Semiconductor Devices

MOS Transistors: Part 2

M. B. Patil
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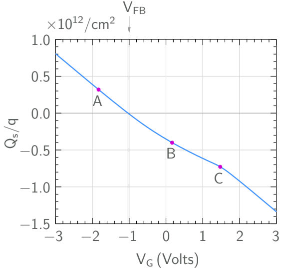
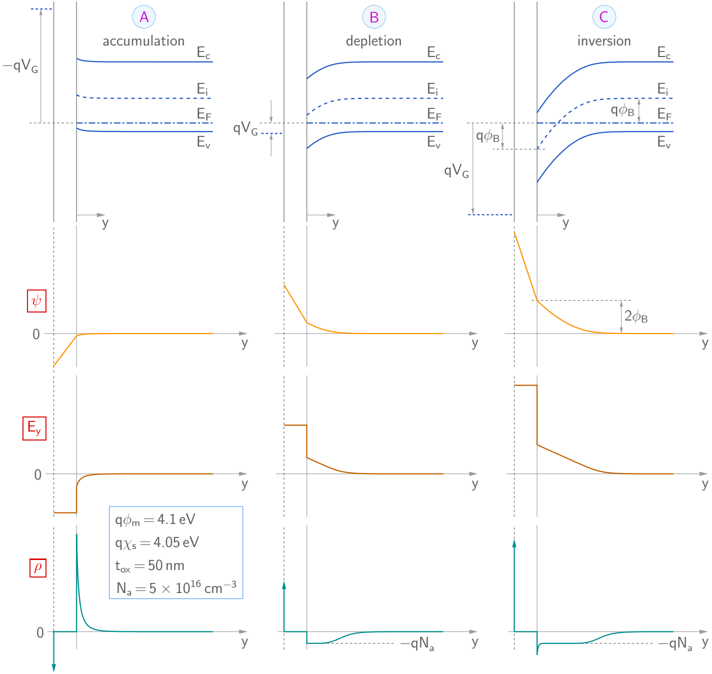
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay

MOS capacitor





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$q\chi_s = 4.05\,\text{eV}$

$t_{ox} = 50\,\text{nm}$

$N_a = 5 \times 10^{16}\,\text{cm}^{-3}$

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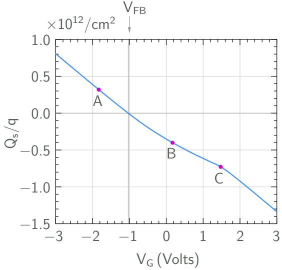

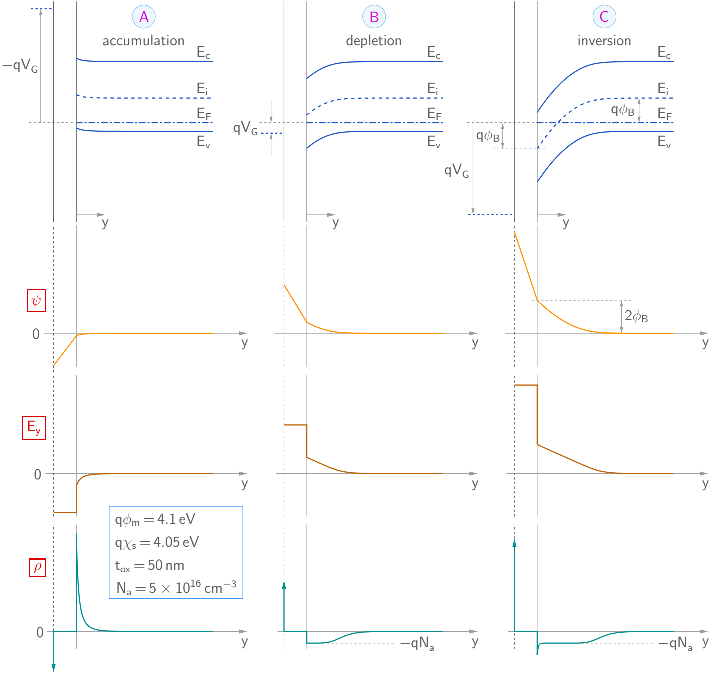

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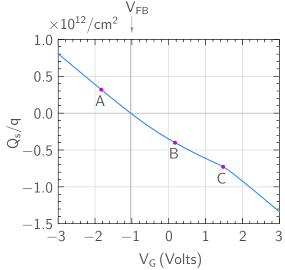

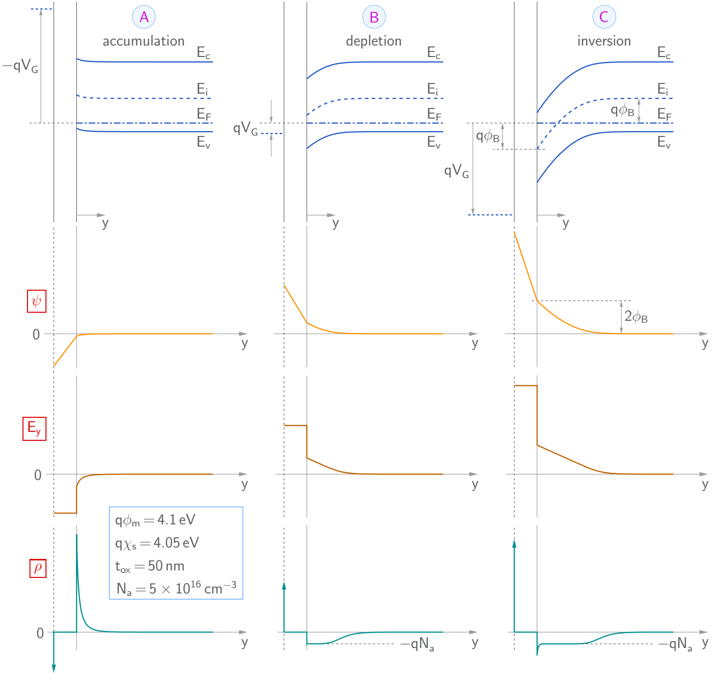

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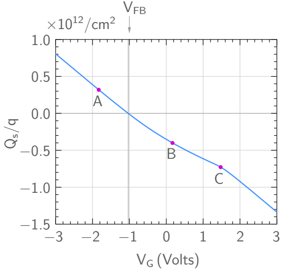

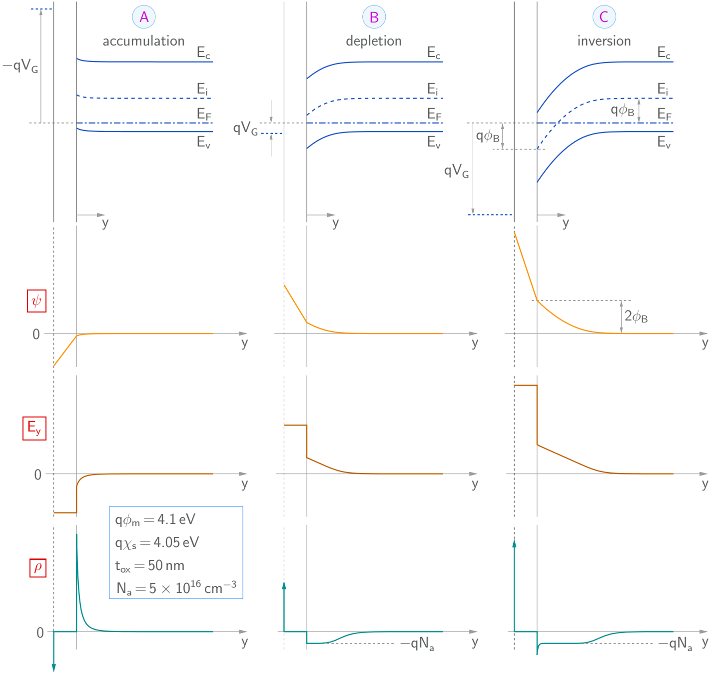

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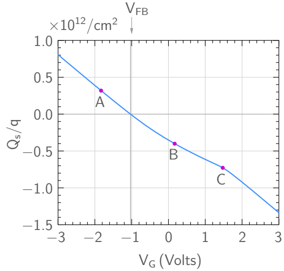

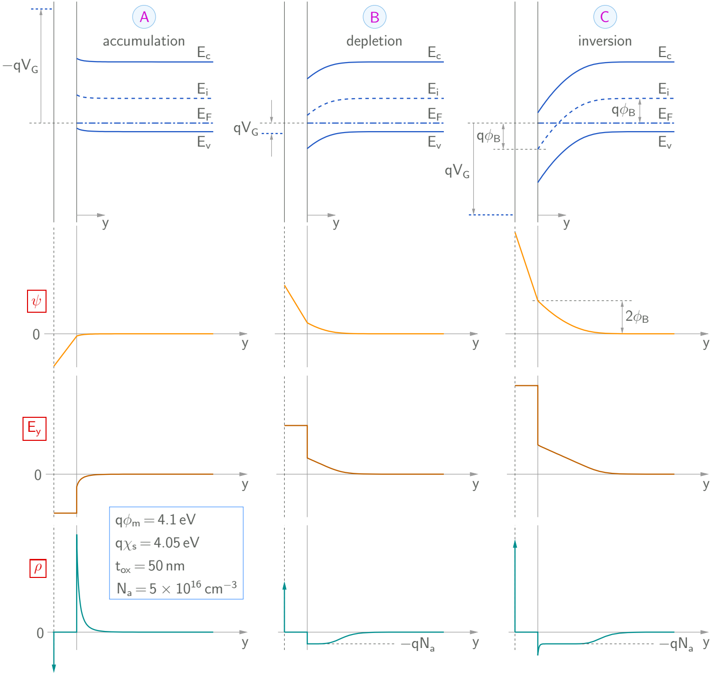

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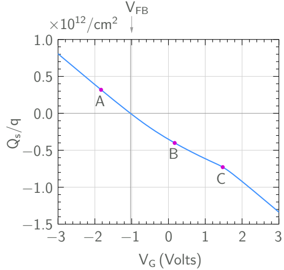
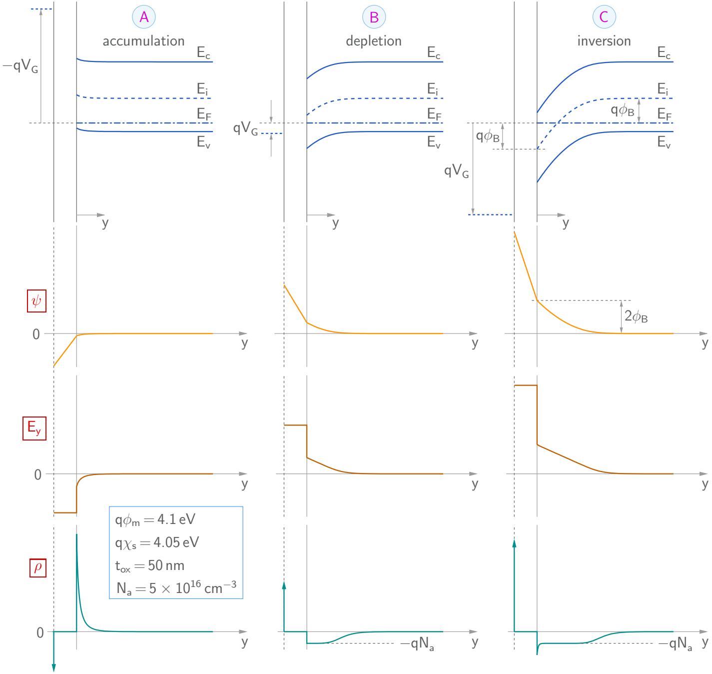
- Home work: Sketch ρ versus y for the case of an n-type substrate.





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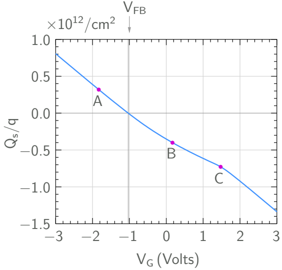

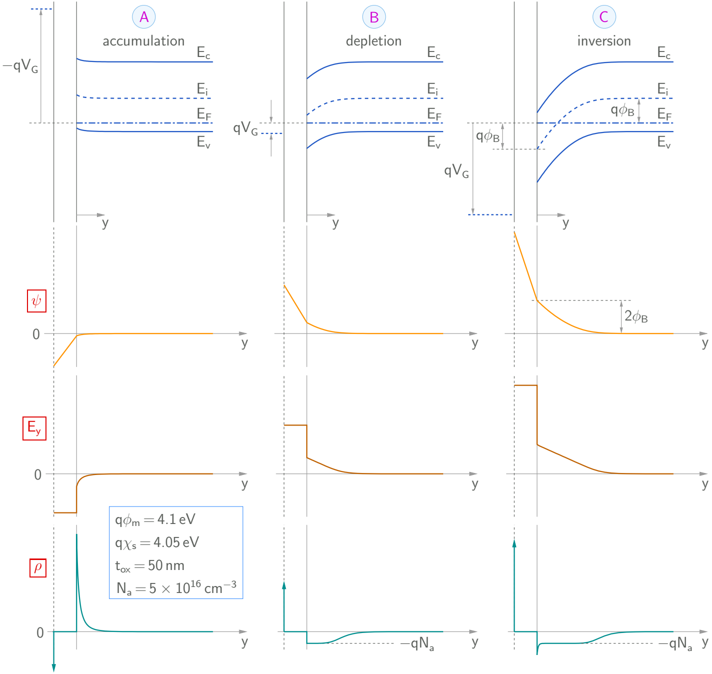

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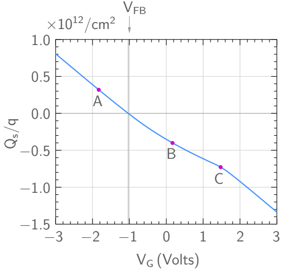

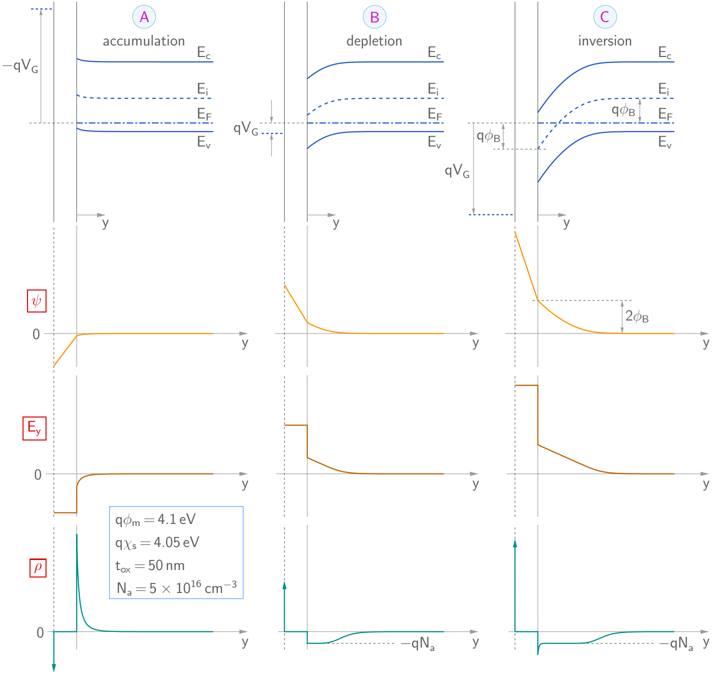

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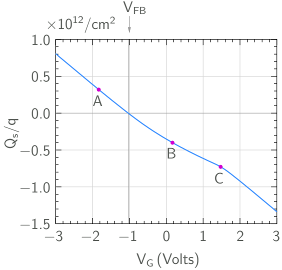

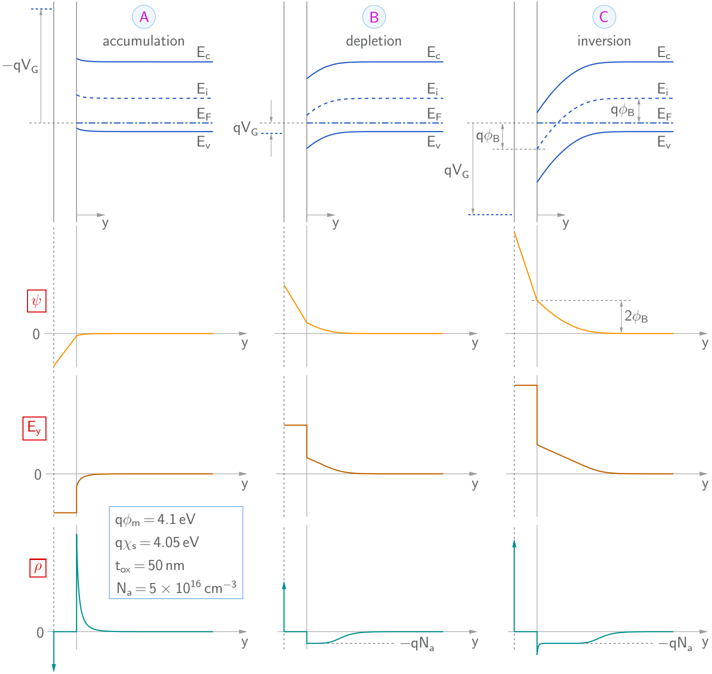

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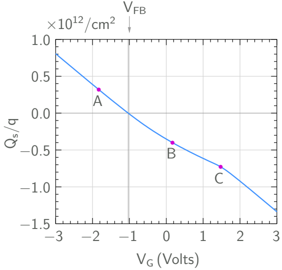
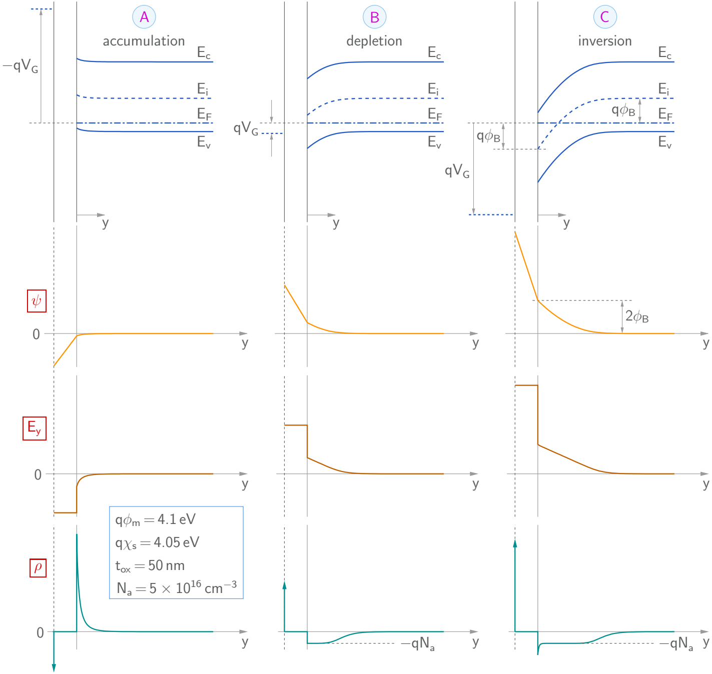
- $Q_I/(-q)$ gives the total number of electrons in the semiconductor per unit area (in the *x*-*z* plane).





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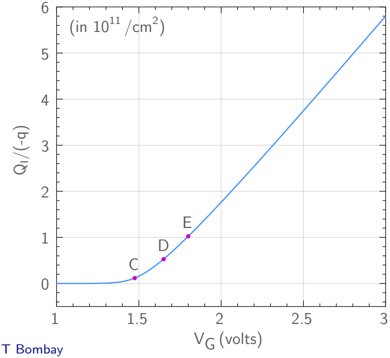
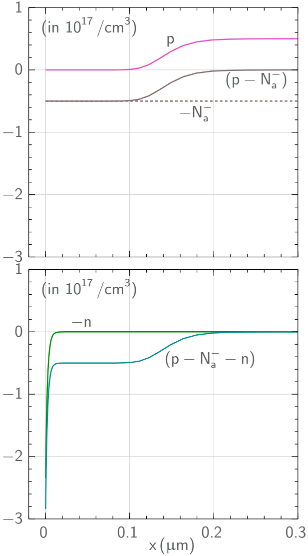
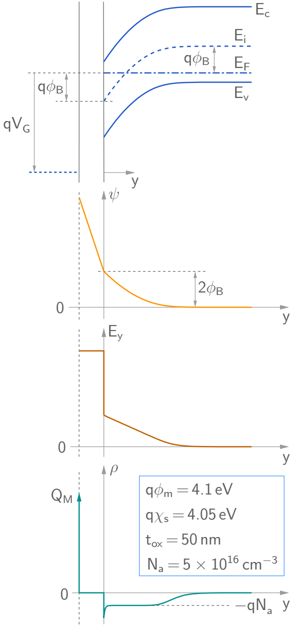




$V_G = 1.65$ V



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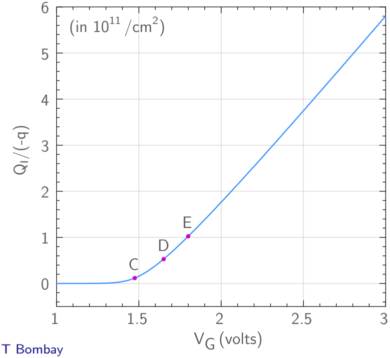

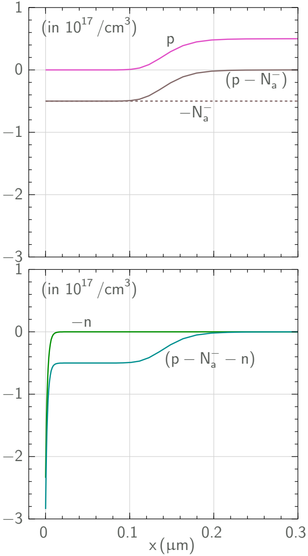

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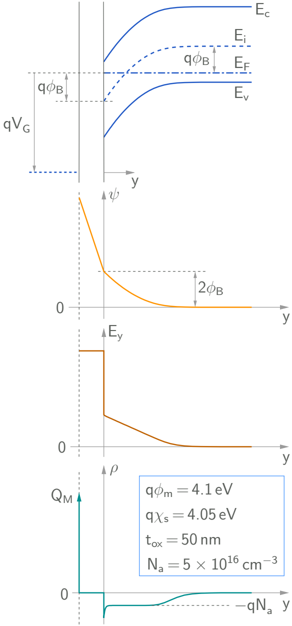

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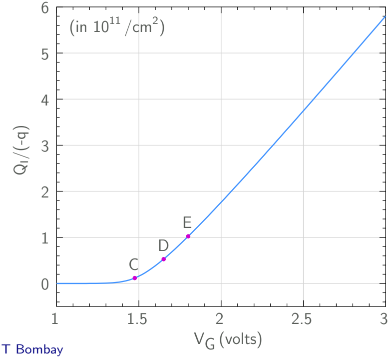

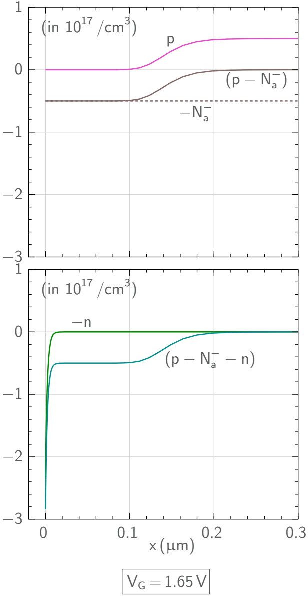

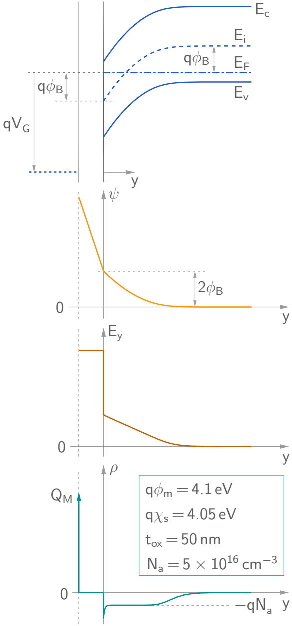

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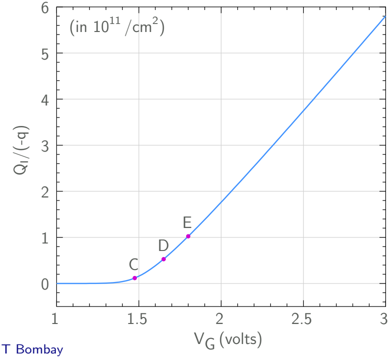

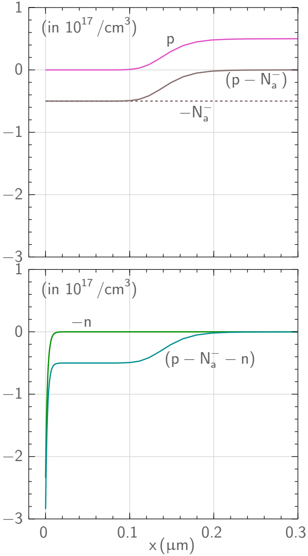

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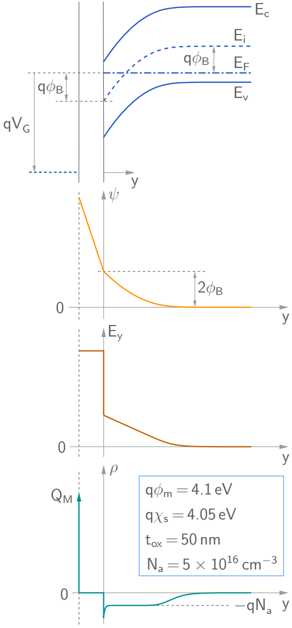

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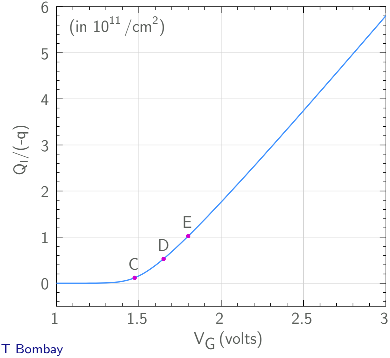
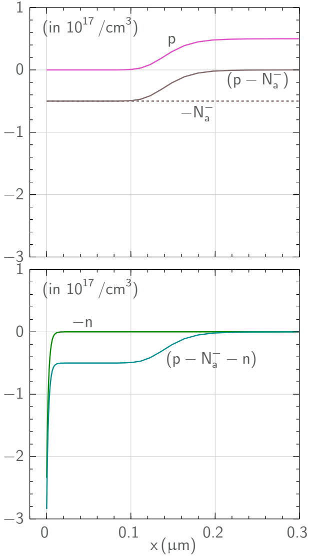
- $n(y) = n_0 = \dfrac{n_i^2}{p_0}$ in most of the device except near the surface where it increases dramatically.





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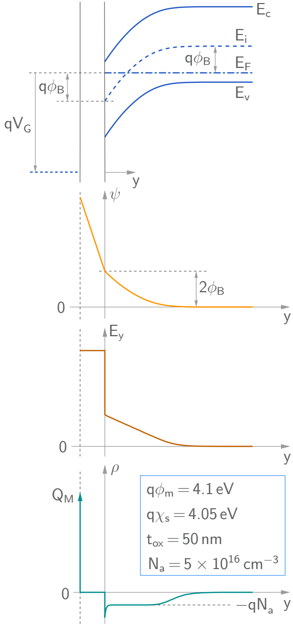

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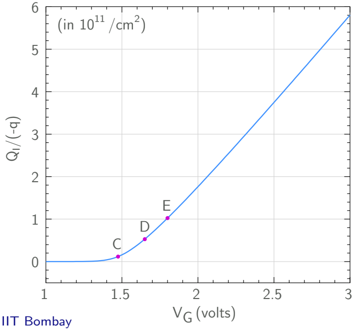

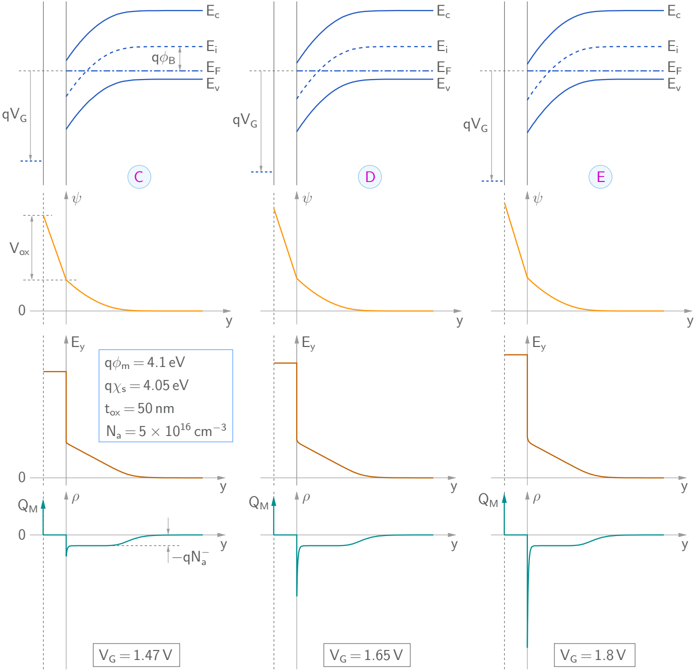

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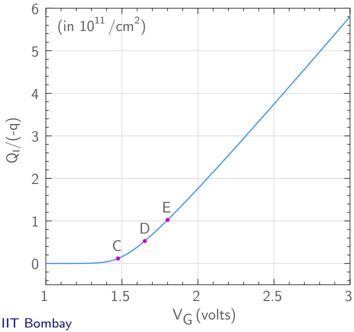

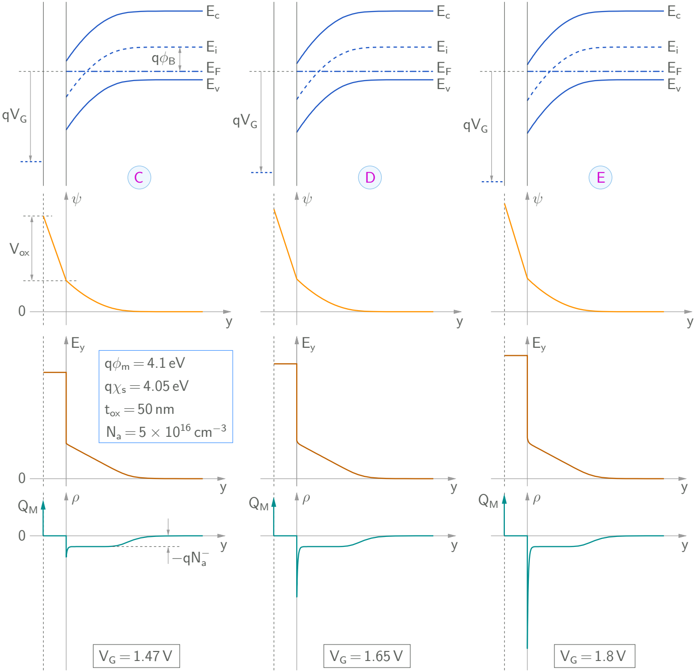

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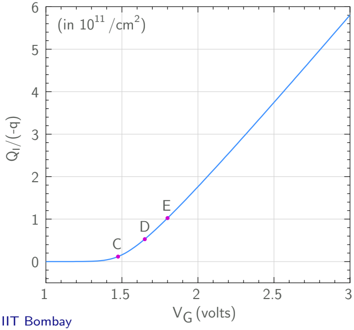

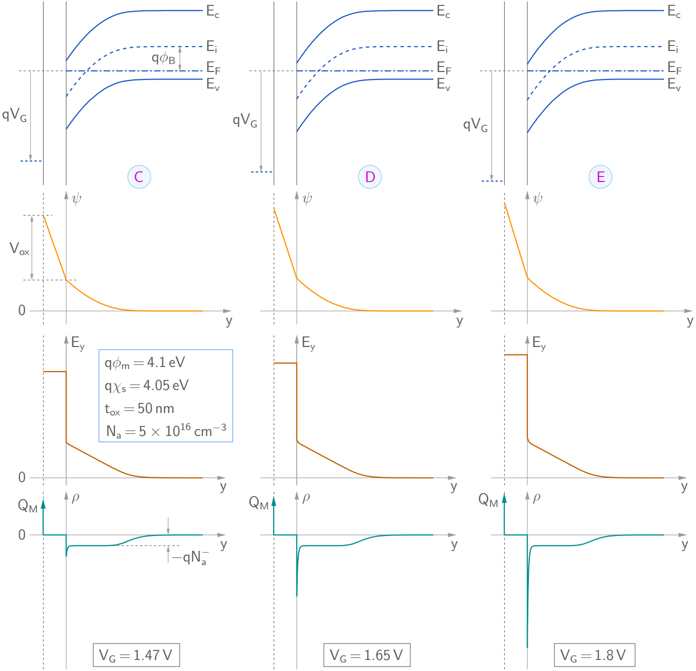

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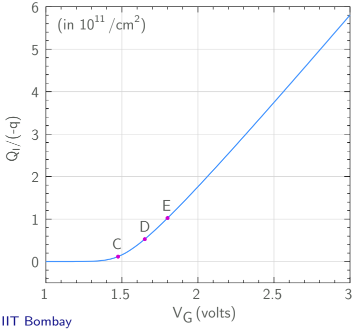

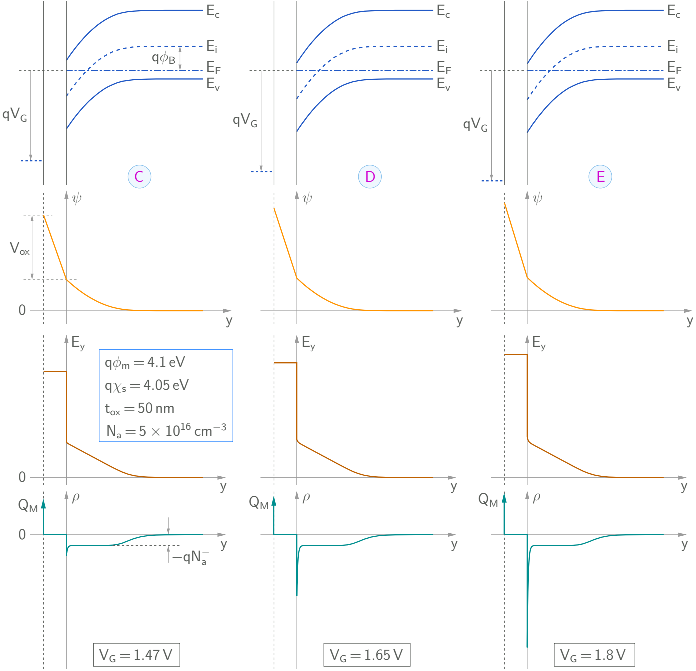

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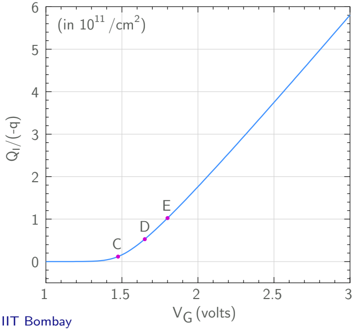
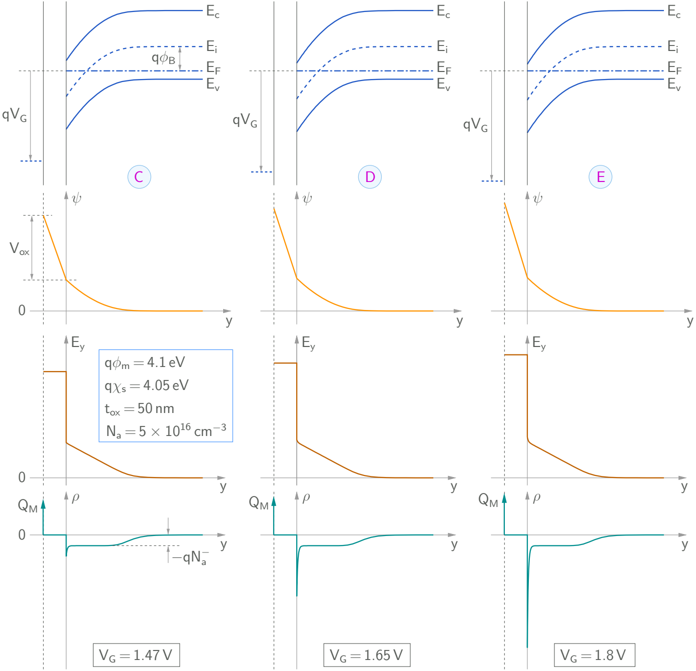
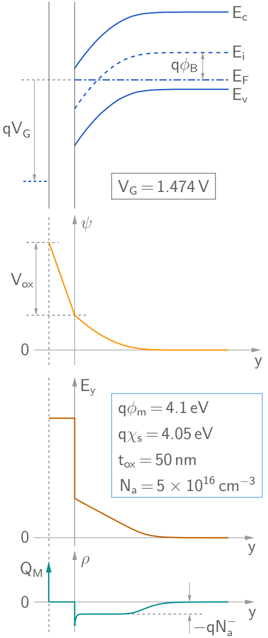
- The inversion charge Q_I is confined to a narrow region near the Si-SiO$_2$ interface, typically about 10 nm (i.e., 100 Å).





MOS capacitor: threshold voltage



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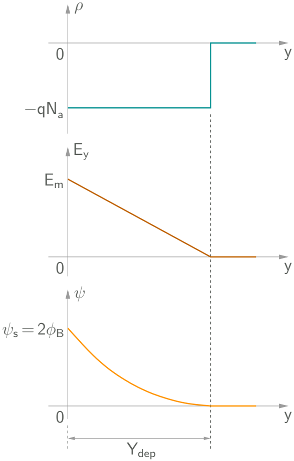

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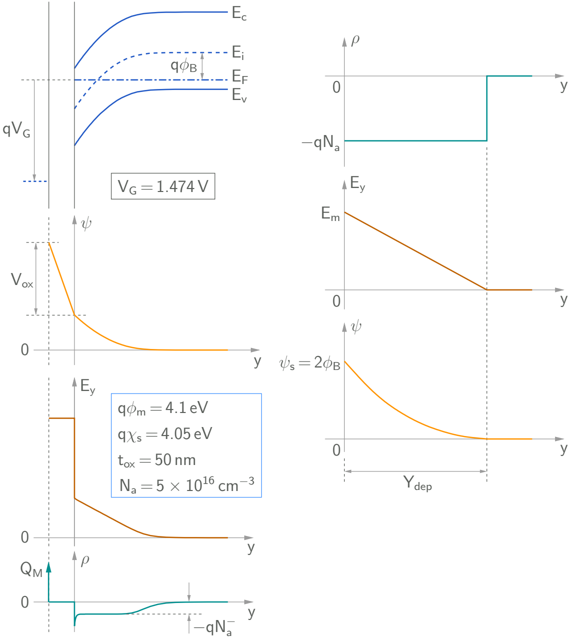

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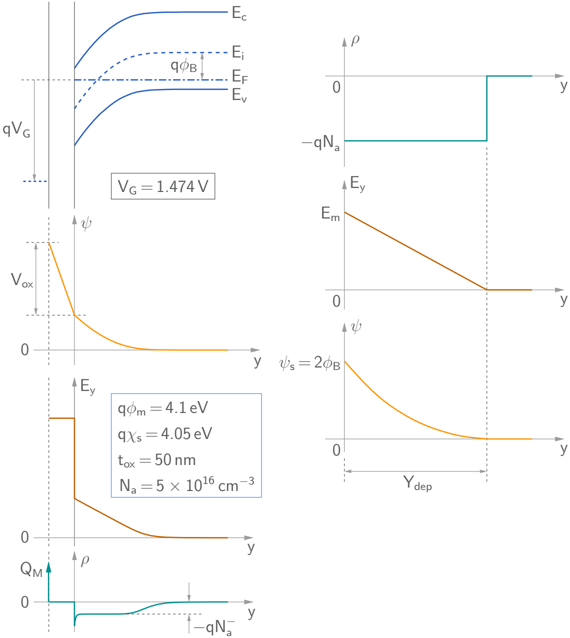
With the depletion approximation, and with Y_{dep} as the depletion width at the onset of inversion, we get

$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0^+}^{Y_{\mathrm{dep}}} d\mathcal{E}_y = \frac{1}{\epsilon_{\mathrm{Si}}} \int_{0^+}^{Y_{\mathrm{dep}}} \rho\, dy$$



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Consider the onset of inversion. We have $\psi_s \approx 2\,\phi_B$, and the electron density at the interface has not yet become significant $\rightarrow n(y) = 0$.

With the depletion approximation, and with Y_{dep} as the depletion width at the onset of inversion, we get

$$\frac{d\mathcal{E}_y}{dy} = \frac{\rho}{\epsilon} \rightarrow \int_{0^+}^{Y_{\mathrm{dep}}} d\mathcal{E}_y = \frac{1}{\epsilon_{\mathrm{Si}}} \int_{0^+}^{Y_{\mathrm{dep}}} \rho\, dy$$

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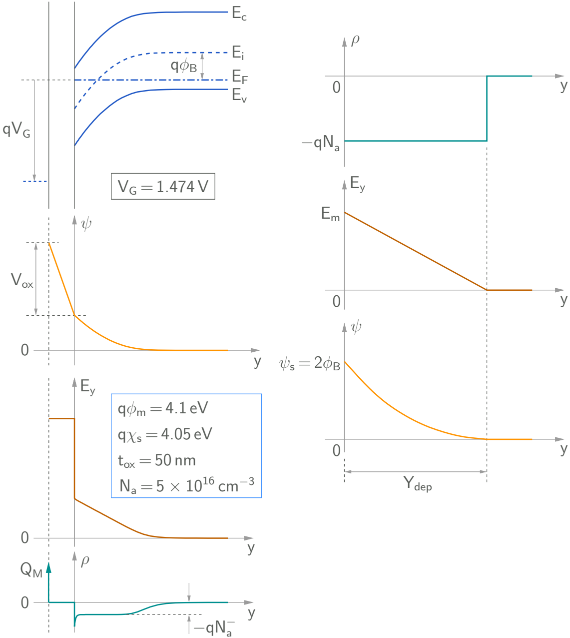

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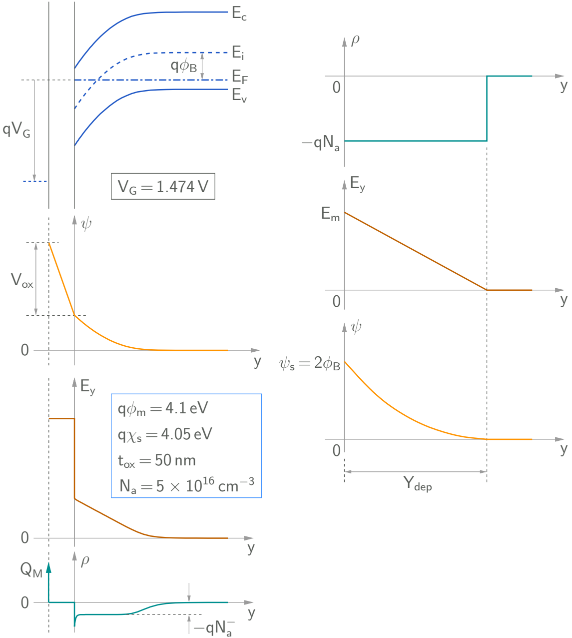

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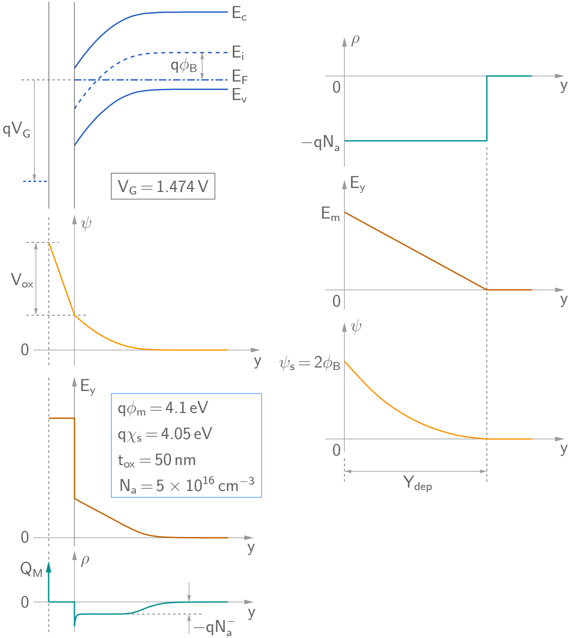

$q\phi_m = 4.1$ eV
$q\chi_s = 4.05$ eV
$t_{ox} = 50$ nm
$N_a = 5 \times 10^{16}\ \text{cm}^{-3}$

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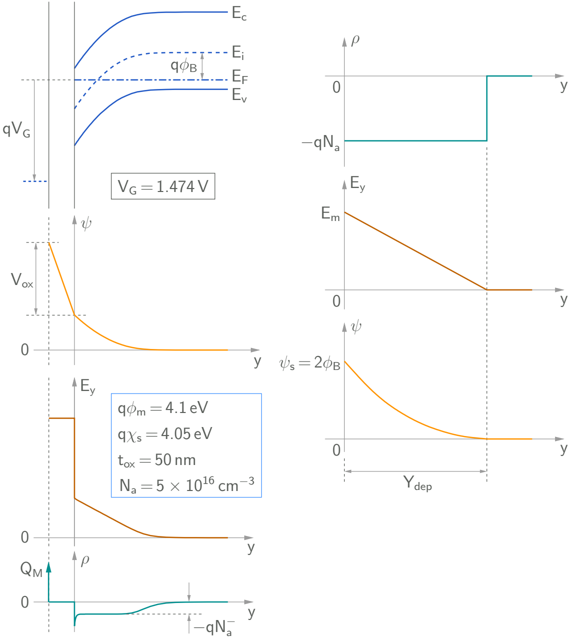

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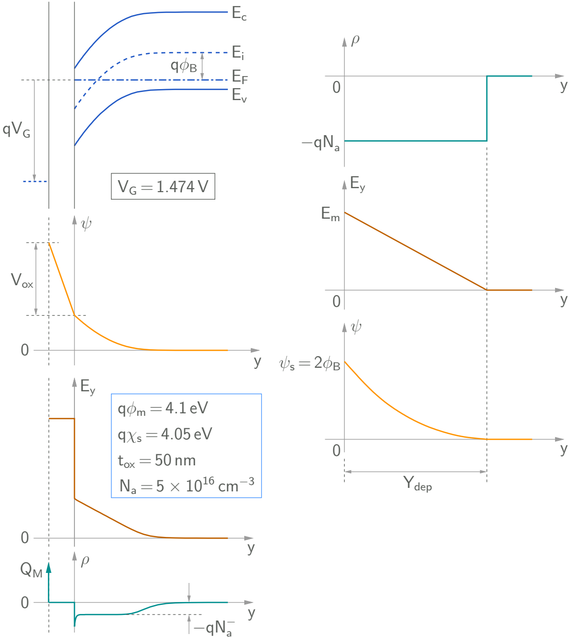
$$\rightarrow Y_{\text{dep}} = \sqrt{\frac{4\epsilon_{\text{Si}}\phi_B}{qN_a}}.$$



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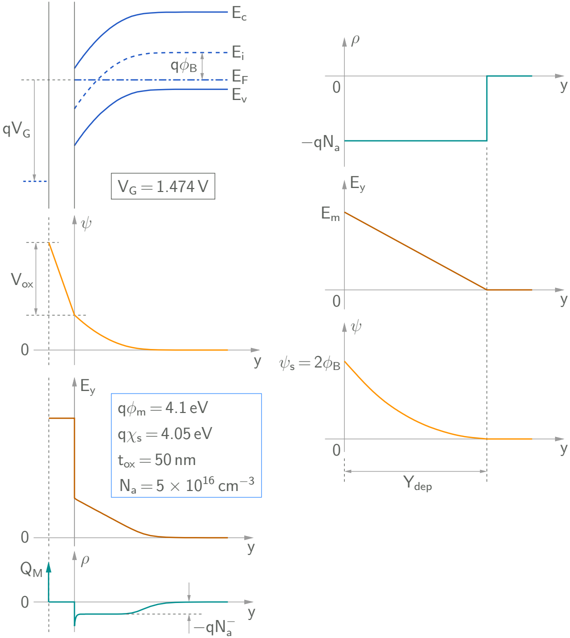

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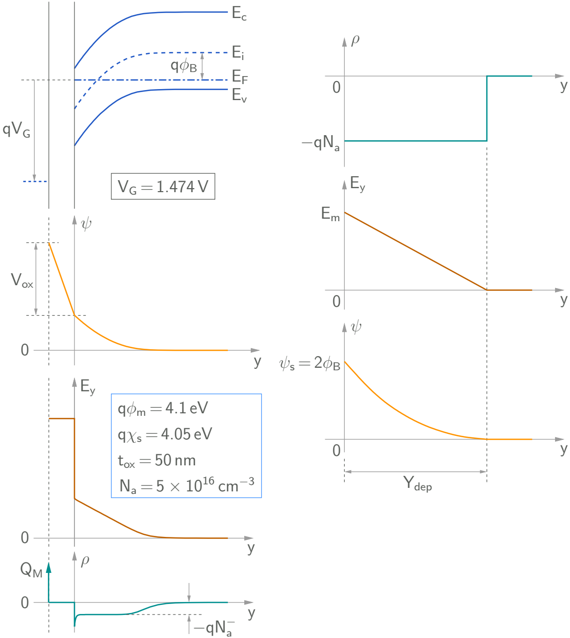

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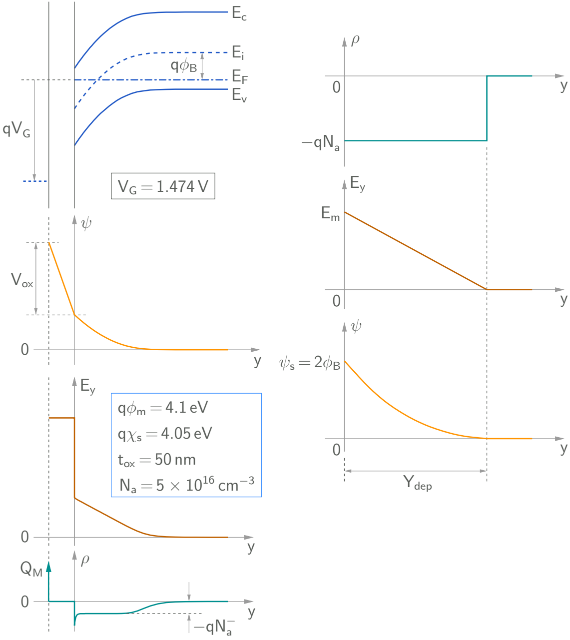

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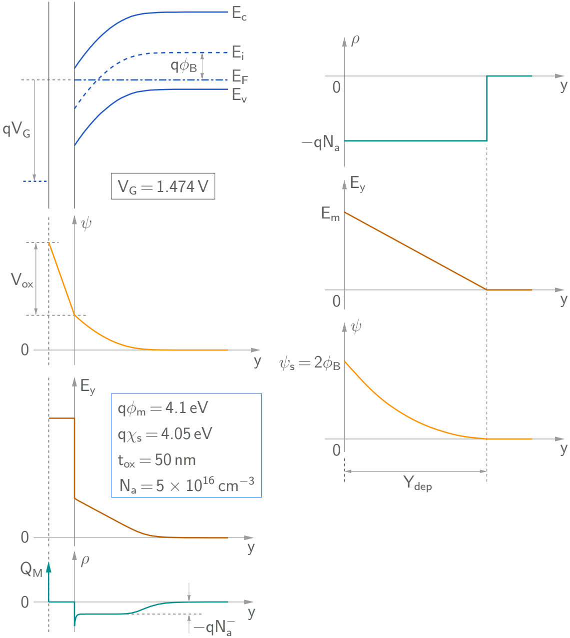

M. B. Patil, IIT Bombay

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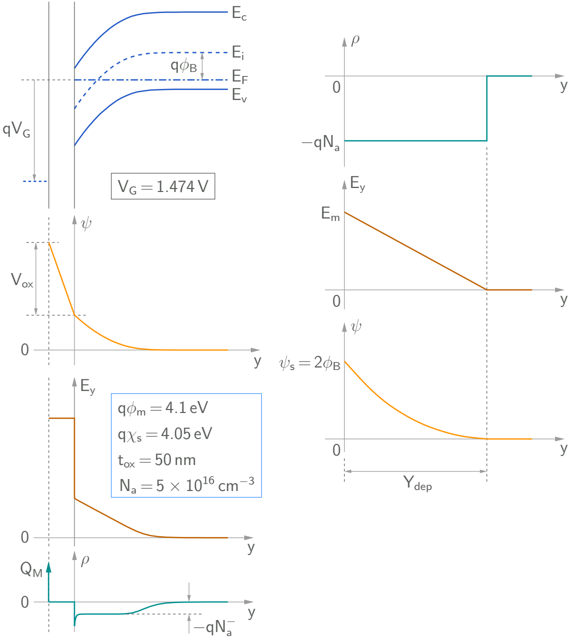

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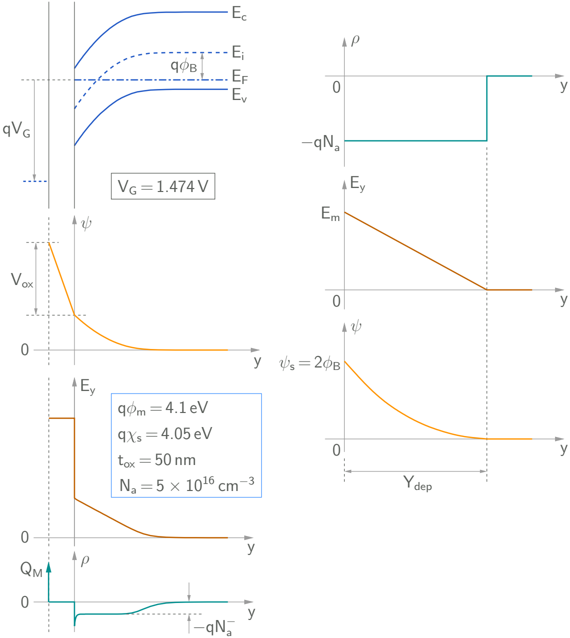

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The voltage drop across the oxide is

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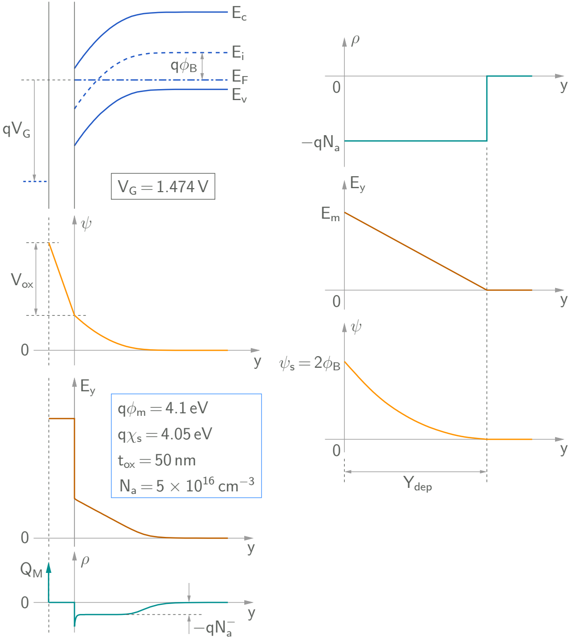
where $C_{ox} = \epsilon_{ox}/t_{ox}$ is the capacitance per unit area of a parallel plate capacitor with SiO$_2$ as the dielectric and a dielectric thickness t_{ox}.



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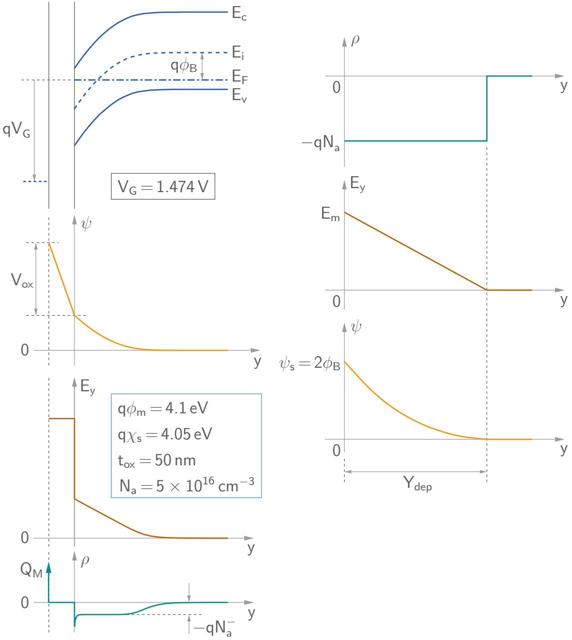
Units of C_{ox}: $\dfrac{\text{F/cm}}{\text{cm}} = \dfrac{\text{F}}{\text{cm}^2}$.



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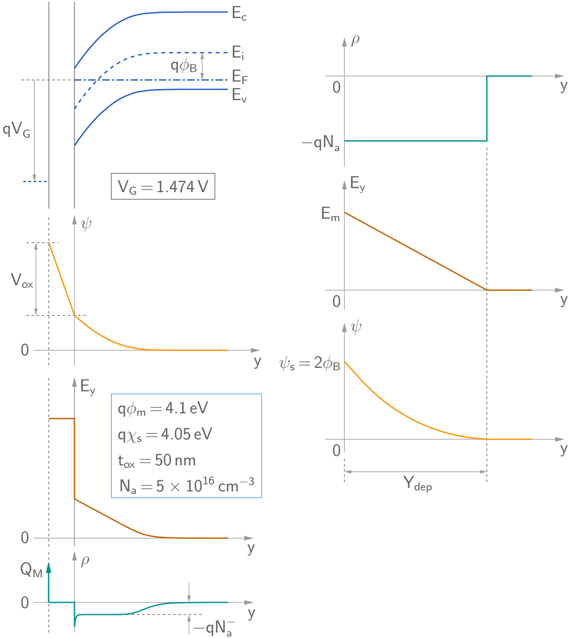

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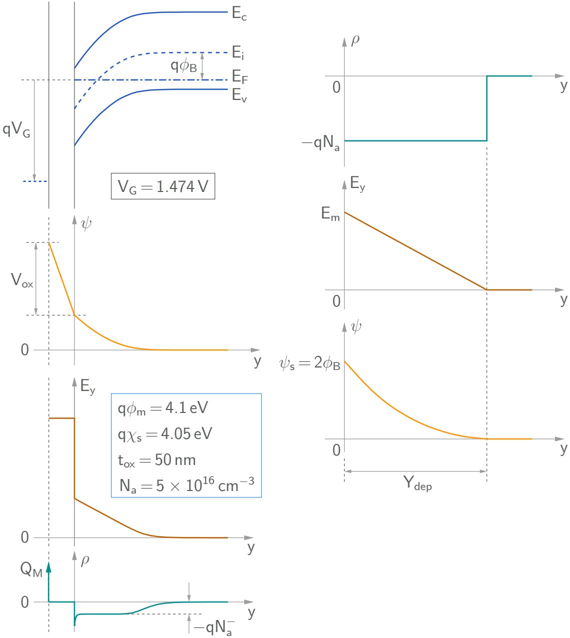
The threshold voltage V_{th} is the gate voltage required to obtain the condition of inversion (i.e., onset of inversion).



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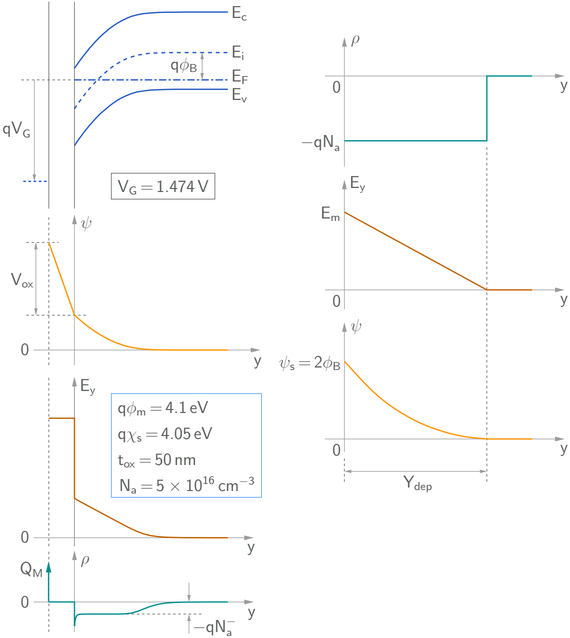

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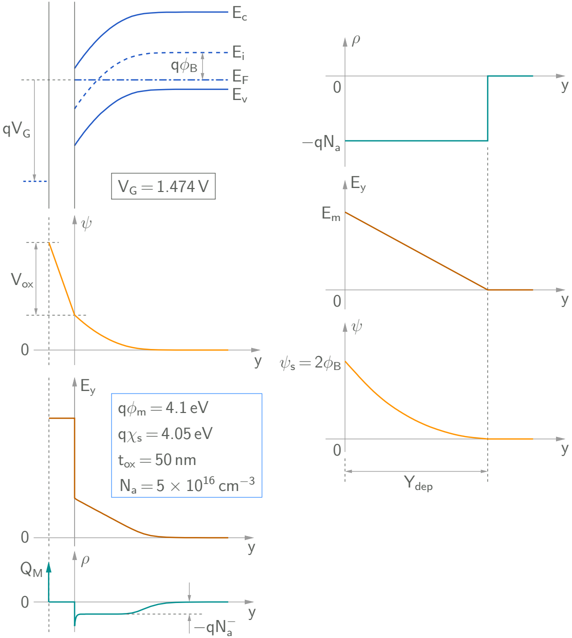

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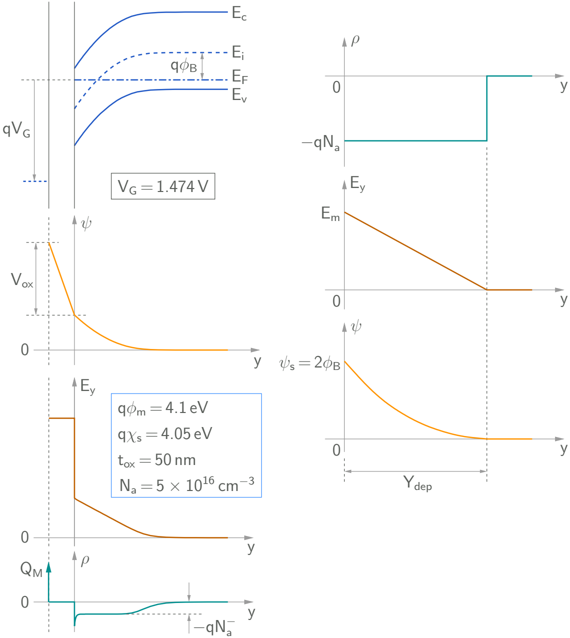

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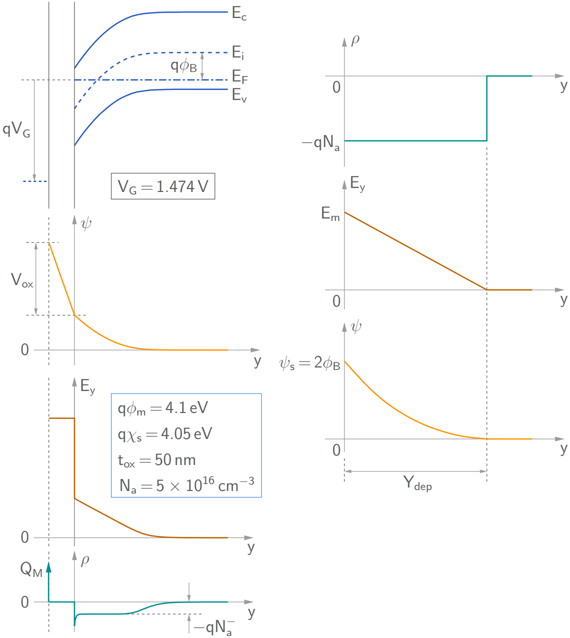

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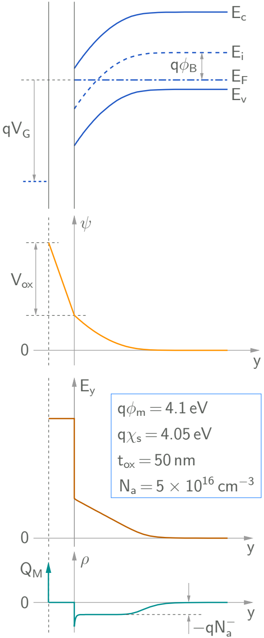
Example

Calculate the threshold voltage for a MOS capacitor with $t_{ox} = 50\,\text{nm}$, $\phi_m = 4.1\,\text{eV}$, $\chi_s = 4.05\,\text{eV}$, $N_a = 5 \times 10^{16}\,\text{cm}^{-3}$.



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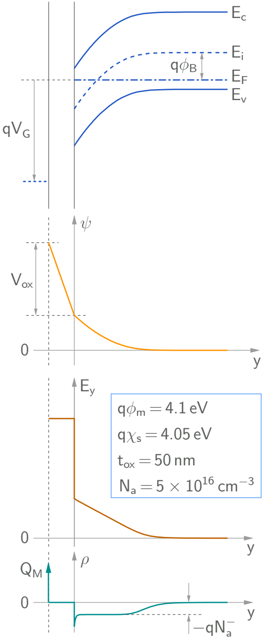

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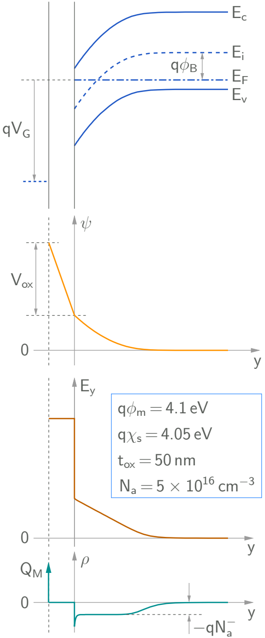

Solution:

$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$

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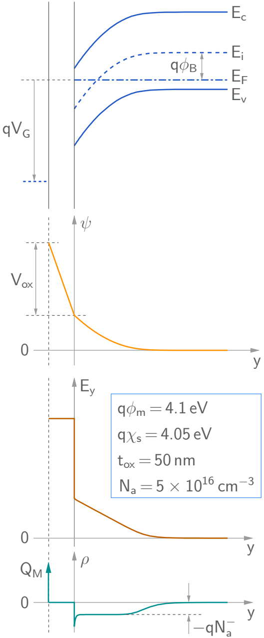

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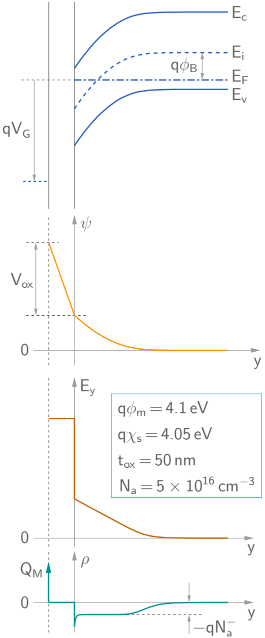

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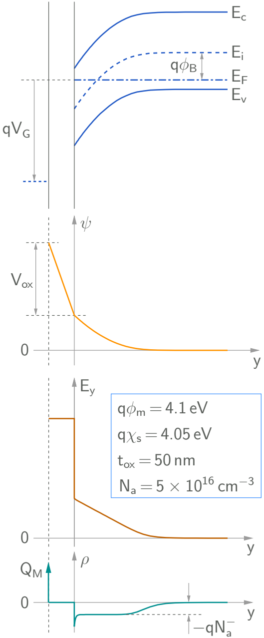
$$p_0 = n_i \exp\left(\frac{\phi_B}{V_T}\right) \;\rightarrow\; \phi_B = V_T \log\left(\frac{p_0}{n_i}\right) = 0.41\,\text{V}.$$



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Solution:

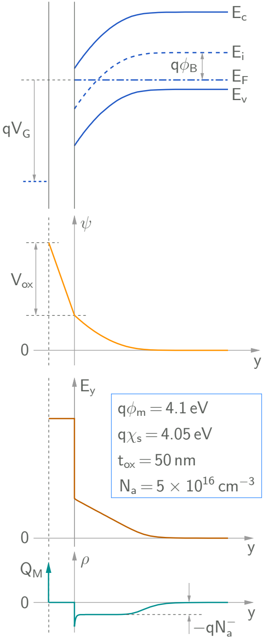
$$q\phi_s = q\chi_s + \left[E_g + k_B T \log\left(\frac{p_0}{N_v}\right)\right]$$

$$= 4.05 + \left[1.12 + 0.0258 \times \log\left(\frac{5 \times 10^{16}}{1.04 \times 10^{19}}\right)\right] = 5.03\,\text{eV}.$$

$$V_{FB} = \phi_{ms} = \phi_m - \phi_s = 4.1 - 5.03 = -0.93\,\text{V}.$$

$$p_0 = n_i \exp\left(\frac{\phi_B}{V_T}\right) \rightarrow \phi_B = V_T \log\left(\frac{p_0}{n_i}\right) = 0.41\,\text{V}.$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}\,\text{F/cm}}{50 \times 10^{-7}\,\text{cm}} = 69\,\text{nF/cm}^2.$$



Example

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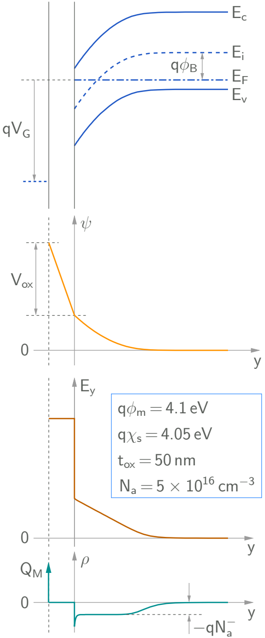

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$$\sqrt{4qN_a\epsilon_{Si}\phi_B} = \sqrt{4 \times 1.6 \times 10^{-19} \times 5 \times 10^{16} \times 11.7 \times 8.85 \times 10^{-14} \times 0.41}$$

Example

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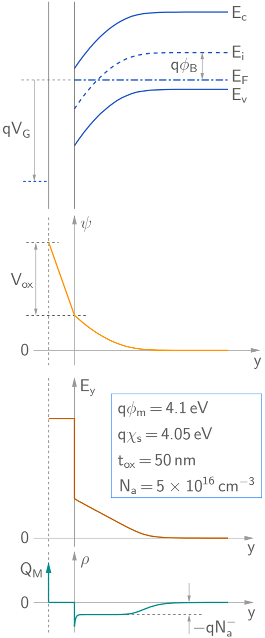
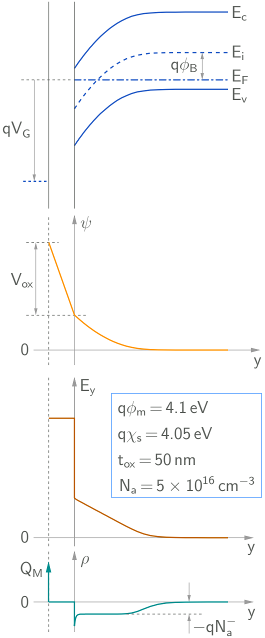
$$= 1.16 \times 10^{-7}\,\text{C/cm}^2.$$



Example (continued)

$$V_{\rm th} = V_{FB} + 2\phi_B + \frac{\sqrt{4qN_a\epsilon_{\rm Si}\phi_B}}{C_{\rm ox}}$$

$$= -0.93 + 2 \times 0.41 + \frac{1.16 \times 10^{-7}}{69 \times 10^{-9}} = 1.57\,{\rm V}.$$



$q\phi_m = 4.1\,{\rm eV}$
$q\chi_s = 4.05\,{\rm eV}$
$t_{ox} = 50\,{\rm nm}$
$N_a = 5 \times 10^{16}\,{\rm cm}^{-3}$

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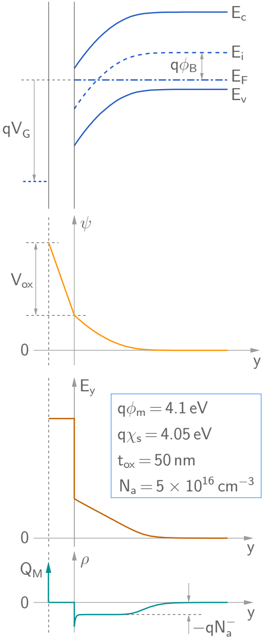
$$Y_{\rm dep}^{\rm inv} = \sqrt{\frac{4\epsilon_{\rm Si}\phi_B}{qN_a}} = \sqrt{\frac{4\times 11.7\times 8.85\times 10^{-14}\times 0.41}{1.6\times 10^{-19}\times 5\times 10^{16}}}$$



$q\phi_m = 4.1$ eV
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Example (continued)

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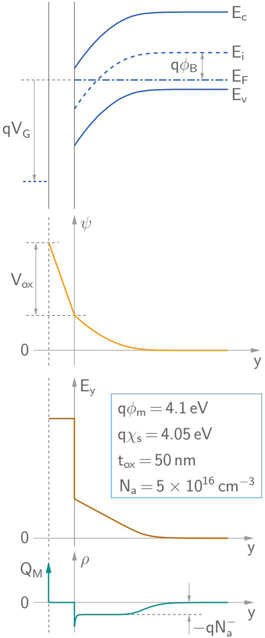
$$= 1.45 \times 10^{-5}\,\text{cm} = 0.145\,\mu\text{m}.$$



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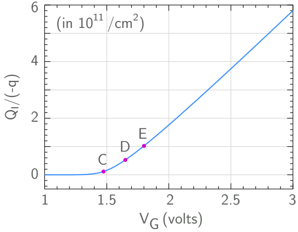
MOS capacitor: inversion charge

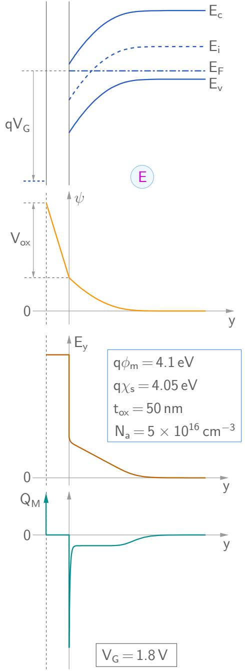
Consider $V_G > V_{\text{th}}$. We have $V_G = V_{FB} + \psi_s + V_{\text{ox}}$.

The surface potential ψ_s stays approximately constant $(=2\phi_B)$ in inversion.

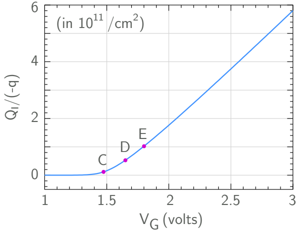
$\rightarrow$ The "excess" gate voltage (beyond V_{th}) can only appear as a change in V_{ox}.

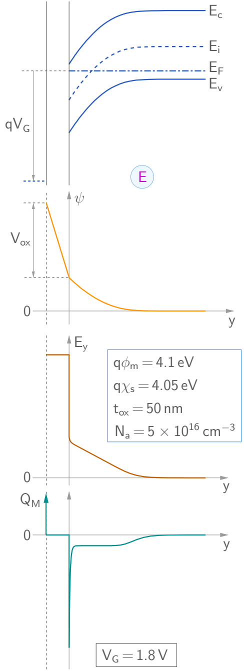
$V_G \approx V_{FB} + 2\phi_B + V_{\text{ox}}$, where $V_{\text{ox}} = \mathcal{E}(0^-)\, t_{\text{ox}} = \mathcal{E}(0^+)\, \dfrac{\epsilon_{\text{Si}}}{\epsilon_{\text{ox}}}\, t_{\text{ox}}$.





MOS capacitor: inversion charge





Consider $V_G > V_{\text{th}}$. We have $V_G = V_{FB} + \psi_s + V_{\text{ox}}$.

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$$\int_{0^+}^{Y_{\text{dep}}} d\mathcal{E}_y = \frac{1}{\epsilon_{\text{Si}}} \int_{0^+}^{Y_{\text{dep}}} \rho\, dy = \frac{1}{\epsilon_{\text{Si}}} \int_{0^+}^{Y_{\text{dep}}} q\,(-N_a^- - n)\, dy$$

$$\rightarrow \mathcal{E}(Y_{\text{dep}}) - \mathcal{E}(0^+) = -\mathcal{E}(0^+) = \frac{1}{\epsilon_{\text{Si}}}\,(-qN_a Y_{\text{dep}} + Q_I) \rightarrow \mathcal{E}(0^+) = \frac{qN_a Y_{\text{dep}}}{\epsilon_{\text{Si}}} - \frac{Q_I}{\epsilon_{\text{Si}}},\ \text{where}$$

$Y_{\text{dep}} \approx Y_{\text{dep}}^{\text{inv}} = \sqrt{\dfrac{4\epsilon_{\text{Si}}\phi_B}{qN_a}}$ (= depletion width at the onset of inversion).

MOS capacitor: inversion charge

Consider $V_G > V_{\text{th}}$. We have $V_G = V_{FB} + \psi_s + V_{\text{ox}}$.

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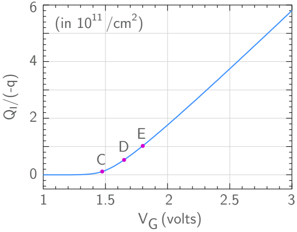
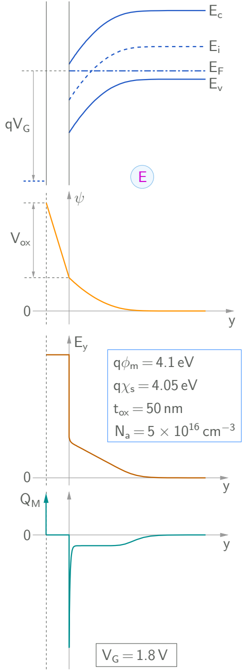
$$Y_{\text{dep}} \approx Y_{\text{dep}}^{\text{inv}} = \sqrt{\frac{4\epsilon_{\text{Si}}\phi_B}{qN_a}}\ (= \text{depletion width at the onset of inversion}).$$

Putting together the various terms, we get

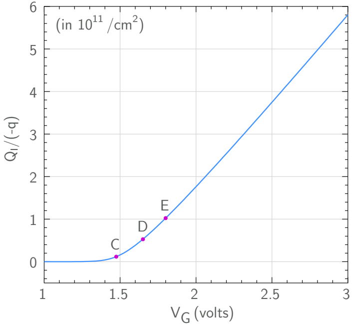
$$V_G = V_{FB} + 2\phi_B + \frac{\sqrt{4qN_a\epsilon_{\text{Si}}\phi_B}}{C_{\text{ox}}} - \frac{Q_I}{C_{\text{ox}}} = V_{\text{th}} - \frac{Q_I}{C_{\text{ox}}}.$$

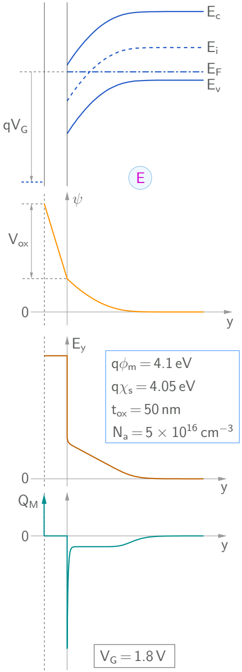
$$\to Q_I = -C_{\text{ox}}(V_G - V_{\text{th}}).$$





MOS capacitor: inversion charge





$q\phi_m = 4.1$ eV

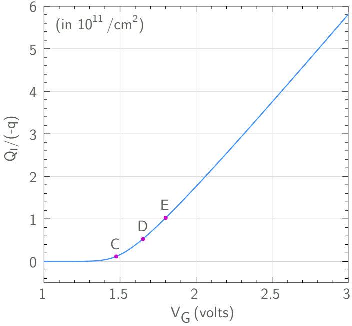
$q\chi_s = 4.05$ eV

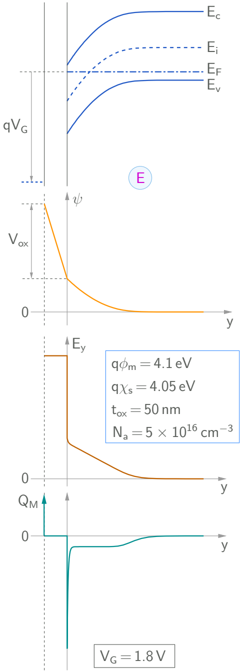
$t_{ox} = 50$ nm

$N_a = 5 \times 10^{16}$ cm^{-3}

$V_G = 1.8$ V

MOS capacitor: inversion charge

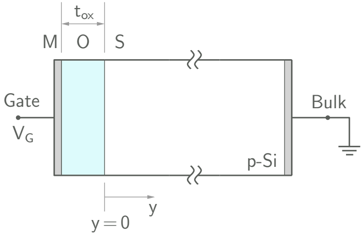




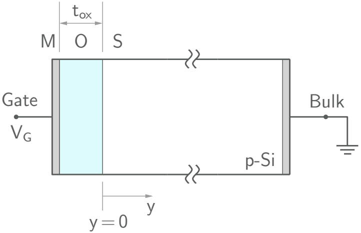
In a MOS capacitor with a uniformly doped *p*-type substrate, we can describe the inversion charge with the following approximate relationship.

$$\begin{aligned} Q_I &= 0, & V_G &\le V_{\mathrm{th}}, \\ &= -C_{\mathrm{ox}}(V_G - V_{\mathrm{th}}), & V_G &> V_{\mathrm{th}}. \end{aligned}$$

MOS capacitor: *C*-*V* relationship

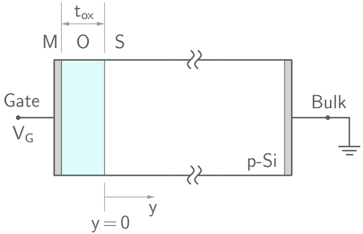


MOS capacitor: C-V relationship



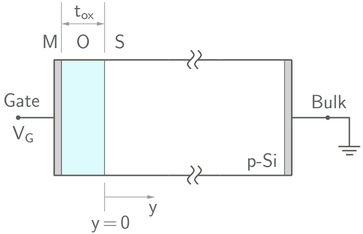
- ∗ The DC current through the MOS structure is zero because of the insulator, and it behaves like a capacitor.

MOS capacitor: *C*-*V* relationship



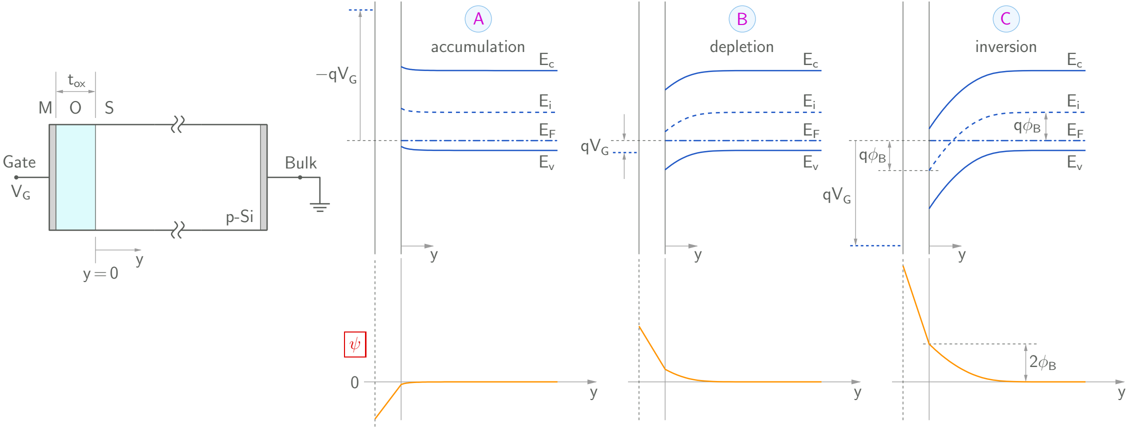
- The DC current through the MOS structure is zero because of the insulator, and it behaves like a capacitor.
- The differential capacitance $C = \dfrac{dQ}{dV_G}$ is of great interest since it contains information about several important parameters, such as the oxide thickness, oxide charge, and doping density in the semiconductor.

MOS capacitor: C-V relationship

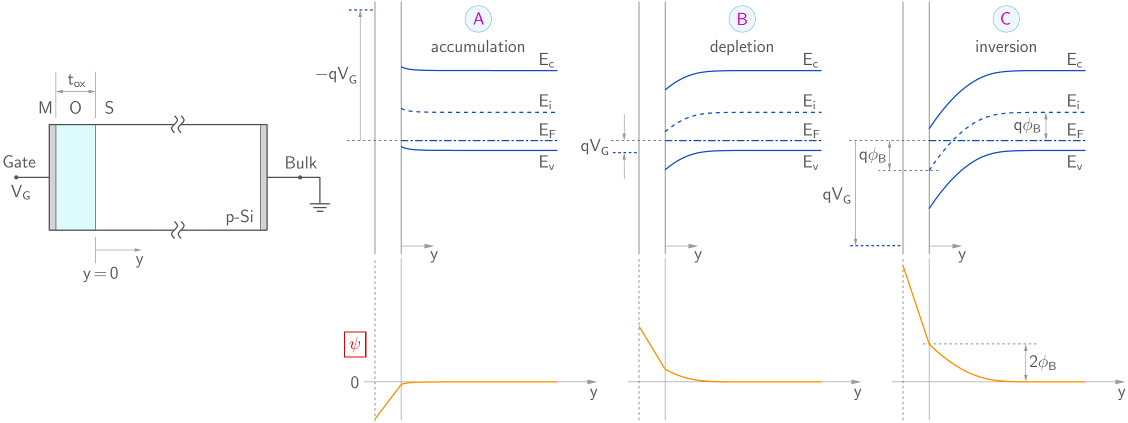


- The DC current through the MOS structure is zero because of the insulator, and it behaves like a capacitor.
- The differential capacitance $C = \dfrac{dQ}{dV_G}$ is of great interest since it contains information about several important parameters, such as the oxide thickness, oxide charge, and doping density in the semiconductor.
- C depends on the bias (DC) value of V_G. A plot of the capacitance C versus the bias voltage is known as the MOS C-V curve, and it serves as an important tool for process evaluation.

MOS capacitor: *C*-*V* relationship

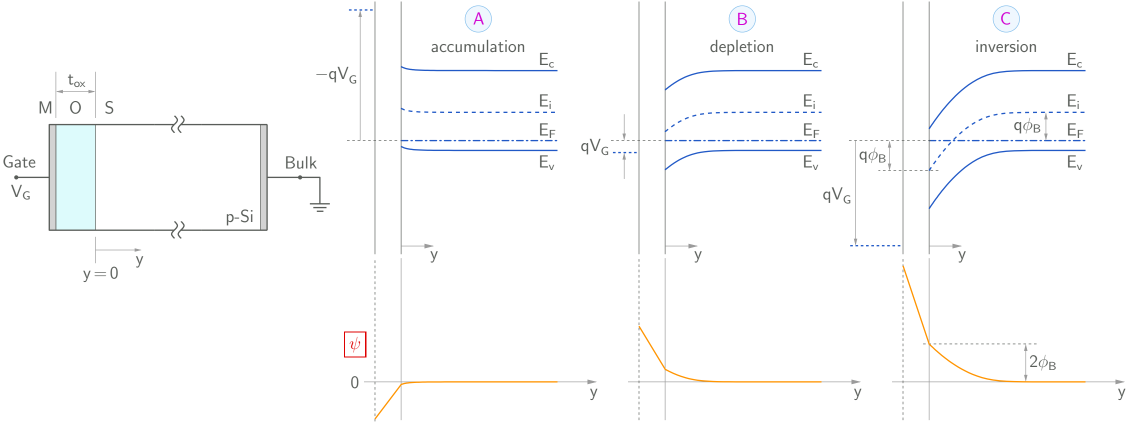


MOS capacitor: *C*-*V* relationship



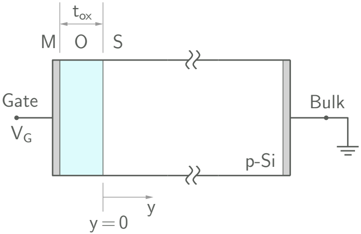
- $V_G = V_{FB} + V_{ox} + \psi_{Si}$.

MOS capacitor: C-V relationship

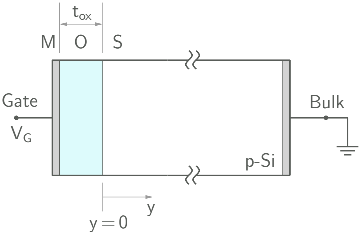


- $V_G = V_{FB} + V_{\mathrm{ox}} + \psi_{\mathrm{Si}}$.
- ψ_{Si}, the voltage drop across the semiconductor, is the same as the surface potential ψ_s if we take $\psi(\infty)$ as 0 V.

MOS capacitor: C-V relationship

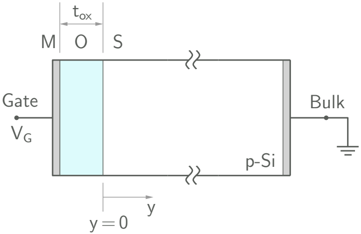


MOS capacitor: C-V relationship



Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^\infty \rho\, dy$.

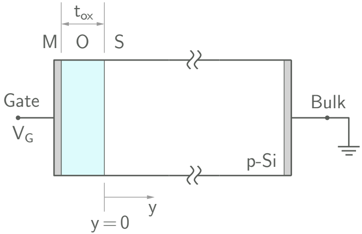
MOS capacitor: C-V relationship



Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^\infty \rho\, dy$.

If $V_G \to V_G + \Delta V_G$, there is a corresponding change ΔQ in the metal charge.

MOS capacitor: C-V relationship



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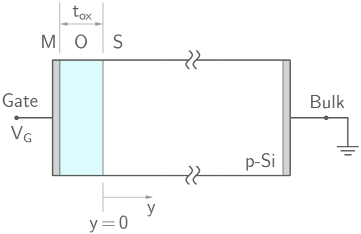
$$\frac{\Delta V_G}{\Delta Q} = \frac{\Delta V_{ox} + \Delta \psi_s}{\Delta Q} = \frac{\Delta V_{ox}}{\Delta Q} + \frac{\Delta \psi_s}{\Delta Q}$$

MOS capacitor: C-V relationship



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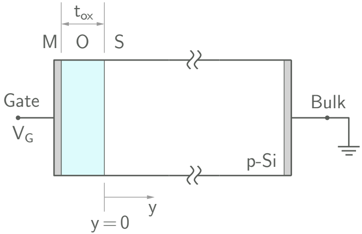
$$\frac{\Delta V_G}{\Delta Q} = \frac{\Delta V_{\text{ox}} + \Delta \psi_s}{\Delta Q} = \frac{\Delta V_{\text{ox}}}{\Delta Q} + \frac{\Delta \psi_s}{\Delta Q} \quad \to \quad \frac{1}{\left(\dfrac{dQ}{dV_G}\right)} = \frac{1}{\left(\dfrac{dQ}{dV_{\text{ox}}}\right)} + \frac{1}{\left(\dfrac{dQ}{d\psi_s}\right)},$$

MOS capacitor: C-V relationship



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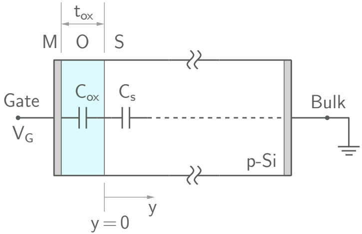
i.e., $C = \dfrac{dQ}{dV_G}$ is given by $\dfrac{1}{C} = \dfrac{1}{C_{ox}} + \dfrac{1}{C_s}$, a series connection of C_{ox} and C_s.

MOS capacitor: C-V relationship



Let Q be the charge per unit area on the metal: $Q = -Q_s = -\int_0^{\infty} \rho\, dy$.

If $V_G \rightarrow V_G + \Delta V_G$, there is a corresponding change ΔQ in the metal charge.

$$\frac{\Delta V_G}{\Delta Q} = \frac{\Delta V_{ox} + \Delta \psi_s}{\Delta Q} = \frac{\Delta V_{ox}}{\Delta Q} + \frac{\Delta \psi_s}{\Delta Q} \quad \rightarrow \quad \frac{1}{\left(\dfrac{dQ}{dV_G}\right)} = \frac{1}{\left(\dfrac{dQ}{dV_{ox}}\right)} + \frac{1}{\left(\dfrac{dQ}{d\psi_s}\right)},$$

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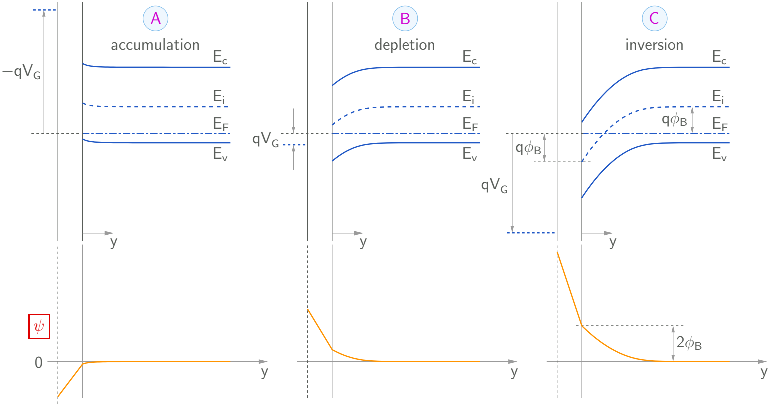
A
accumulation
B
depletion
C
inversion
E_c
E_i
E_F
E_v
$-qV_G$
qV_G
qV_G
$q\phi_B$
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y
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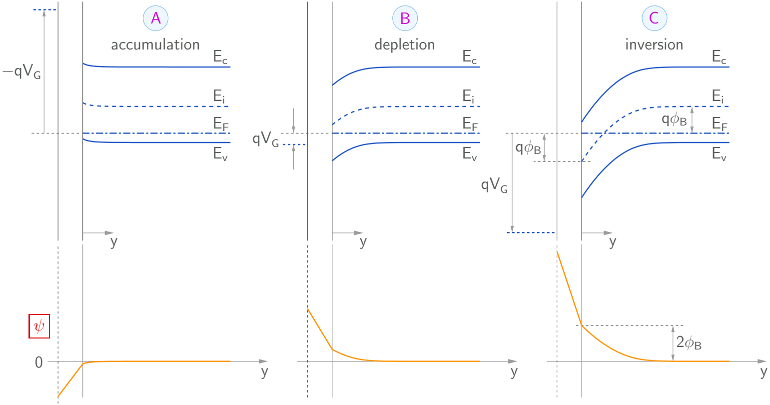
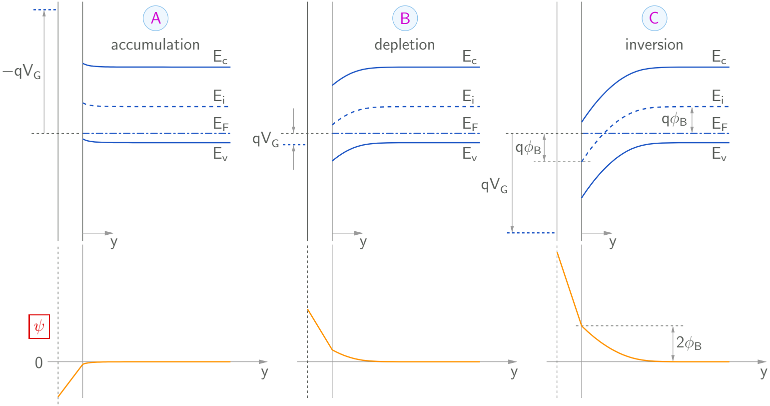
0
$2\phi_B$
y



To obtain $Q_s(\psi_s)$, we start with $n = n_0 e^{\psi/V_T}$, $p = p_0 e^{-\psi/V_T}$, $N_a^- \approx N_a = p_0 - n_0$.



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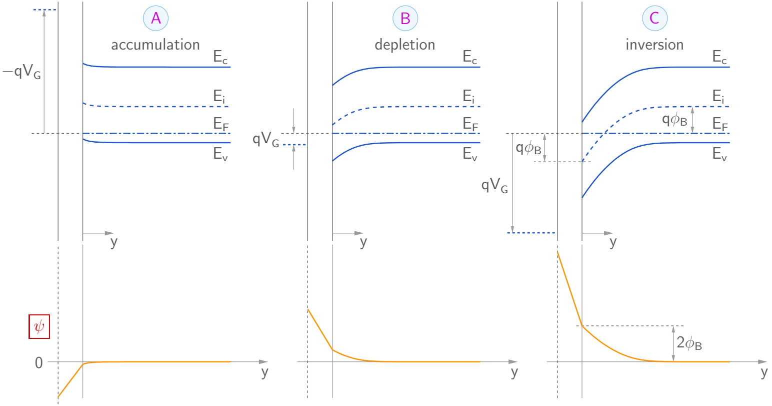
Sufficiently far from the interface, $\psi = 0\,\text{V}$, $n = n_0$, $p = p_0$, $\rho = p - n - N_a^- = 0$.



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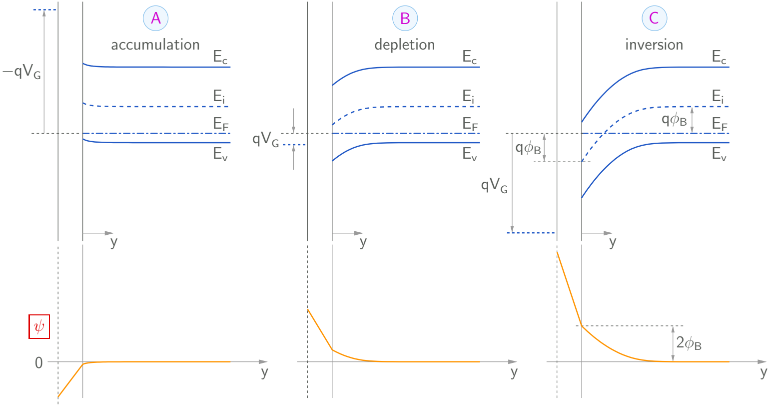
Poisson's equation: $\dfrac{d\mathcal{E}}{dy} = \dfrac{\rho}{\epsilon_{\text{Si}}} = \dfrac{q}{\epsilon_{\text{Si}}}\,(p - n - N_a^-)$.



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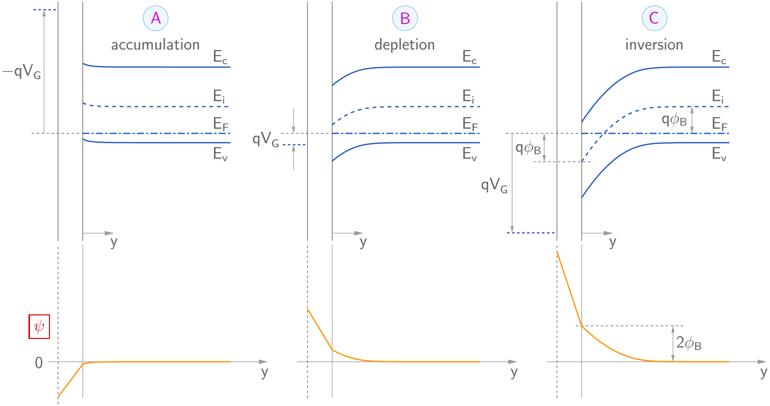
$$\frac{d\mathcal{E}}{dy} = \frac{d\mathcal{E}}{d\psi}\,\frac{d\psi}{dy} = -\,\mathcal{E}\,\frac{d\mathcal{E}}{d\psi}$$



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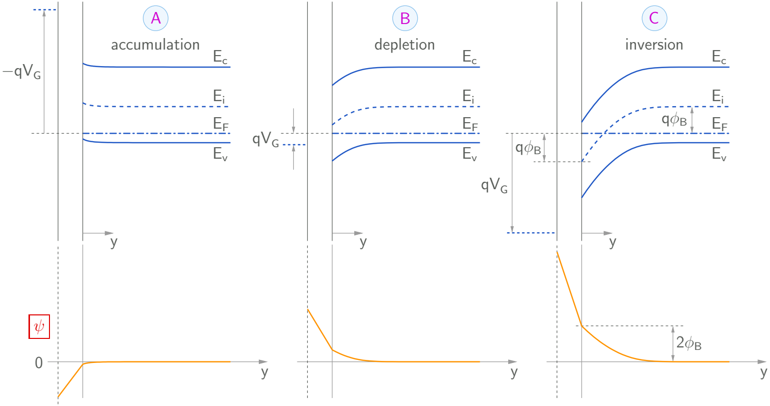
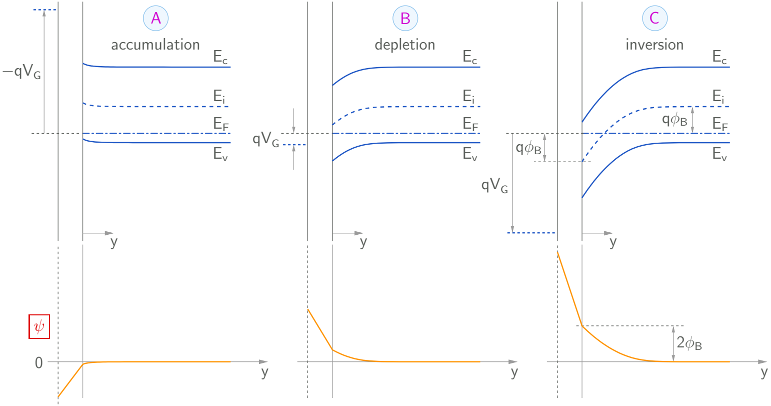
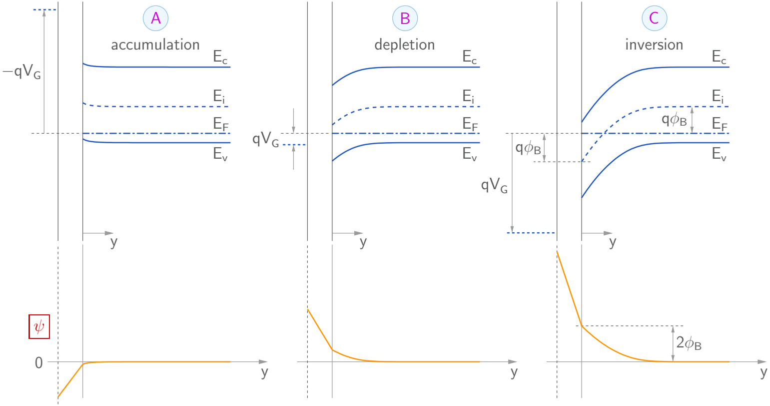
$$\frac{d\mathcal{E}}{dy} = \frac{d\mathcal{E}}{d\psi}\,\frac{d\psi}{dy} = -\,\mathcal{E}\,\frac{d\mathcal{E}}{d\psi} \quad \rightarrow \quad \mathcal{E}\,d\mathcal{E} = \frac{q}{\epsilon_{\text{Si}}}\left[n_0\left(e^{\psi/V_T} - 1\right) - p_0\left(e^{-\psi/V_T} - 1\right)\right]d\psi.$$


A
accumulation
E_c
E_i
E_F
E_v
$-qV_G$
y
B
depletion
E_c
E_i
E_F
E_v
qV_G
y
C
inversion
E_c
E_i
$q\phi_B$
E_F
E_v
$q\phi_B$
qV_G
y
ψ
0
y
y
$2\phi_B$
y

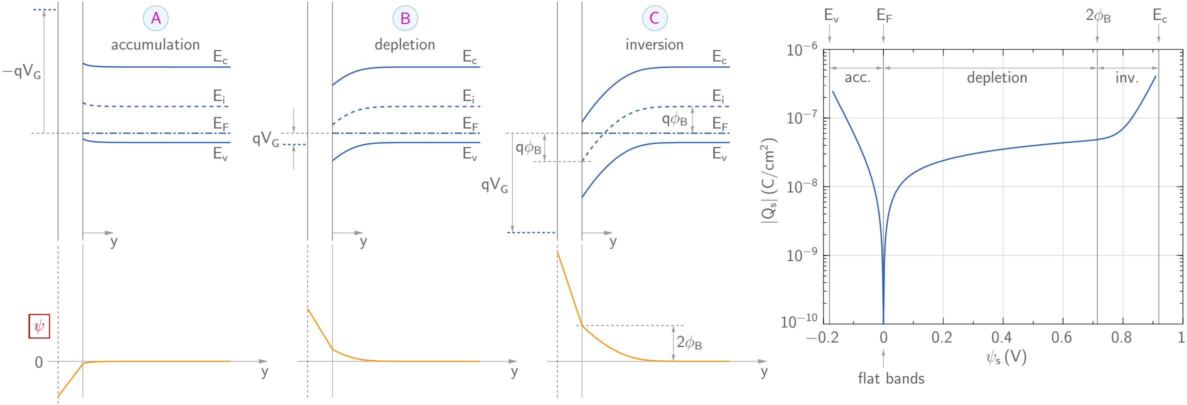


$$\int_{y=0^+}^{\infty} \mathcal{E}\, d\mathcal{E} = \frac{q}{\epsilon_{\mathrm{Si}}} \int_{\psi_s}^{0} \left[n_0 \left(e^{\psi/V_T} - 1 \right) - p_0 \left(e^{-\psi/V_T} - 1 \right) \right] d\psi.$$



$$\int_{y=0^+}^{\infty} \mathcal{E}\, d\mathcal{E} = \frac{q}{\epsilon_{\rm Si}} \int_{\psi_s}^{0} \left[n_0 \left(e^{\psi/V_T} - 1 \right) - p_0 \left(e^{-\psi/V_T} - 1 \right) \right] d\psi.$$

We now obtain $\mathcal{E}(0^+)$ as a function of ψ_s and then $Q_s = -\epsilon_{\rm Si}\, \mathcal{E}(0^+)$ as the total charge in the semiconductor per unit area.



$$\int_{y=0^+}^{\infty} \mathcal{E}\, d\mathcal{E} = \frac{q}{\epsilon_{\text{Si}}} \int_{\psi_s}^{0} \left[n_0 \left(e^{\psi/V_T} - 1 \right) - p_0 \left(e^{-\psi/V_T} - 1 \right) \right] d\psi.$$

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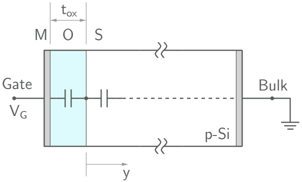
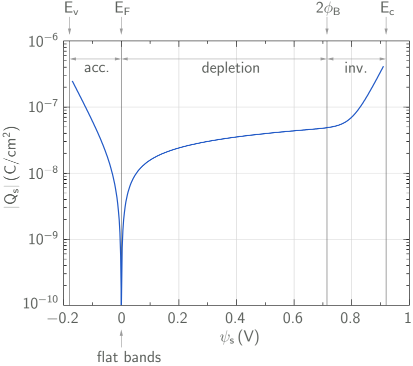
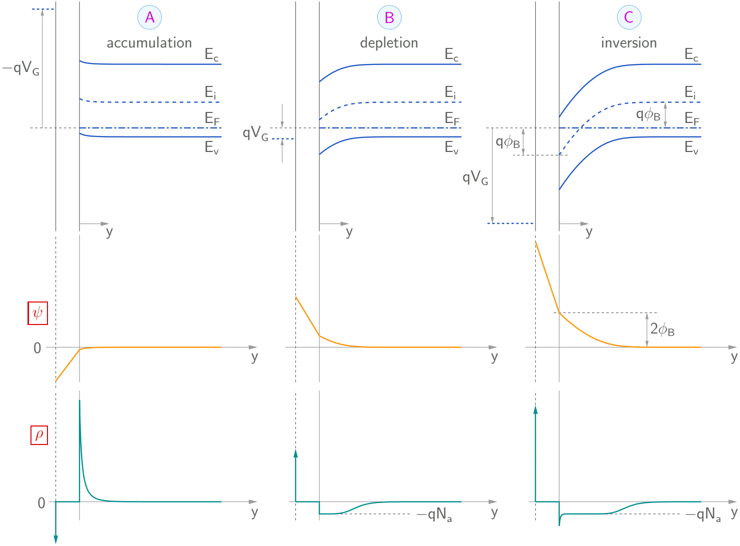
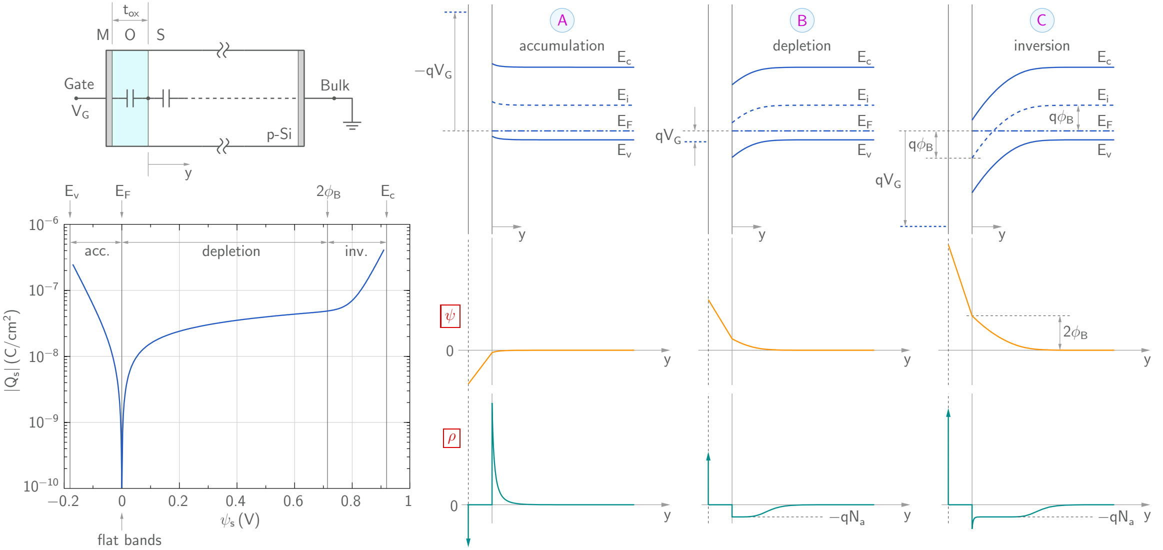
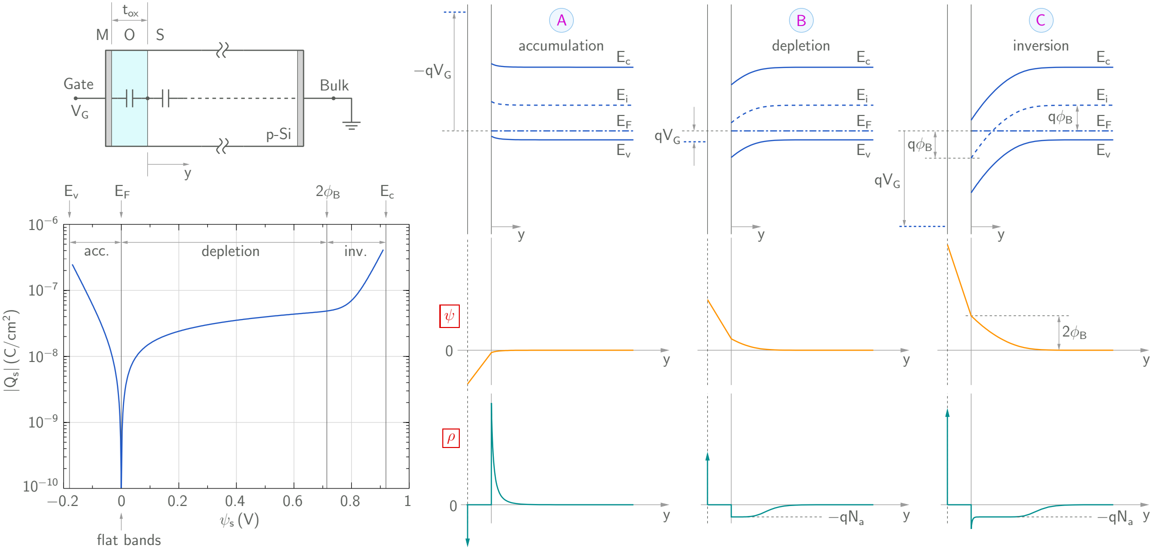

t_ox
M
O
S
Gate
V_G
Bulk
p-Si
y

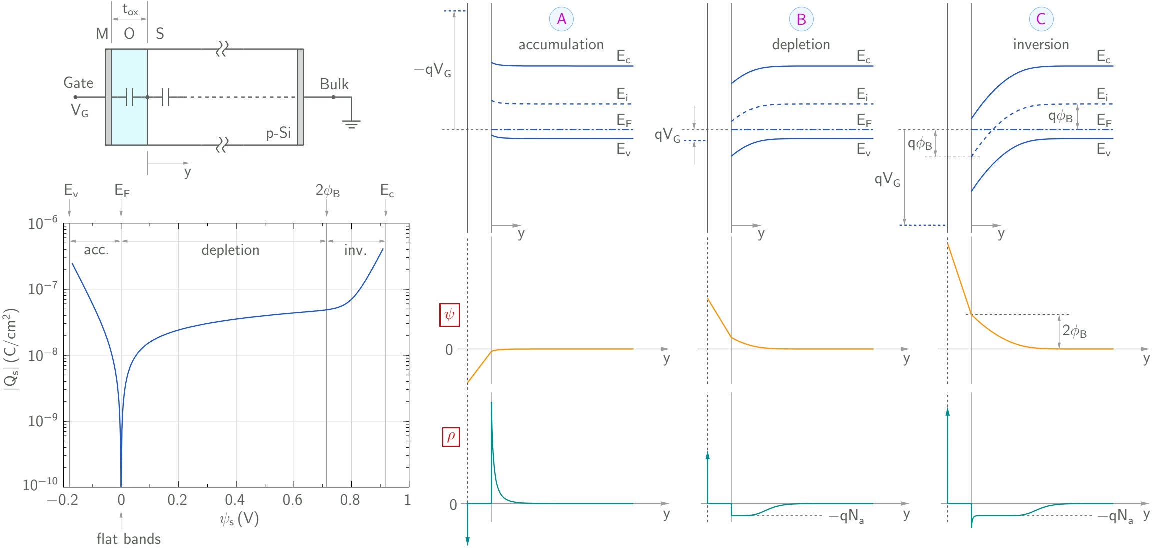

E_v
E_F
2φ_B
E_c
acc.
depletion
inv.
10^−6
10^−7
10^−8
10^−9
10^−10
|Q_s| (C/cm^2)
−0.2
0
0.2
0.4
0.6
0.8
1
ψ_s (V)
flat bands


A
accumulation
B
depletion
C
inversion
−qV_G
qV_G
qV_G
E_c
E_i
E_F
E_v
qφ_B
qφ_B
y
ψ
0
2φ_B
ρ
0
−qN_a
−qN_a
y

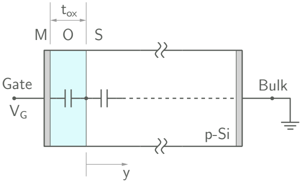


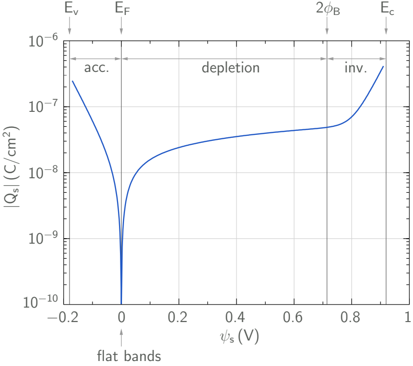
- In accumulation ($\psi_s < 0\,\text{V}$) and inversion ($\psi_s > 2\phi_B$), Q_s changes rapidly with ψ_s because $p \propto e^{-(E_F-E_v)/kT}$, $n \propto e^{-(E_c-E_F)/kT}$.


t$_{ox}$
M
O
S
Gate
V$_G$
Bulk
p-Si
y
E$_v$
E$_F$
2ϕ_B
E$_c$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
acc.
depletion
inv.
$|Q_s|$ (C/cm^2)
−0.2
0
0.2
0.4
0.6
0.8
1
ψ_s (V)
flat bands
A
accumulation
B
depletion
C
inversion
E$_c$
E$_i$
E$_F$
E$_v$
−qV$_G$
qV$_G$
qϕ_B
y
ψ
0
2ϕ_B
ρ
−qN$_a$



- In the depletion regime, the region near the surface is depleted of electrons and holes, and the variation of Q_s with ψ_s comes from the change in the ionised acceptor charge, i.e., the change in the depletion width with ψ_s.


t_ox
M
O
S
Gate
V_G
Bulk
p-Si
y


E_v
E_F
2φ_B
E_c
acc.
depletion
inv.
10^−6
10^−7
10^−8
10^−9
10^−10
|Q_s| (C/cm²)
−0.2
0
0.2
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0.6
0.8
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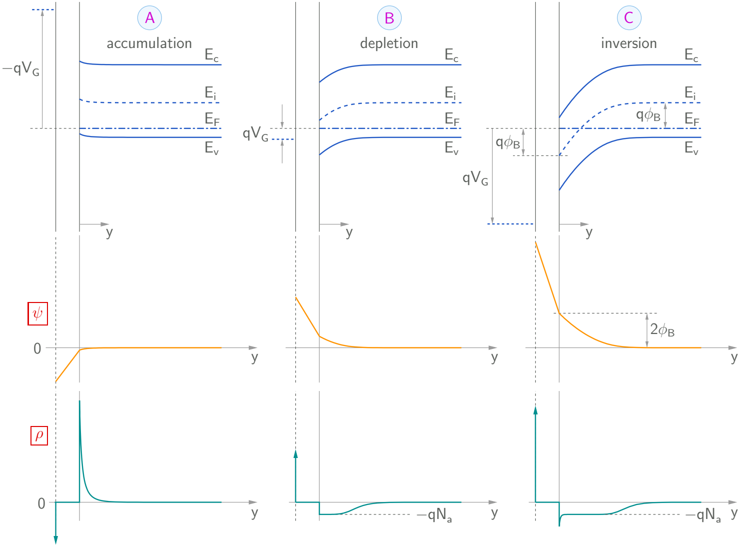
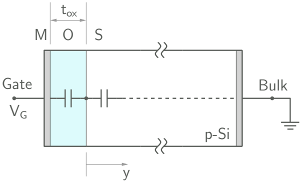
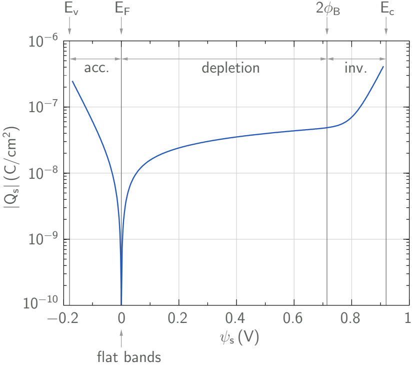
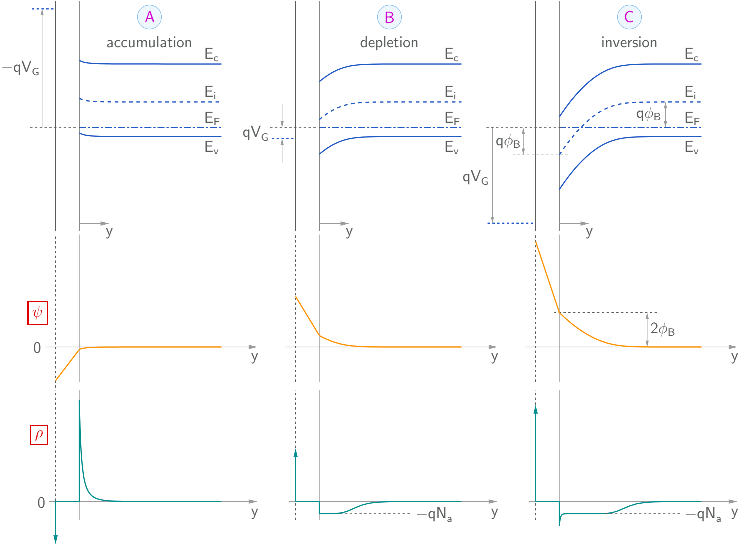
ψ_s (V)
flat bands

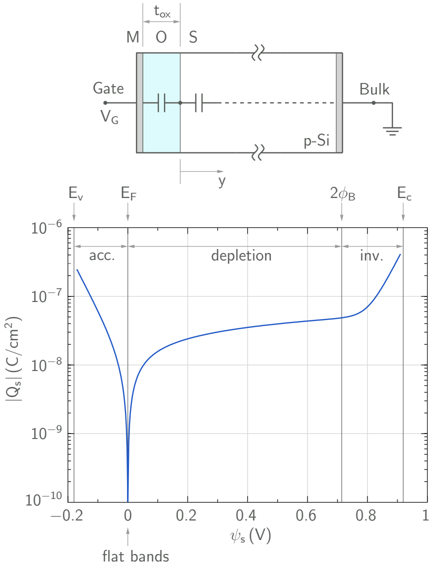

A
accumulation
B
depletion
C
inversion
E_c
E_i
E_F
E_v
−qV_G
qV_G
qφ_B
y
ψ
0
2φ_B
ρ
−qN_a

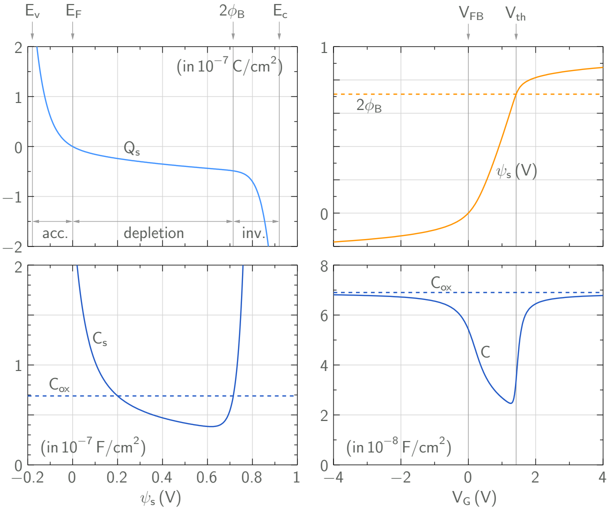




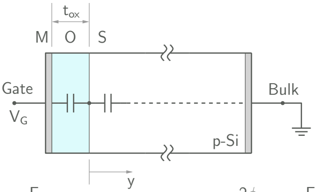


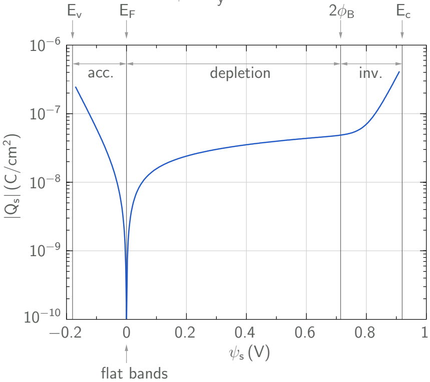
* Since the depletion width varies relatively slowly with ψ_s (as $\sqrt{\psi_s}$), $\dfrac{dQ}{d\psi_s}$ is relatively small in the depletion regime.

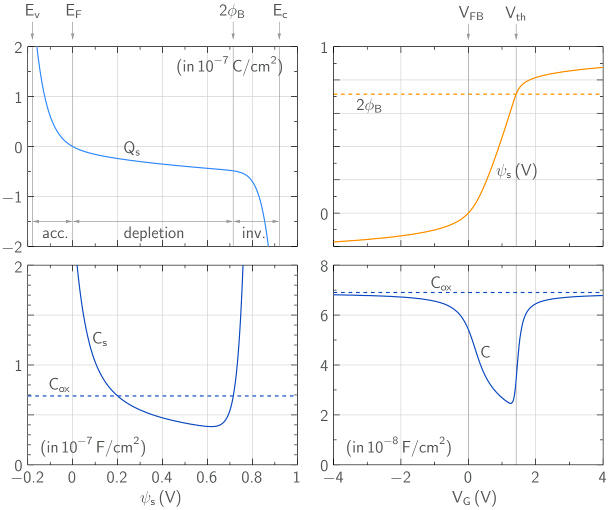




($N_a = 10^{16}\,\text{cm}^{-3}$, $t_{ox} = 50\,\text{nm}$, $V_{FB} = 0\,\text{V}$, $T = 300\,\text{K}$)

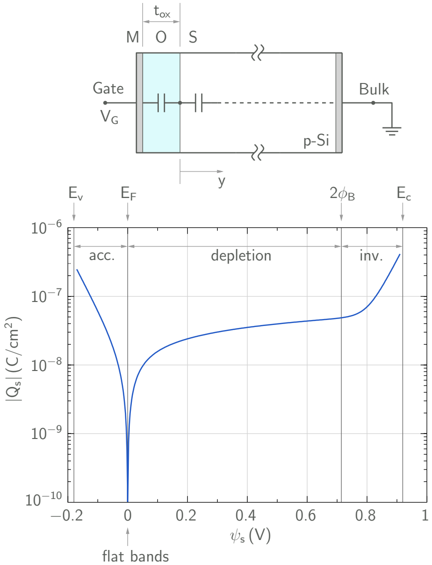


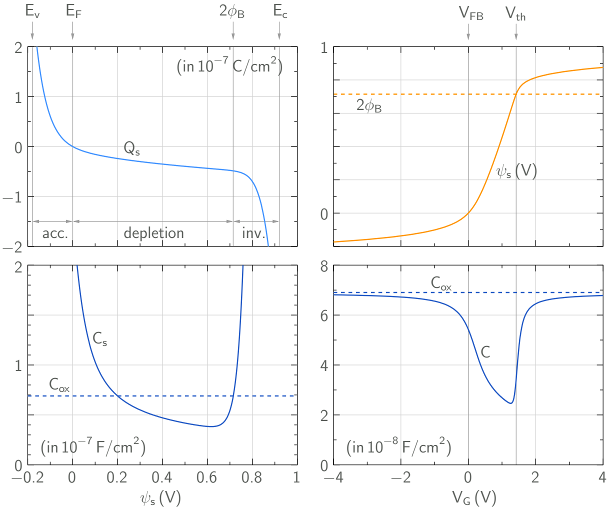




$(N_a = 10^{16}\,\text{cm}^{-3},\ t_{ox} = 50\,\text{nm},\ V_{FB} = 0\,\text{V},\ T = 300\,\text{K})$

$$* \quad Q_M = -Q_s \;\rightarrow\; C_s \equiv \frac{dQ_M}{d\psi_s} = -\frac{dQ_s}{d\psi_s}.$$





$(N_a = 10^{16}\,\text{cm}^{-3},\ t_{ox} = 50\,\text{nm},\ V_{FB} = 0\,\text{V},\ T = 300\,\text{K})$

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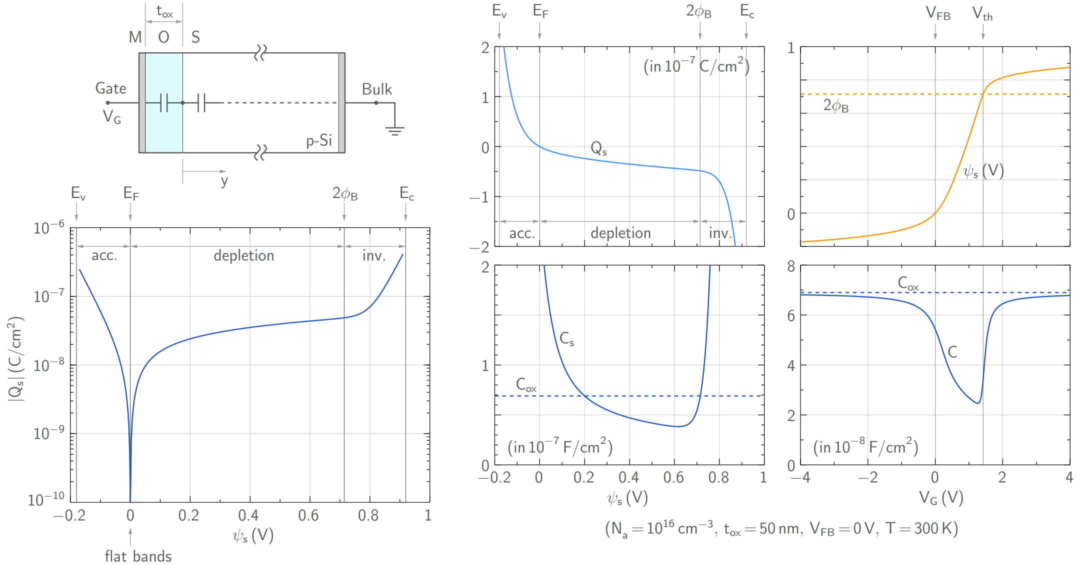
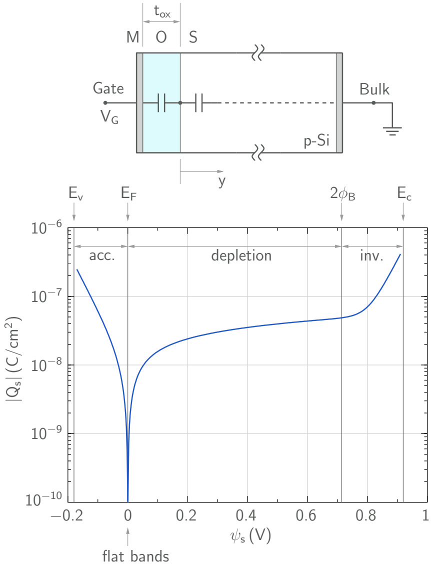
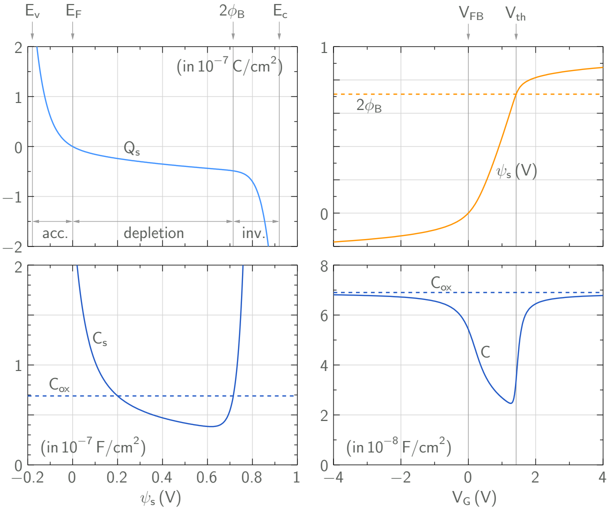


$(N_a = 10^{16}\,\text{cm}^{-3},\ t_{ox} = 50\,\text{nm},\ V_{FB} = 0\,\text{V},\ T = 300\,\text{K})$

- Accumulation and inversion: C_s is large compared to C_{ox}. Since $\dfrac{1}{C} = \dfrac{1}{C_{ox}} + \dfrac{1}{C_s}$, $C \rightarrow C_{ox}$.





($N_a = 10^{16}\,\text{cm}^{-3}$, $t_{ox} = 50\,\text{nm}$, $V_{FB} = 0\,\text{V}$, $T = 300\,\text{K}$)

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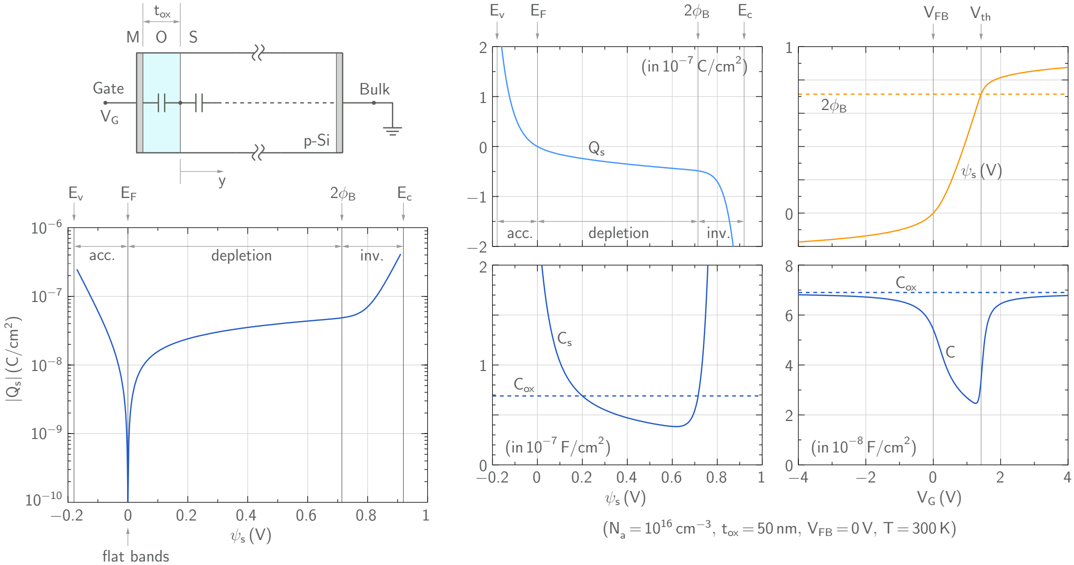
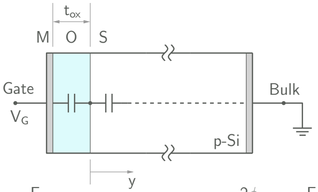
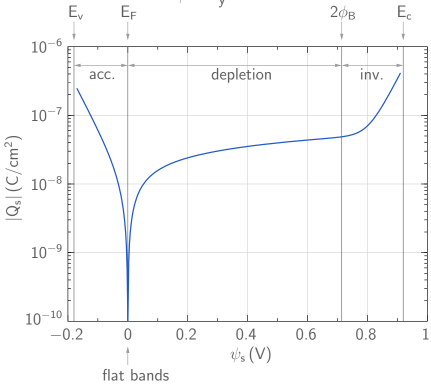


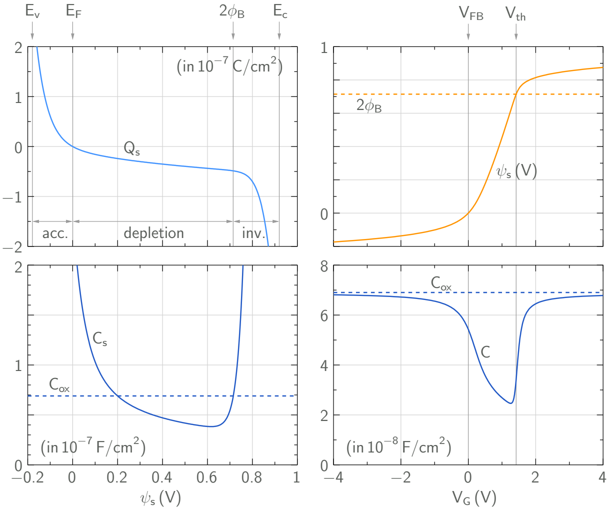
($N_a = 10^{16}\ \text{cm}^{-3}$, $t_{ox} = 50\ \text{nm}$, $V_{FB} = 0\ \text{V}$, $T = 300\ \text{K}$)

- To map the surface potential ψ_s to the gate voltage V_G, we use

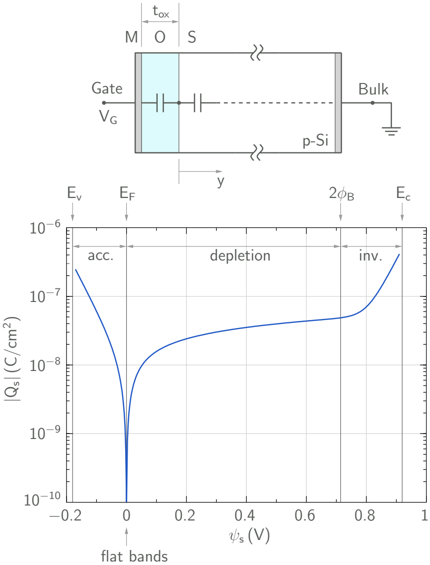
$$V_G = V_{FB} + \psi_s + \mathcal{E}_{ox} t_{ox} = V_{FB} + \psi_s + \frac{(-Q_s)}{\epsilon_{ox}}\, t_{ox} = V_{FB} + \psi_s + \frac{(-Q_s)}{C_{ox}}.$$

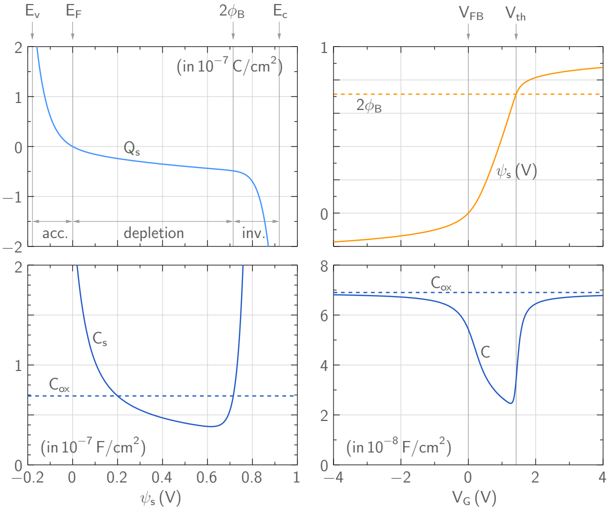






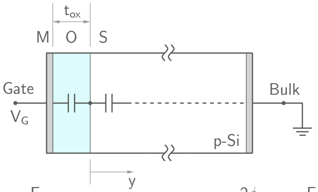
$(N_a = 10^{16}\,\text{cm}^{-3},\ t_{ox} = 50\,\text{nm},\ V_{FB} = 0\,\text{V},\ T = 300\,\text{K})$

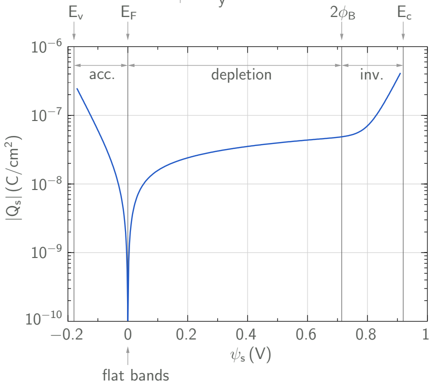


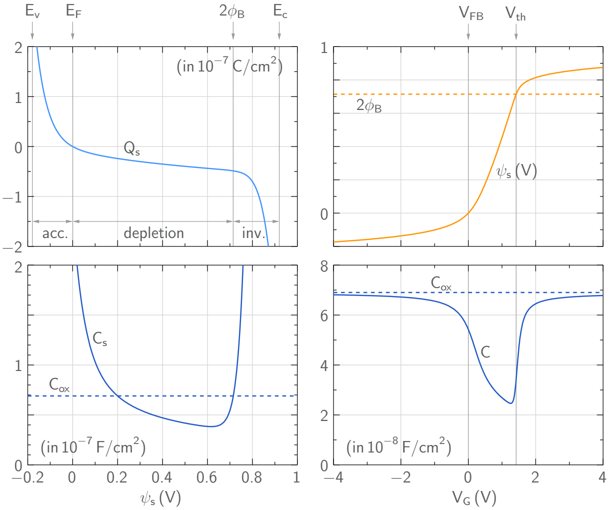


($N_a = 10^{16}\,\text{cm}^{-3}$, $t_{ox} = 50\,\text{nm}$, $V_{FB} = 0\,\text{V}$, $T = 300\,\text{K}$)

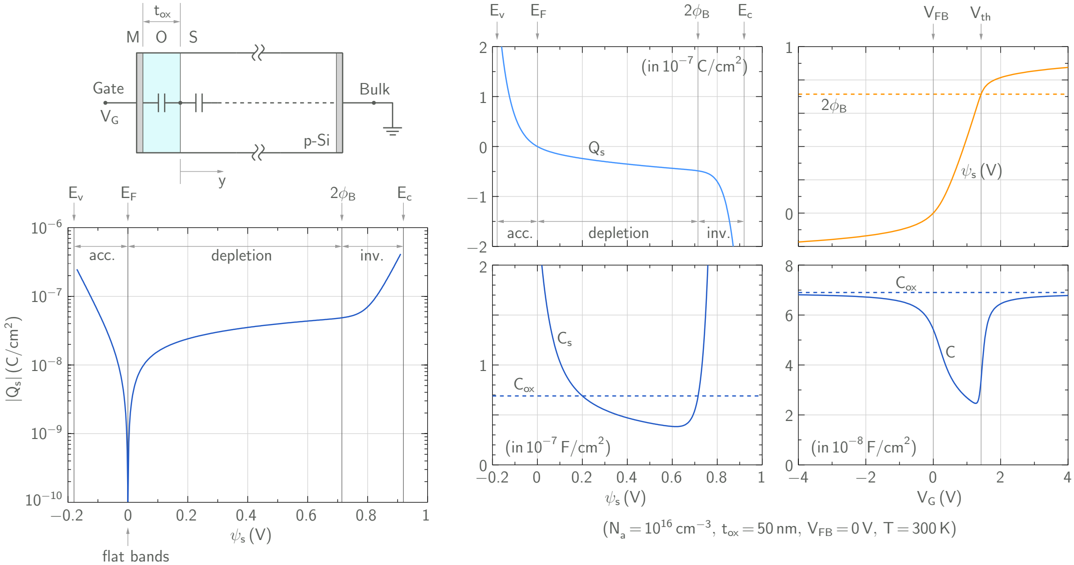
- In accumulation and inversion, $C \rightarrow C_{ox}$.





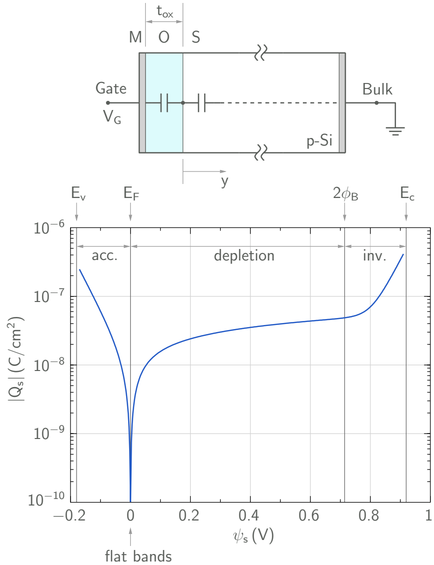


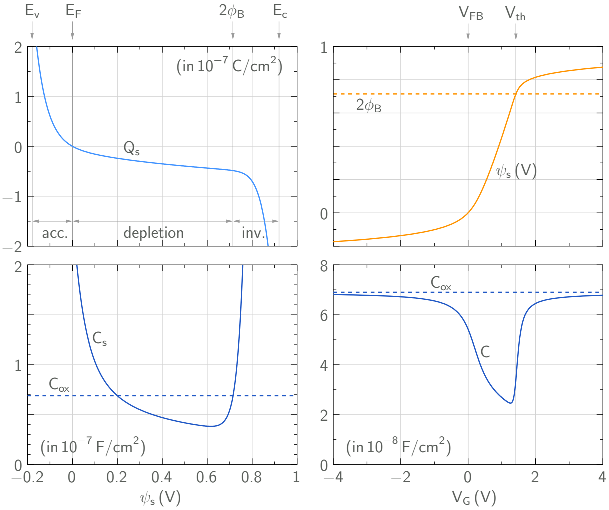
($N_a = 10^{16}\,\mathrm{cm}^{-3}$, $t_{ox} = 50\,\mathrm{nm}$, $V_{FB} = 0\,\mathrm{V}$, $T = 300\,\mathrm{K}$)



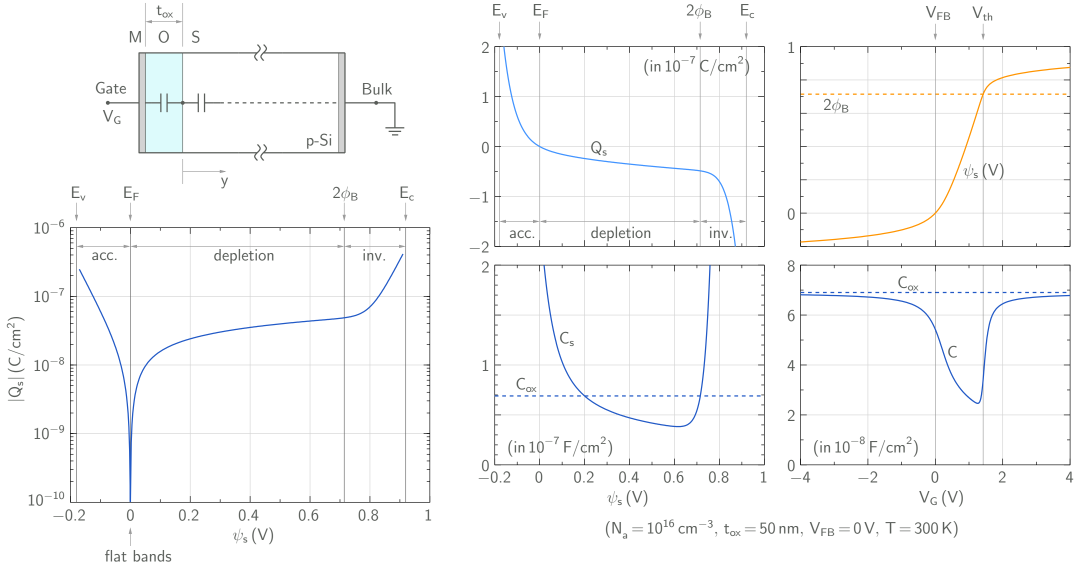
($N_a = 10^{16}\,\text{cm}^{-3}$, $t_{ox} = 50\,\text{nm}$, $V_{FB} = 0\,\text{V}$, $T = 300\,\text{K}$)

- In depletion, C is smaller than C_{ox} and is minimum when C_s is minimum. This corresponds to the situation where there is no inversion charge yet, but the depletion width has reached its maximum value which happens at the onset of inversion, i.e., $V_G \approx V_{th}$.



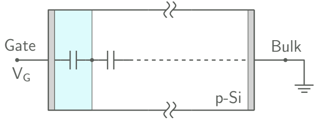


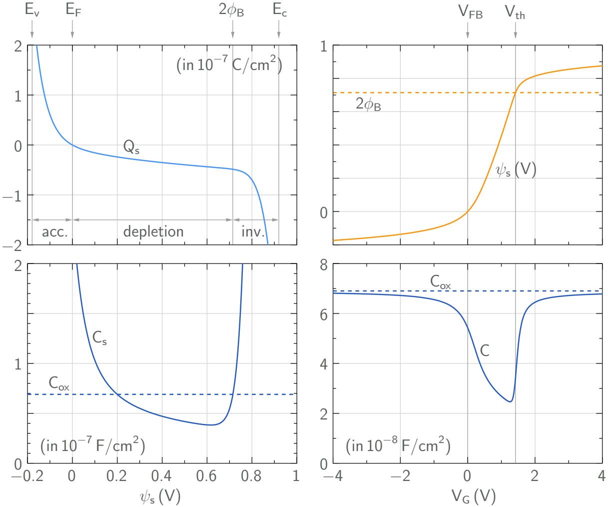
($N_a = 10^{16}\,\text{cm}^{-3}$, $t_{ox} = 50\,\text{nm}$, $V_{FB} = 0\,\text{V}$, $T = 300\,\text{K}$)

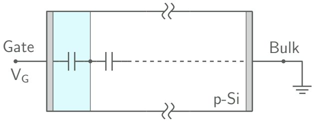


($N_a = 10^{16}\,\text{cm}^{-3}$, $t_{ox} = 50\,\text{nm}$, $V_{FB} = 0\,\text{V}$, $T = 300\,\text{K}$)

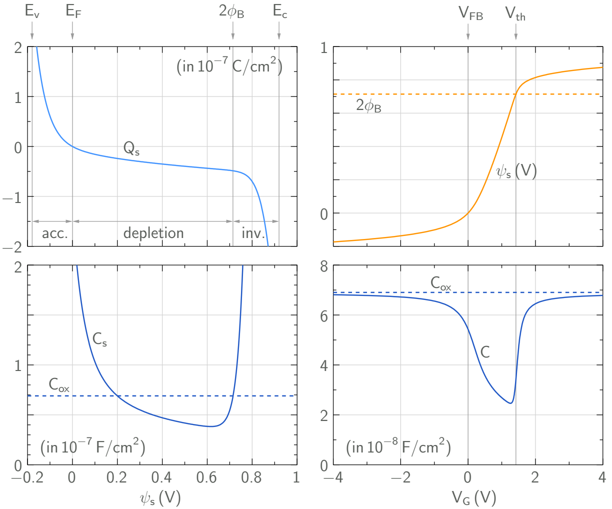
- Since $V_G = V_{FB} + \psi_s + \mathcal{E}_{ox} t_{ox}$, a change in V_{FB} by ΔV_{FB} causes the C-V curve to shift horizontally by ΔV_{FB}.

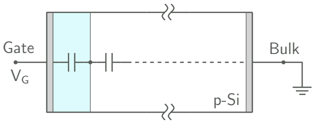

Gate
V_G
Bulk
p-Si


E_v
E_F
2φ_B
E_c
V_FB
V_th
(in 10^−7 C/cm^2)
Q_s
acc.
depletion
inv.
2φ_B
ψ_s (V)
C_s
C_ox
(in 10^−7 F/cm^2)
C_ox
C
(in 10^−8 F/cm^2)
2
1
0
−1
−2
1
0
8
6
4
2
0
−0.2
0
0.2
0.4
0.6
0.8
1
ψ_s (V)
−4
−2
0
2
4
V_G (V)

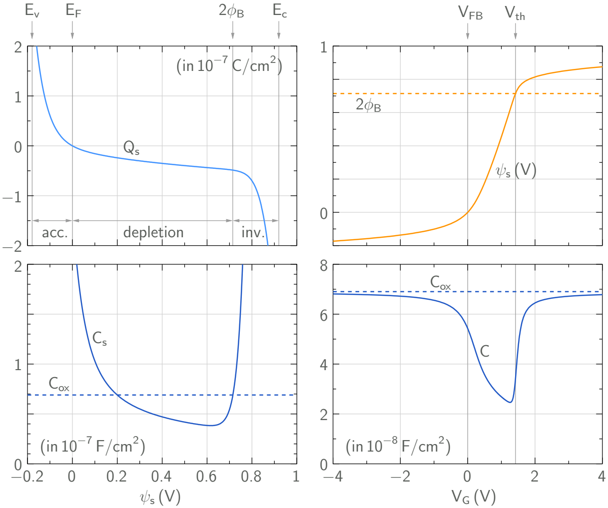


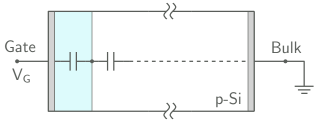
- ∗ We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.





- We have assumed so far that the variation in the gate voltage is slow enough for the carriers to respond.
- The C-V measurement is made by applying $v_G(t) = V_G + v_g \sin \omega t$. We require $f < 100\,\text{Hz}$ for the above assumption to hold.





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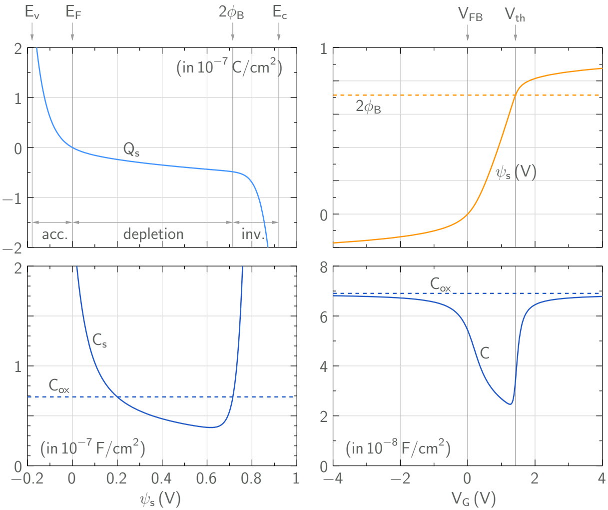

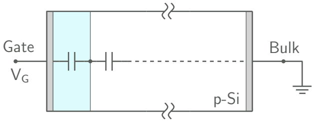

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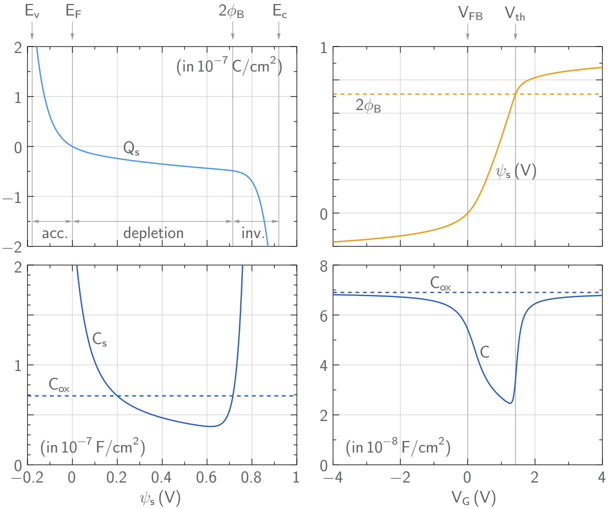

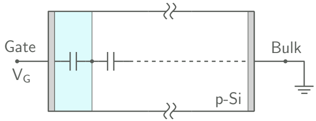

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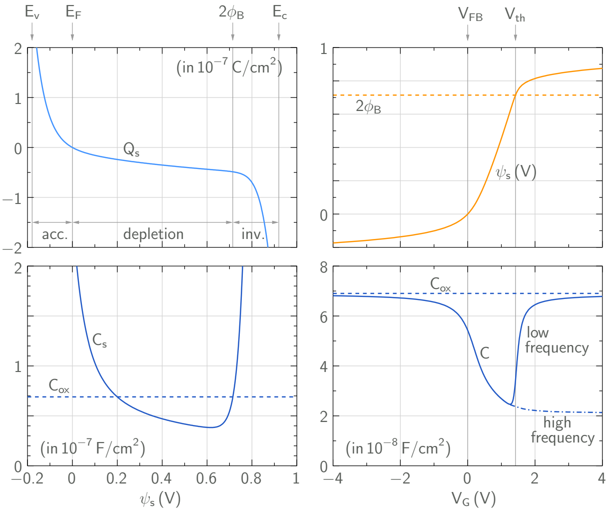

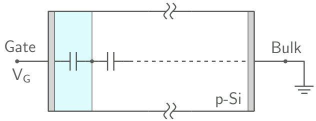

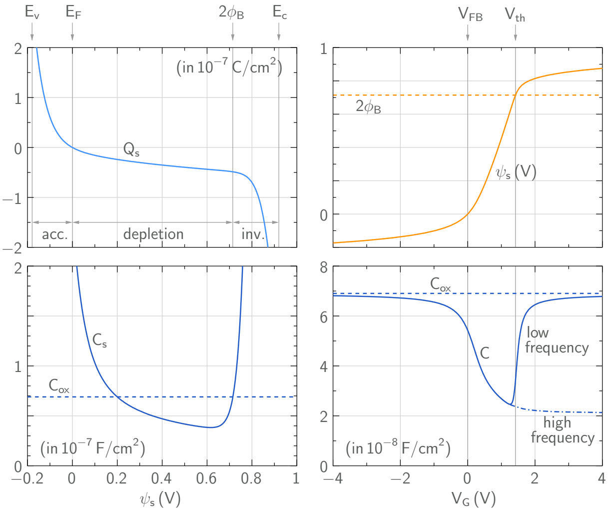

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- The low-frequency ($f < 100\,\text{Hz}$) and high-freq ($f > 1\,\text{MHz}$) C-V curves offer an excellent "diagnostic" tool during processing since they can be used to find the oxide thickness, flat-band voltage, etc.