $V_{oc} = V_{Th}$, $I_{sc} = \dfrac{V_{Th}}{R_{Th}} = \dfrac{V_{oc}}{R_{Th}} \rightarrow R_{Th} = \dfrac{V_{oc}}{I_{sc}}$.

* In the original circuit, find V_{oc} and I_{sc} $\rightarrow R_{Th} = \dfrac{V_{oc}}{I_{sc}}$.

* Note: We do not deactivate any sources in this case.

Thevenin's theorem: example



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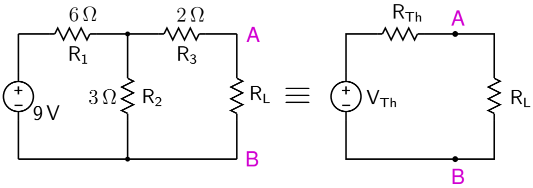

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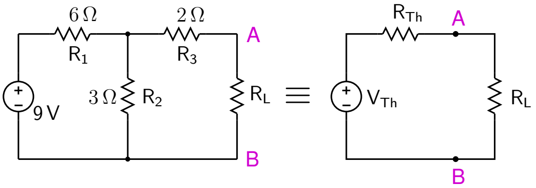
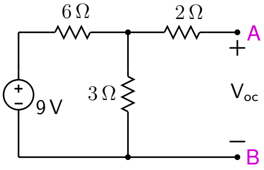


V_{Th} :



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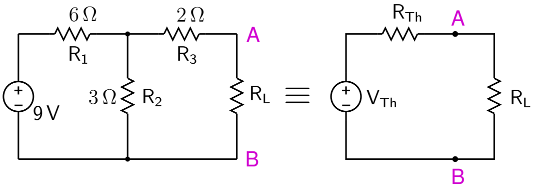

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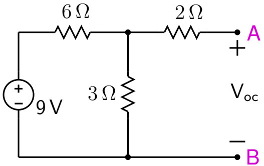
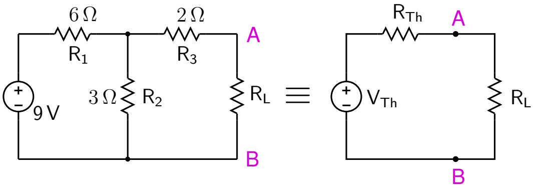


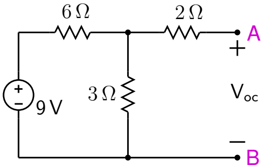
$$V_{oc} = 9\,\text{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

$$= 9\,\text{V} \times \frac{1}{3} = 3\,\text{V}$$

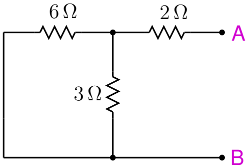
Thevenin's theorem: example



V_{Th} :

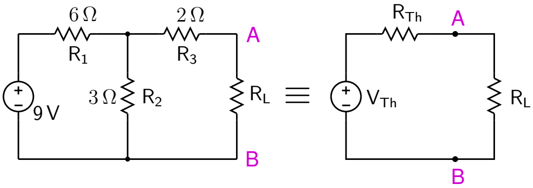


R_{Th} :

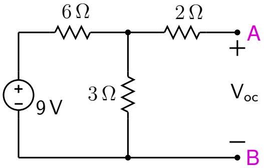


$$
\begin{aligned}
V_{oc} &= 9\,\text{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega} \\
&= 9\,\text{V} \times \frac{1}{3} = 3\,\text{V}
\end{aligned}
$$

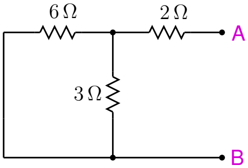
Thevenin's theorem: example



V_{Th} :



R_{Th} :



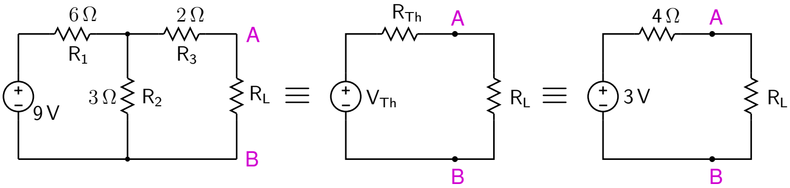
$$V_{oc} = 9\,\mathrm{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

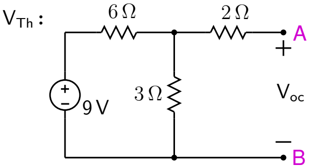
$$= 9\,\mathrm{V} \times \frac{1}{3} = 3\,\mathrm{V}$$

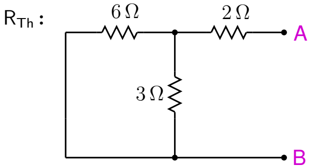
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

Thevenin's theorem: example







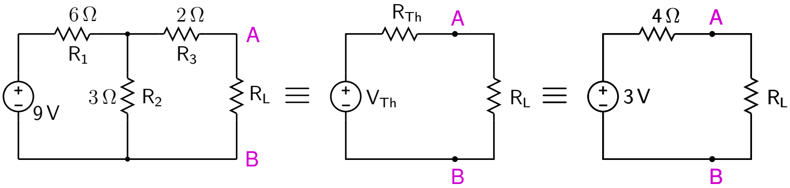
$$V_{oc} = 9\,V \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

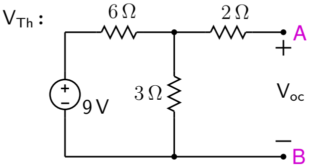
$$= 9\,V \times \frac{1}{3} = 3\,V$$

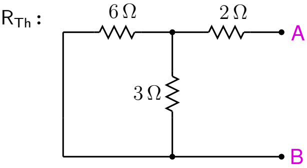
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

Thevenin's theorem: example







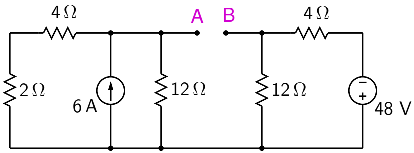
$$V_{oc} = 9\,\text{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

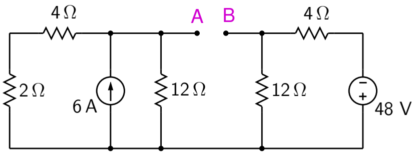
$$= 9\,\text{V} \times \frac{1}{3} = 3\,\text{V}$$

$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

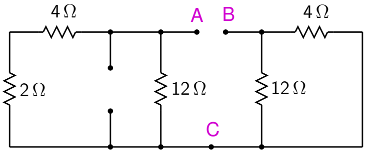
$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

Thevenin's theorem: example

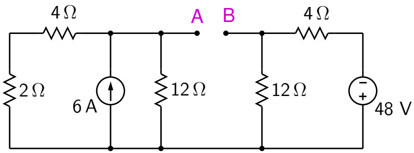




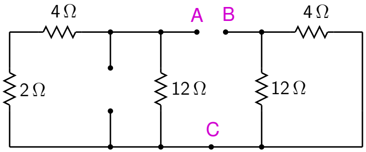
R_{Th}:



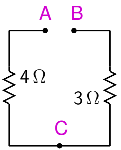
Thevenin's theorem: example

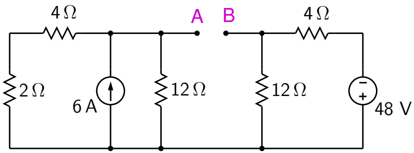


R_{Th}:

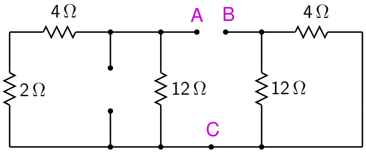


$\equiv$

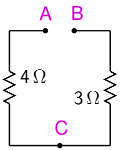




R_{Th}:

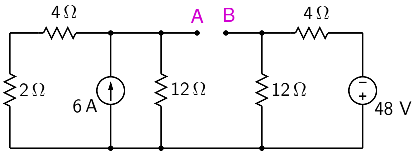


$\equiv$

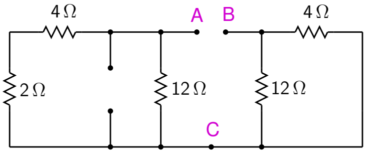


$\Rightarrow \quad R_{Th} = 7\,\Omega$

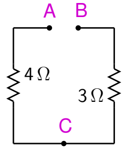
Thevenin's theorem: example



R_{Th}:

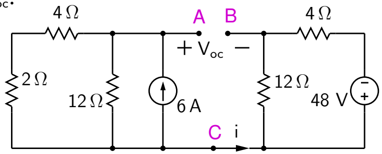


$\equiv$

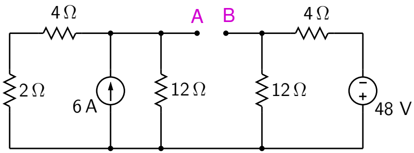


$\Rightarrow \quad R_{Th} = 7\,\Omega$

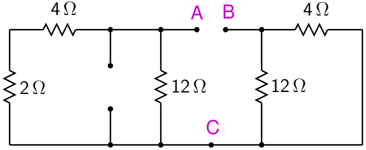
V_{oc}:



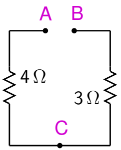
Thevenin's theorem: example



R_{Th}:

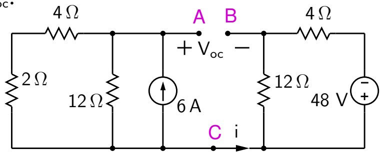


$\equiv$



$\Rightarrow$ $R_{Th} = 7\,\Omega$

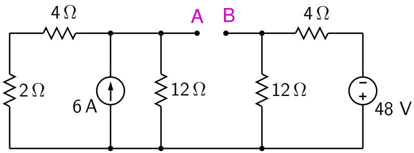
V_{oc}:



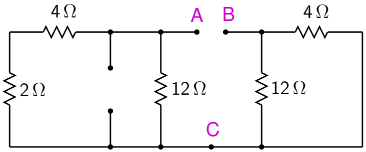
Note: $i = 0$ (since there is no return path).

$$
\begin{aligned}
V_{AB} &= V_A - V_B \\
&= (V_A - V_C) + (V_C - V_B) \\
&= V_{AC} + V_{CB} \\
&= 24\,\mathrm{V} + 36\,\mathrm{V} = 60\,\mathrm{V}
\end{aligned}
$$

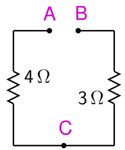
Thevenin's theorem: example



R_{Th}:

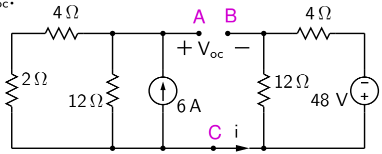


$\equiv$



$\Rightarrow \; R_{Th} = 7\,\Omega$

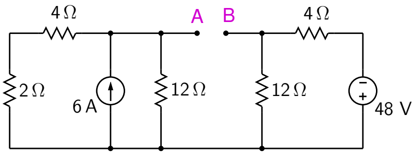
V_{oc}:



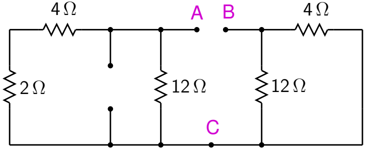
Note: $i = 0$ (since there is no return path).

$$\begin{aligned} V_{AB} &= V_A - V_B \\ &= (V_A - V_C) + (V_C - V_B) \\ &= V_{AC} + V_{CB} \\ &= 24\,\text{V} + 36\,\text{V} = 60\,\text{V} \end{aligned}$$

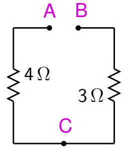
$$\boxed{\begin{aligned} V_{Th} &= 60\,\text{V} \\ R_{Th} &= 7\,\Omega \end{aligned}}$$



R_{Th}:

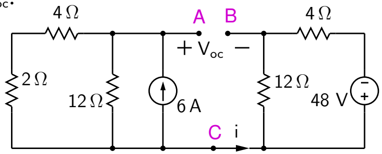


$\equiv$



$\Rightarrow \quad R_{Th} = 7\,\Omega$

V_{oc}:

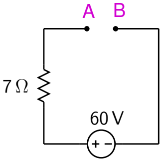


Note: $i = 0$ (since there is no return path).

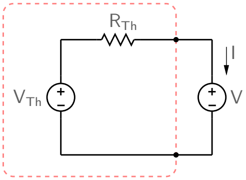
$$
\begin{aligned}
V_{AB} &= V_A - V_B \\
&= (V_A - V_C) + (V_C - V_B) \\
&= V_{AC} + V_{CB} \\
&= 24\,\mathrm{V} + 36\,\mathrm{V} = 60\,\mathrm{V}
\end{aligned}
$$

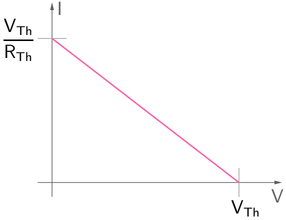
$V_{Th} = 60\,\mathrm{V}$

$R_{Th} = 7\,\Omega$

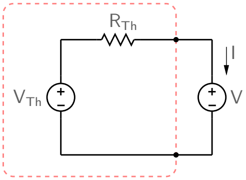


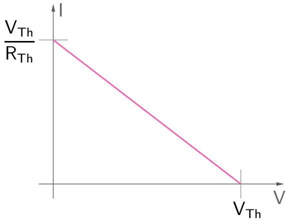
Graphical method for finding V_{Th} and R_{Th}





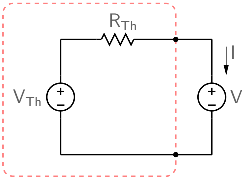
$I = \dfrac{V_{\text{Th}} - V}{R_{\text{Th}}}$ (Note: negative slope for I versus V plot)

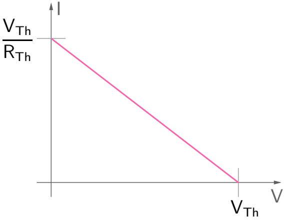




$I = \dfrac{V_{\text{Th}} - V}{R_{\text{Th}}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\text{Th}}$ (same as V_{oc})



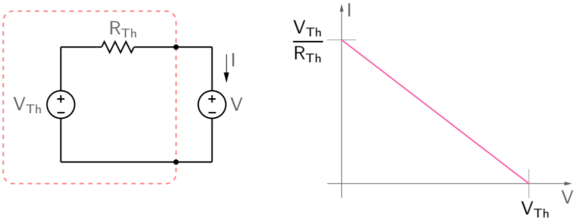


$I = \dfrac{V_{\mathrm{Th}} - V}{R_{\mathrm{Th}}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\mathrm{Th}}$ (same as V_{oc})

$V = 0 \rightarrow I = \dfrac{V_{\mathrm{Th}}}{R_{\mathrm{Th}}}$ (same as I_{sc})

Graphical method for finding V_{Th} and R_{Th}



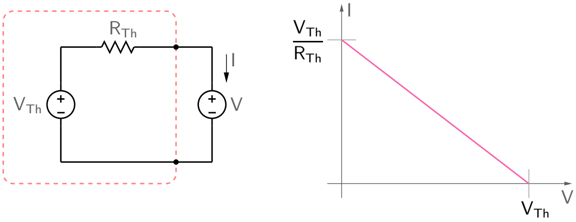
$I = \dfrac{V_{\rm Th} - V}{R_{\rm Th}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\rm Th}$ (same as $V_{\rm oc}$)

$V = 0 \rightarrow I = \dfrac{V_{\rm Th}}{R_{\rm Th}}$ (same as $I_{\rm sc}$)

i.e., a plot of I versus V can be used to find $V_{\rm Th}$ and $R_{\rm Th}$.

Graphical method for finding V_{Th} and R_{Th}



$I = \dfrac{V_{\text{Th}} - V}{R_{\text{Th}}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\text{Th}}$ (same as V_{oc})

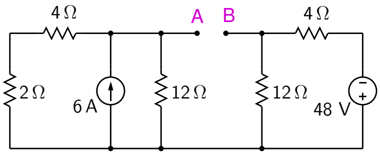
$V = 0 \rightarrow I = \dfrac{V_{\text{Th}}}{R_{\text{Th}}}$ (same as I_{sc})

i.e., a plot of I versus V can be used to find V_{Th} and R_{Th}.

(Instead of a voltage source, we could also connect a resistor load (R), vary R, and then plot I versus V.)

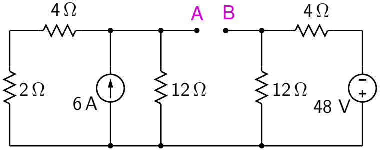
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



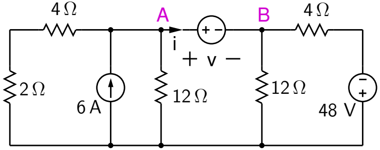
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



Connect a voltage source between A and B.

Plot i versus v.

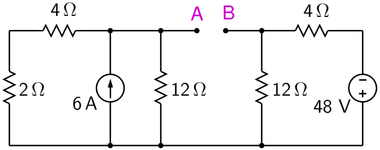


V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.

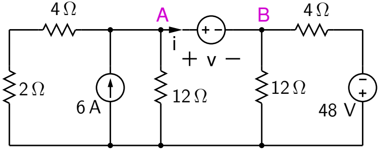
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



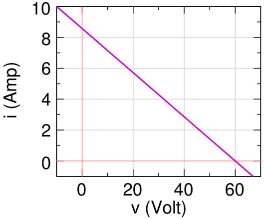
Connect a voltage source between A and B.

Plot i versus v.



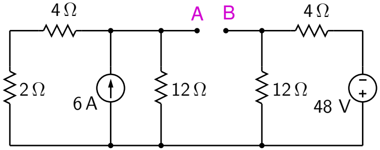
V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.



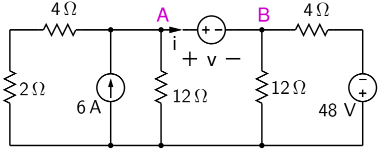
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



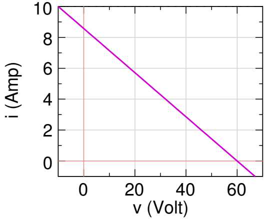
Connect a voltage source between A and B.

Plot i versus v.



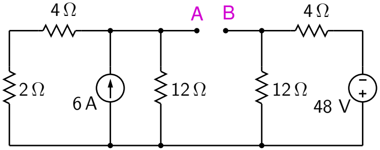
V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.



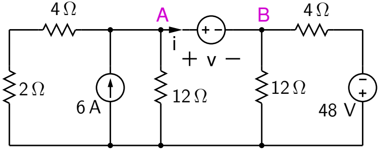
$V_{oc} = 60$ V, $I_{sc} = 8.57$ A

$R_{Th} = V_{oc}/I_{sc} = 7\ \Omega$



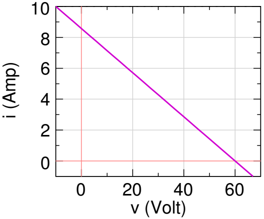
Connect a voltage source between A and B.

Plot i versus v.



V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.

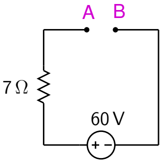


$V_{oc} = 60$ V, $I_{sc} = 8.57$ A

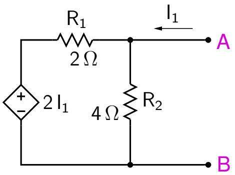
$R_{Th} = V_{oc}/I_{sc} = 7\ \Omega$

$V_{Th} = 60\,\text{V}$

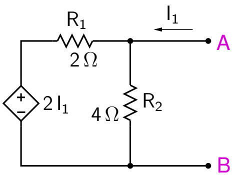
$R_{Th} = 7\,\Omega$



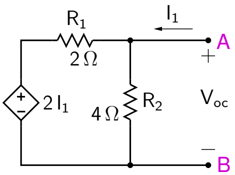
Thevenin's theorem: example



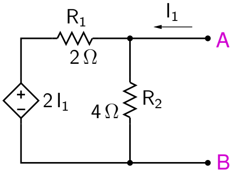
Thevenin's theorem: example



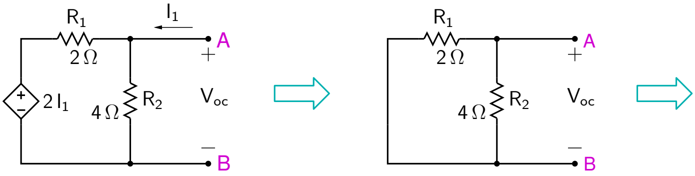
$V_{Th} = V_{oc}$



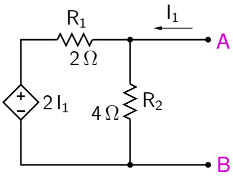
Thevenin's theorem: example



$V_{Th} = V_{oc}$

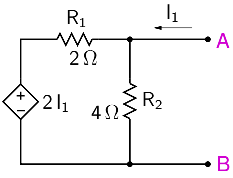


$\boxed{V_{Th} = 0}$



R_{Th}: Deactivate independent sources, connect a test source.

Thevenin's theorem: example



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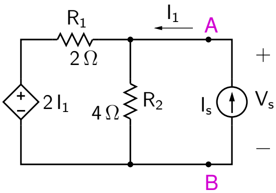

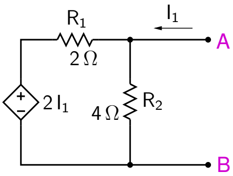

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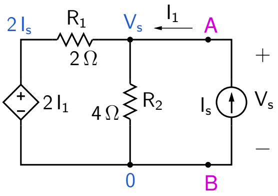
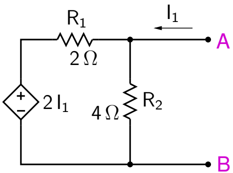


Thevenin's theorem: example



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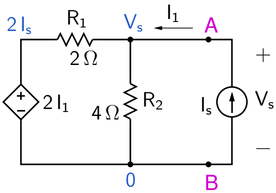
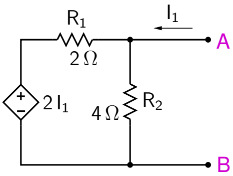


We need to compute $R_{Th} = \dfrac{V_s}{I_s}$.

KCL: $-I_s + \dfrac{V_s}{R_2} + \dfrac{V_s - 2\,I_s}{R_1} = 0$



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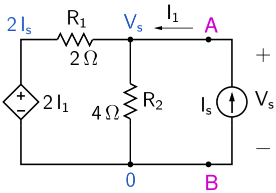

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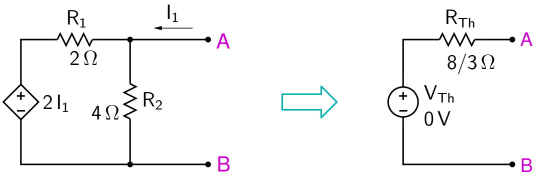
$\rightarrow V_s \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} \right) = I_s \left(1 + \dfrac{2}{R_1} \right)$

$\rightarrow R_{Th} = \dfrac{V_s}{I_s} = \dfrac{8}{3}\,\Omega$

Thevenin's theorem: example



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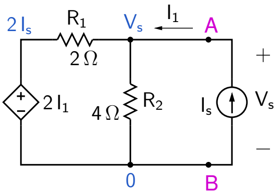

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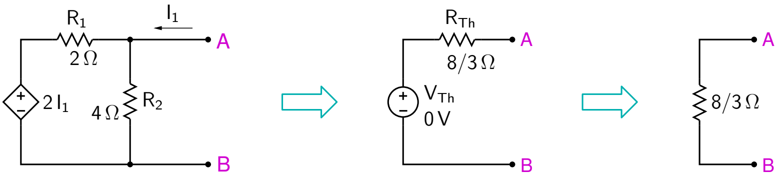
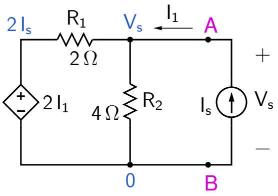
Thevenin's theorem: example



R_{Th}: Deactivate independent sources, connect a test source.



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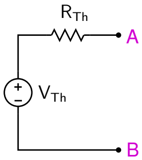
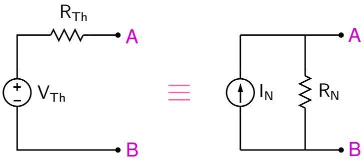
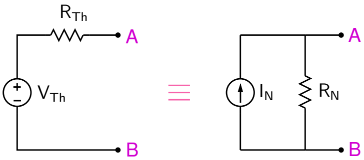
Norton equivalent circuit (source transformation)



Norton equivalent circuit (source transformation)

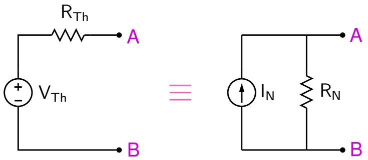


Norton equivalent circuit (source transformation)



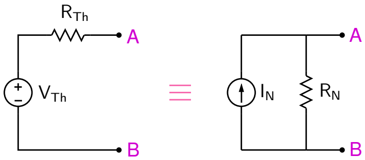
* Consider the open circuit case.

Norton equivalent circuit (source transformation)



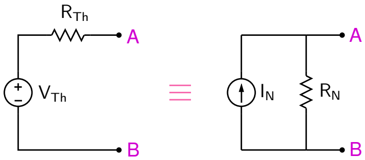
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.

Norton equivalent circuit (source transformation)



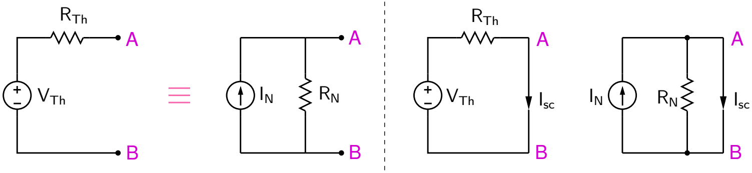
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.

Norton equivalent circuit (source transformation)



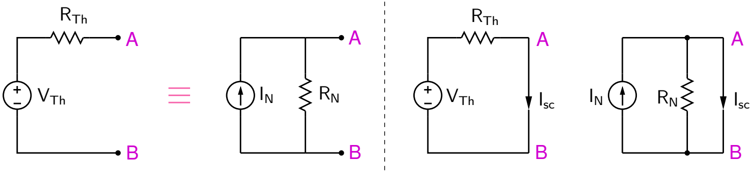
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N \, R_N$.
  $\Rightarrow V_{Th} = I_N \, R_N$.

Norton equivalent circuit (source transformation)



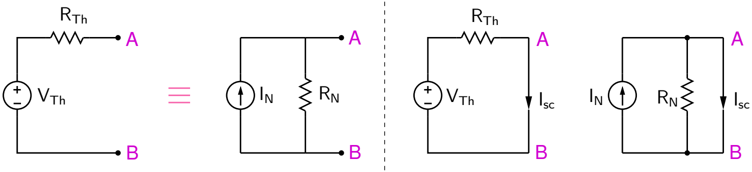
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.
  $\Rightarrow V_{Th} = I_N\, R_N$.
* Consider the short circuit case.

Norton equivalent circuit (source transformation)



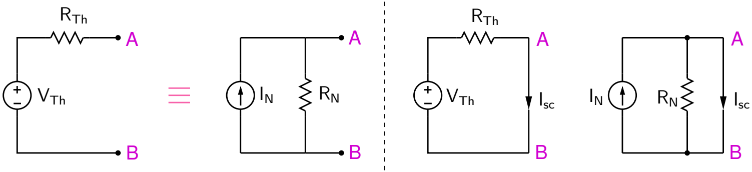
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N \, R_N$.
  $\Rightarrow V_{Th} = I_N \, R_N$.
* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.

Norton equivalent circuit (source transformation)



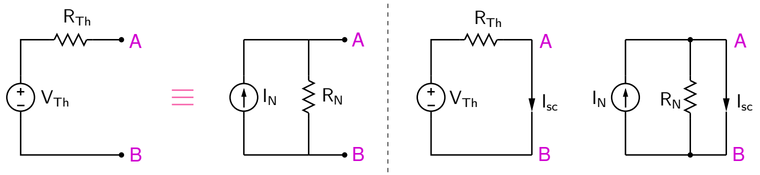
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.
  $\Rightarrow V_{Th} = I_N\, R_N$.
* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.
  $\Rightarrow V_{Th} = I_N\, R_N$.
* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.
  $\Rightarrow V_{Th} = \dfrac{V_{Th}}{R_{Th}}\, R_N$

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.
  $\Rightarrow V_{Th} = I_N\, R_N$.
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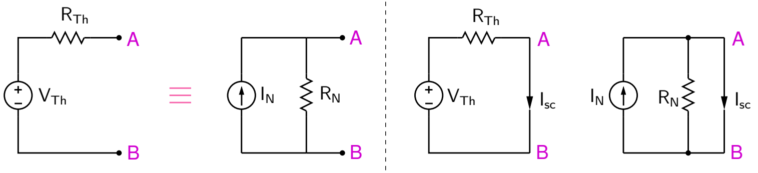
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.
  $\Rightarrow V_{Th} = \dfrac{V_{Th}}{R_{Th}}\, R_N \;\rightarrow\; R_{Th} = R_N$.

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
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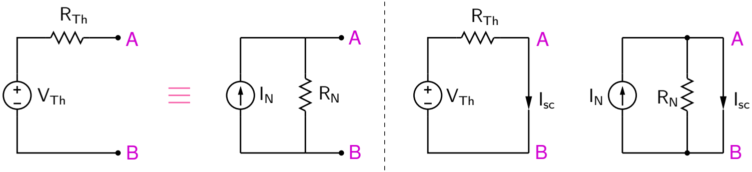
* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.
  $\Rightarrow V_{Th} = \dfrac{V_{Th}}{R_{Th}}\, R_N \;\rightarrow\; R_{Th} = R_N$.

$$\boxed{R_N = R_{Th},\; I_N = \frac{V_{Th}}{R_{Th}}}$$

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
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* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
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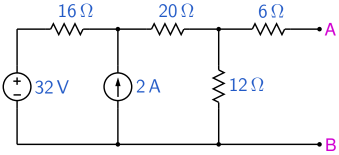
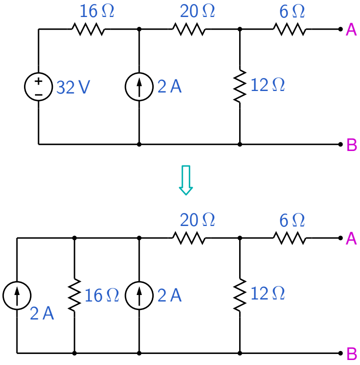
$\boxed{R_N = R_{Th},\ I_N = \dfrac{V_{Th}}{R_{Th}}}$ $\boxed{R_{Th} = R_N,\ V_{Th} = I_N R_N}$

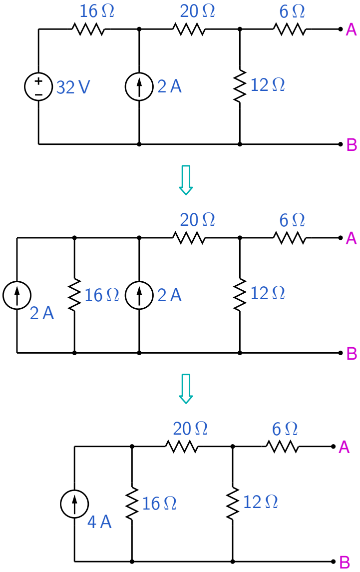
Source transformation: example



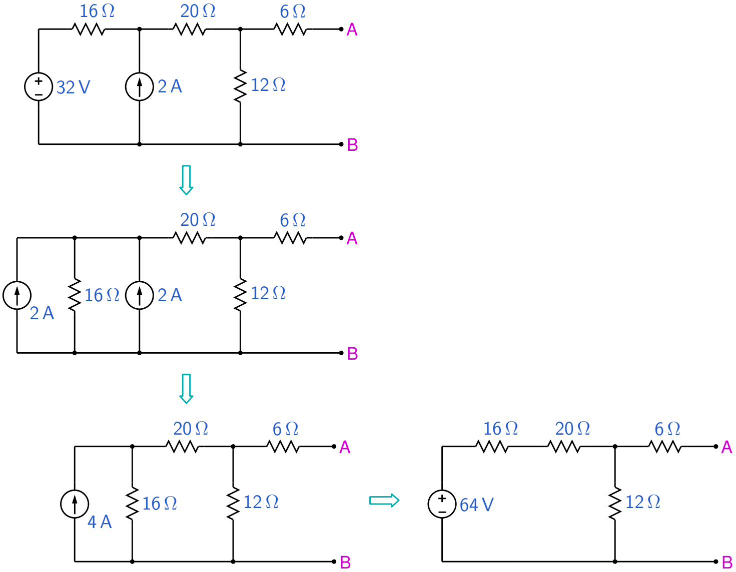
Source transformation: example



Source transformation: example



Source transformation: example



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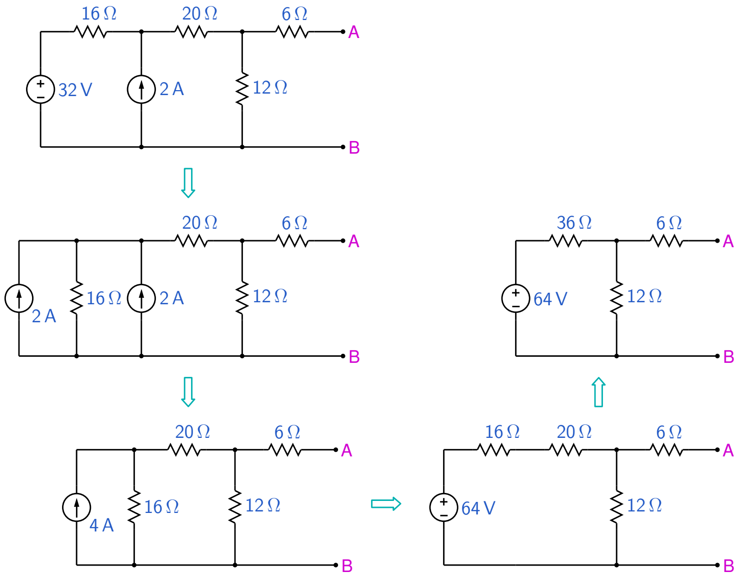

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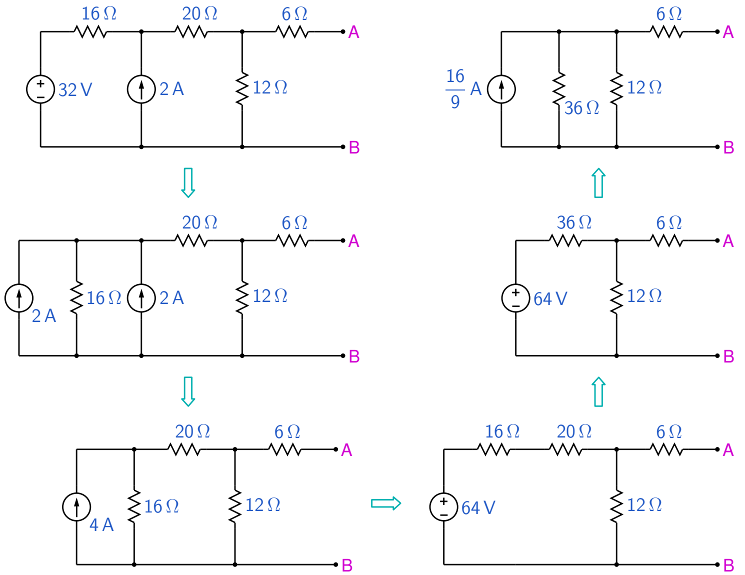

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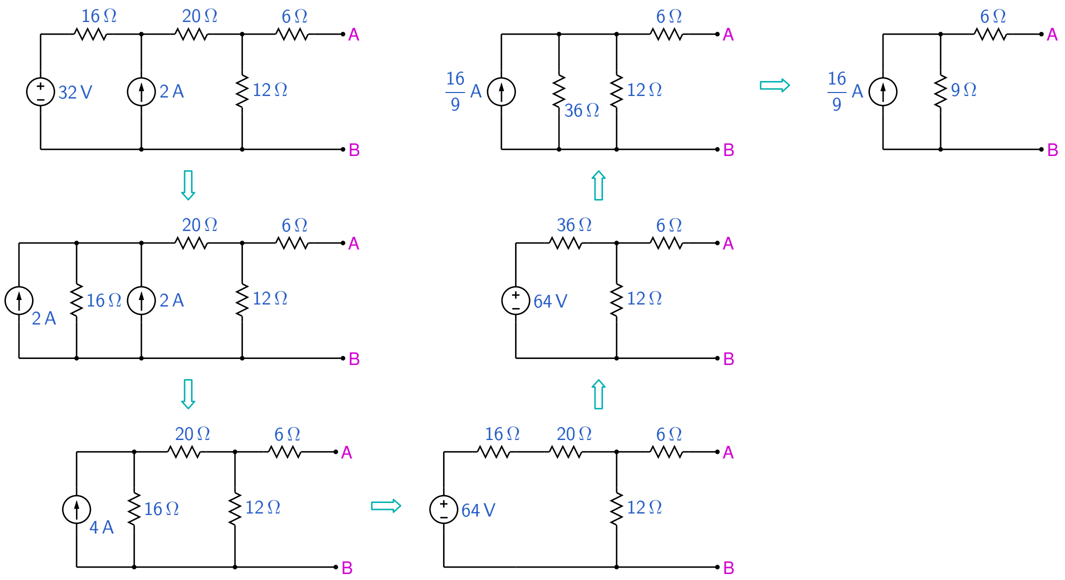

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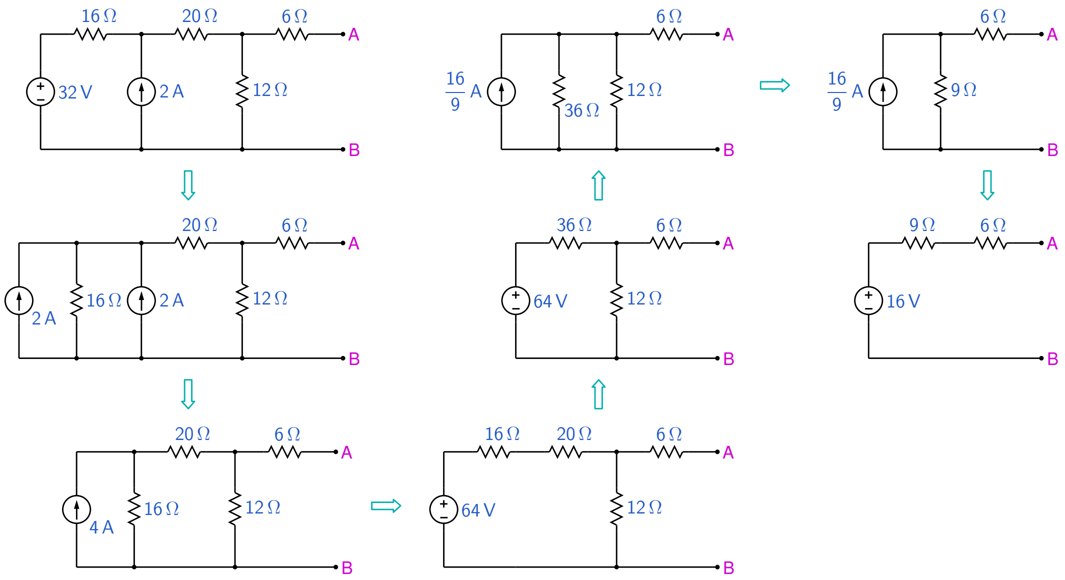

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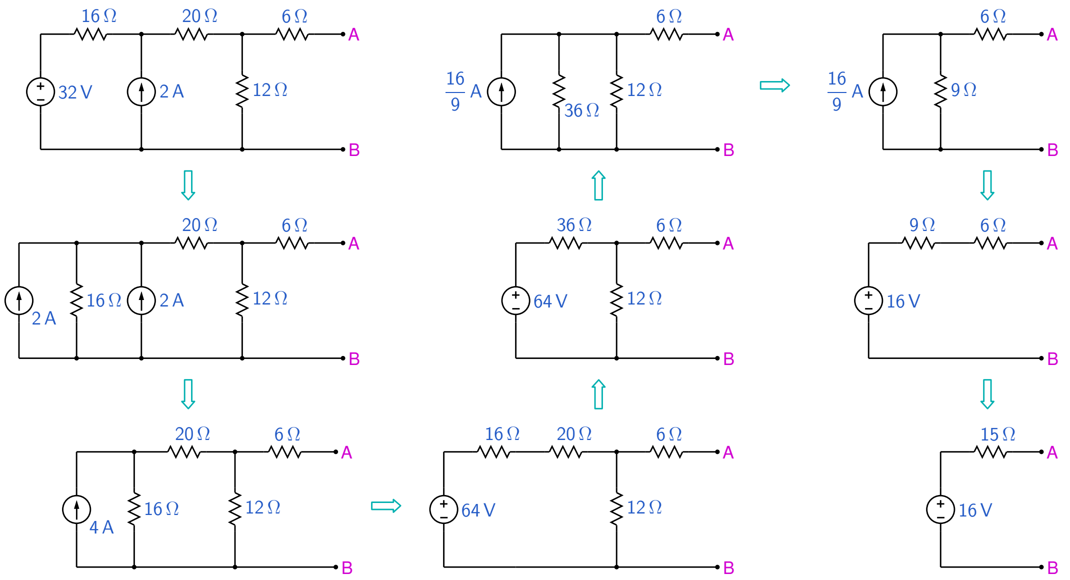
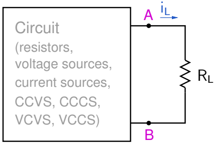
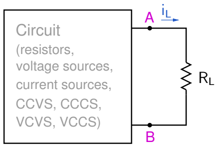


Maximum power transfer

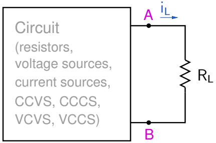


Maximum power transfer



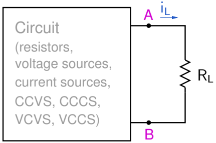
* Power "transferred" to load is, $P_L = i_L^2 \, R_L$.

Maximum power transfer



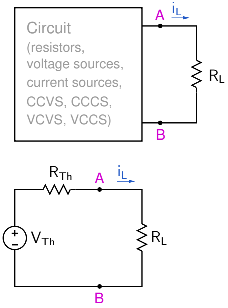
* Power "transferred" to load is, $P_L = i_L^2 \, R_L$.
* For a given black box, what is the value of R_L for which P_L is maximum?

Maximum power transfer



* Power "transferred" to load is, $P_L = i_L^2 \, R_L$.
* For a given black box, what is the value of R_L for which P_L is maximum?
* Replace the black box with its Thevenin equivalent.

Maximum power transfer



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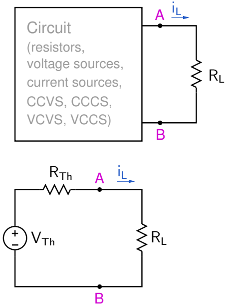
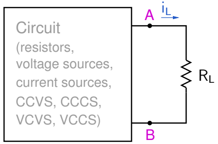
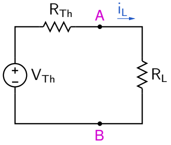


* Power "transferred" to load is, $P_L = i_L^2\, R_L$.
* For a given black box, what is the value of R_L for which P_L is maximum?
* Replace the black box with its Thevenin equivalent.
* $i_L = \dfrac{V_{Th}}{R_{Th} + R_L}$, $P_L = V_{Th}^2 \times \dfrac{R_L}{(R_{Th} + R_L)^2}$.

Maximum power transfer

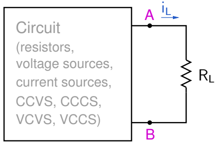


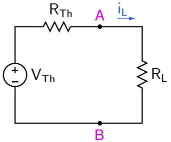


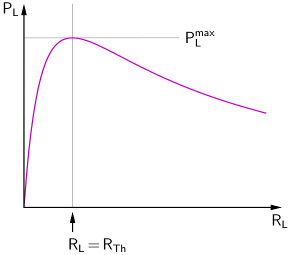
* Power "transferred" to load is, $P_L = i_L^2\, R_L$.
* For a given black box, what is the value of R_L for which P_L is maximum?
* Replace the black box with its Thevenin equivalent.
* $i_L = \dfrac{V_{Th}}{R_{Th} + R_L}$, $P_L = V_{Th}^2 \times \dfrac{R_L}{(R_{Th} + R_L)^2}$.
* For $\dfrac{dP_L}{dR_L} = 0$, we need

$$\frac{(R_{Th} + R_L)^2 - R_L \times 2\,(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0\,,$$

i.e., $R_{Th} + R_L = 2\,R_L \Rightarrow R_L = R_{Th}$.







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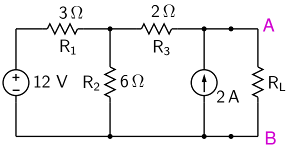
* For $\dfrac{dP_L}{dR_L} = 0$, we need

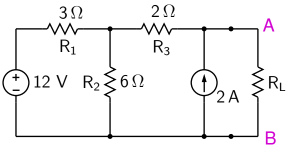
$$\frac{(R_{Th} + R_L)^2 - R_L \times 2\,(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0\,,$$

i.e., $R_{Th} + R_L = 2\,R_L \Rightarrow R_L = R_{Th}$.

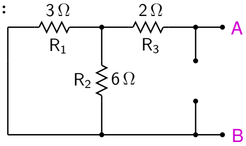
Find R_L for which P_L is maximum.



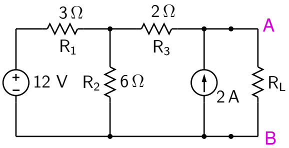
Find R_L for which P_L is maximum.



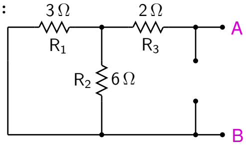
R_{Th}:



Find R_L for which P_L is maximum.



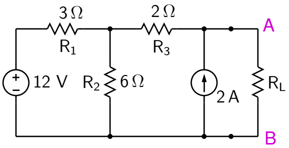
R_{Th}:



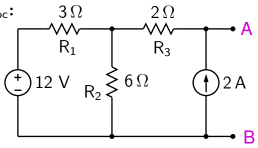
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

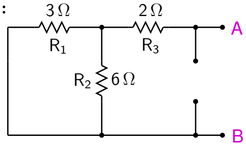
Find R_L for which P_L is maximum.



V_{oc}:



R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

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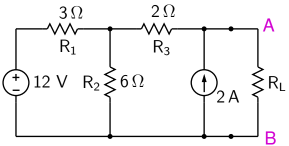
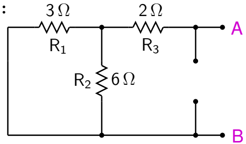
Find R_L for which P_L is maximum.



R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

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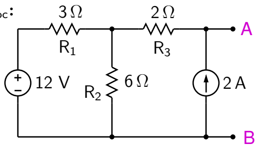
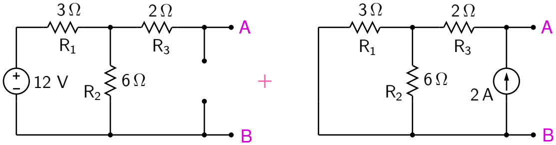
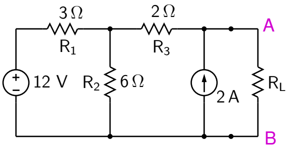
V_{oc}:



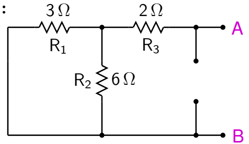
Use superposition to find V_{oc}:



Find R_L for which P_L is maximum.



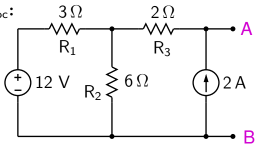
R_{Th}:



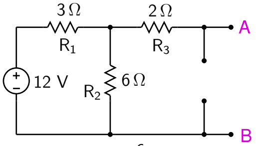
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

V_{oc}:



Use superposition to find V_{oc}:



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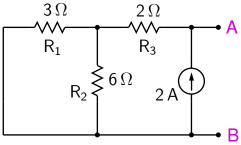
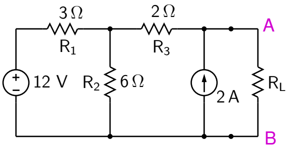
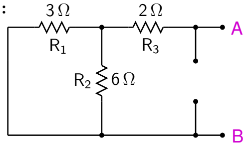


$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8 \text{ V}$$

Find R_L for which P_L is maximum.



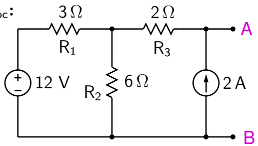
R_{Th}:



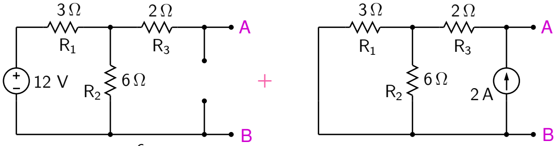
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

V_{oc}:



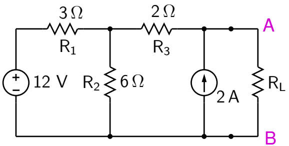
Use superposition to find V_{oc}:



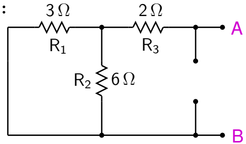
$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8 \text{ V}$$

$$V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

Find R_L for which P_L is maximum.



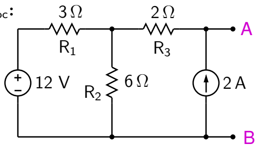
R_{Th}:



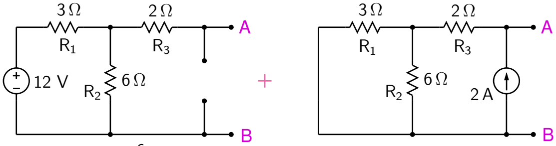
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

V_{oc}:



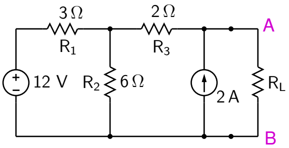
Use superposition to find V_{oc}:



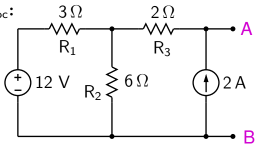
$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8 \text{ V} \qquad V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16\,\text{V}$$

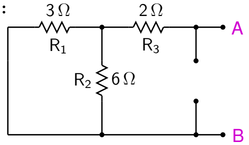
Find R_L for which P_L is maximum.



V_{oc}:



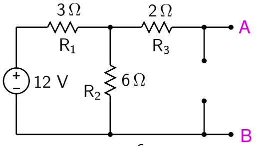
R_{Th}:



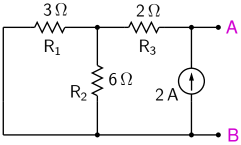
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

Use superposition to find V_{oc}:

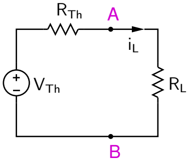


+



$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8\ \text{V} \qquad V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16\,\text{V}$$



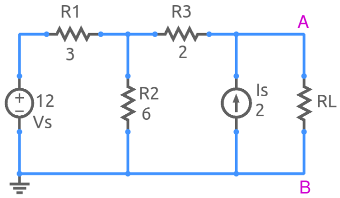
P_L is maximum when $R_L = R_{Th} = 4\,\Omega$

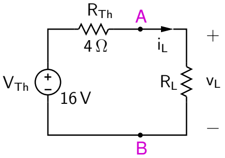
$\Rightarrow i_L = V_{Th}/(2\,R_{Th}) = 2\,\text{A}$

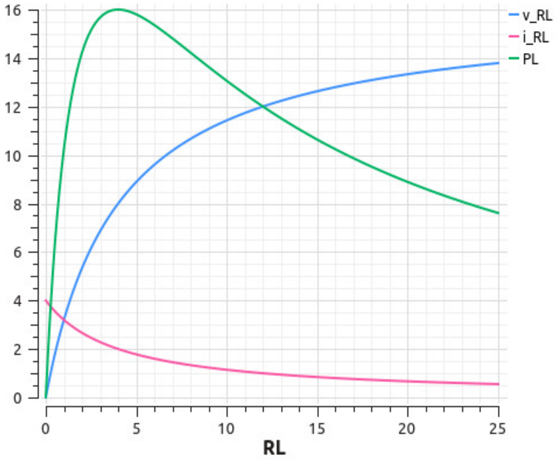
$P_L^{max} = 2^2 \times 4 = 16\,\text{W}$.

Maximum power transfer: simulation results

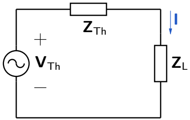
SEQUEL file: ee101_maxpwr_1.sqproj





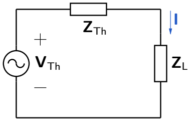


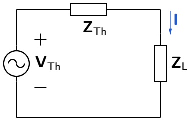
Maximum power transfer (sinusoidal steady state)



Maximum power transfer (sinusoidal steady state)

Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.



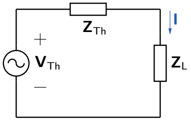


Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by $\mathbf{Z}_L$ is,

$$
\begin{aligned}
P &= \frac{1}{2}\, I_m^2 R_L \\
&= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L \\
&= \frac{1}{2}\, \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\, R_L \,.
\end{aligned}
$$

Maximum power transfer (sinusoidal steady state)



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

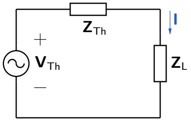
The power absorbed by $\mathbf{Z}_L$ is,

$$P = \frac{1}{2}\, I_m^2 R_L$$

$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$

$$= \frac{1}{2}\, \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\, R_L \,.$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by $\mathbf{Z}_L$ is,

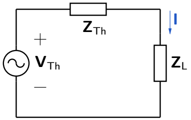
$$P = \frac{1}{2}\, I_m^2 R_L$$
$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$
$$= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\, R_L\,.$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.

With $X_L = -X_{Th}$, we have,

$$P = \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2}\, R_L\,,$$

which is maximum for $R_L = R_{Th}$.



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by $\mathbf{Z}_L$ is,

$$P = \frac{1}{2}\, I_m^2 R_L$$

$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$

$$= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\, R_L\,.$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.

With $X_L = -X_{Th}$, we have,

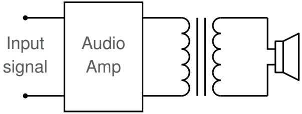
$$P = \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2}\, R_L\,,$$

which is maximum for $R_L = R_{Th}$.

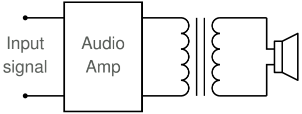
Therefore, for maximum power transfer to the load $\mathbf{Z}_L$, we need,

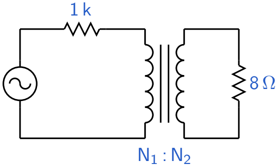
$R_L = R_{Th}$, $X_L = -X_{Th}$, i.e., $\boxed{\mathbf{Z}_L = \mathbf{Z}_{Th}^*.}$

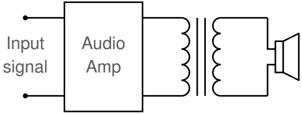
Impedance matching

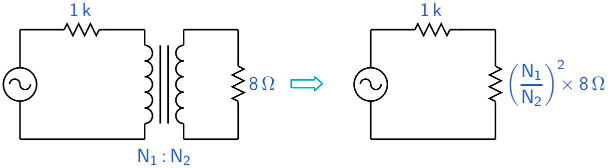


Impedance matching

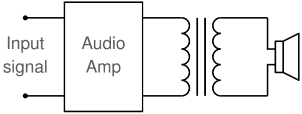


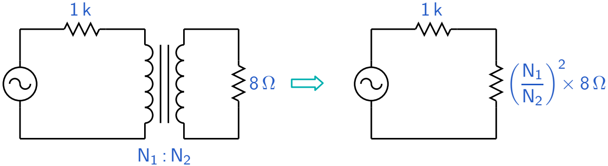



Input
signal
Audio
Amp

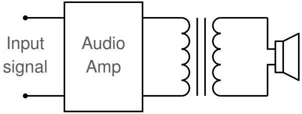

1 k
8 Ω
$N_1 : N_2$
1 k
$\left(\frac{N_1}{N_2}\right)^2 \times 8\,\Omega$

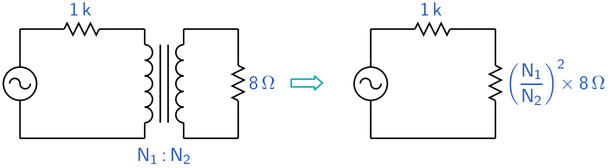
Impedance matching





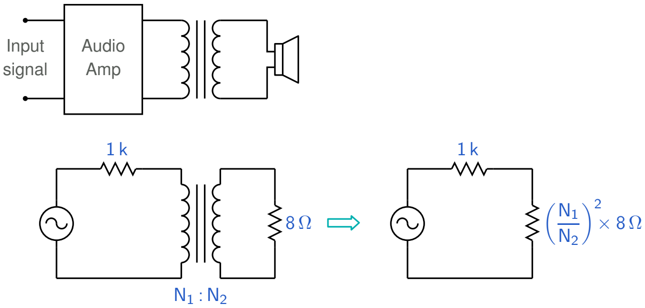
Calculate the turns ratio to provide maximum power transfer of the audio signal.





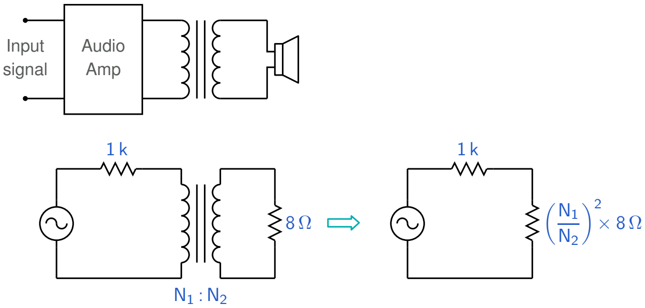
Calculate the turns ratio to provide maximum power transfer of the audio signal.

$Z_L = Z_{Th}^*$



Calculate the turns ratio to provide maximum power transfer of the audio signal.

$$Z_L = Z_{Th}^* \;\rightarrow\; \left(\frac{N_1}{N_2}\right)^2 \times 8\,\Omega = 1\,\text{k}\Omega$$



Calculate the turns ratio to provide maximum power transfer of the audio signal.

$$Z_L = Z^*_{Th} \;\rightarrow\; \left(\frac{N_1}{N_2}\right)^2 \times 8\,\Omega = 1\,\text{k}\Omega \;\rightarrow\; \frac{N_1}{N_2} = \sqrt{\frac{1000}{8}} = 11.2$$