Semiconductor Devices

p-*n* Junctions: Part 5

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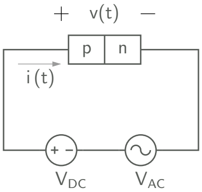
pn junction diode: AC and transient conditions

* We have looked at the DC behaviour of a *pn* junction diode so far. We now want to consider V_a (the applied voltage) varying with time.
* Two situations are of interest:
  * Small-signal behaviour (AC): With $V_a(t) = V_{DC} + V_m \sin \omega t$, how does the current vary with time when V_m is "small?"
  * Large-signal behaviour: The variation in the applied voltage is not small. In particular, we are interested in the turn-off and turn-on transients.

Small-signal behaviour



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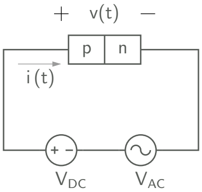

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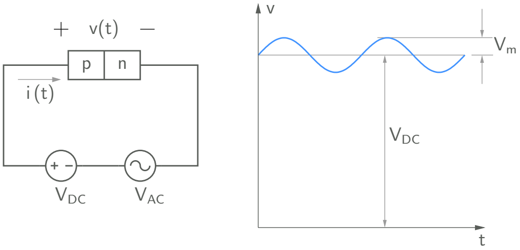

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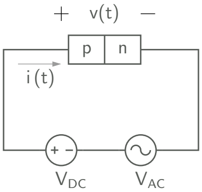

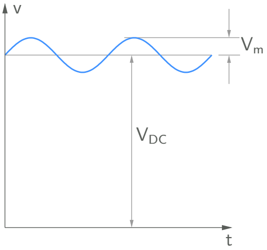

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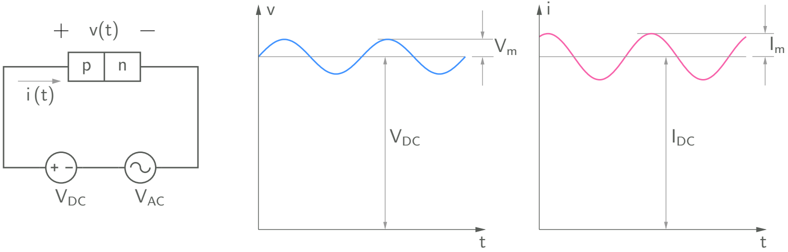

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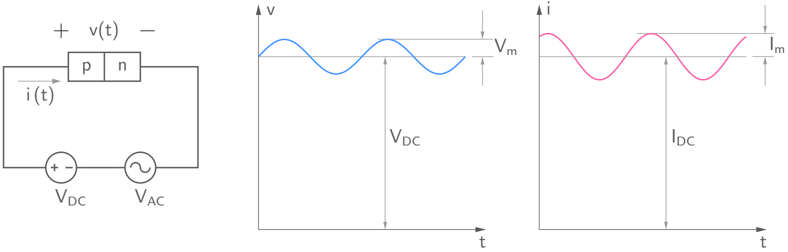

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* In small-signal analysis, we are interested in the relationship between the sinusoidal parts of the current and voltage, in particular, the ratio of the current and voltage phasors, $I_m \angle \phi / V_m \angle 0$.

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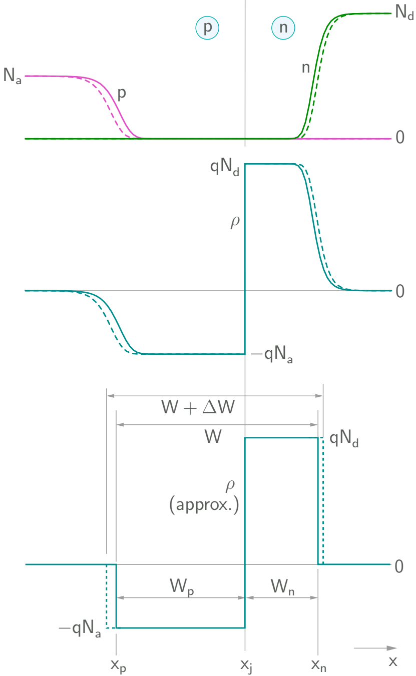

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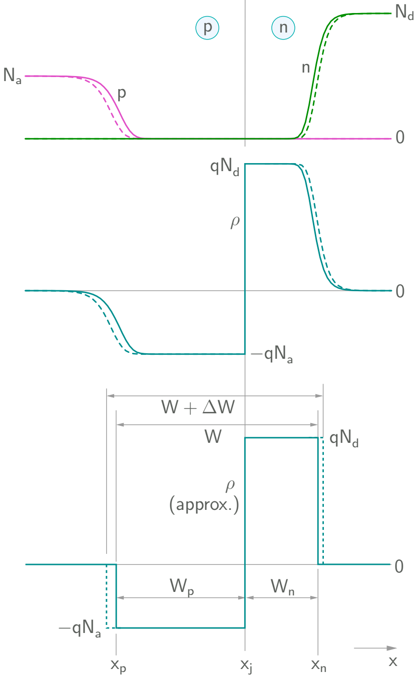
* This change is made possible by removal of majority carriers.



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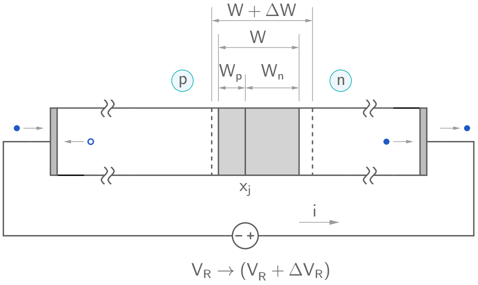

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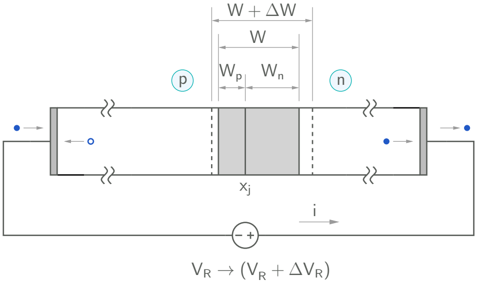

* Movement of majority carriers is relatively fast, and the time scale involved is $\sim \tau = \dfrac{\epsilon_s}{q\mu_n n}$ for electrons.

  For $n = 10^{16}\,\text{cm}^{-3}$, $\mu_n = 1000\,\text{cm}^2/\text{V-s}$, $\epsilon_s = 11.7\epsilon_0$, $\tau = 0.6\,\text{ps}$, which is negligibly small for all practical purposes.

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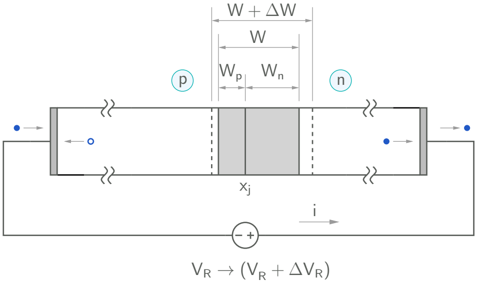

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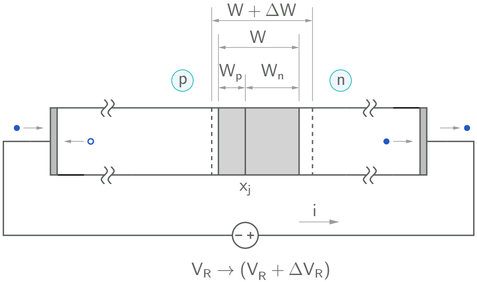

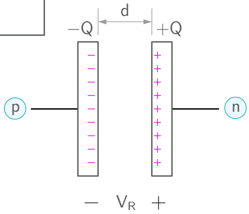

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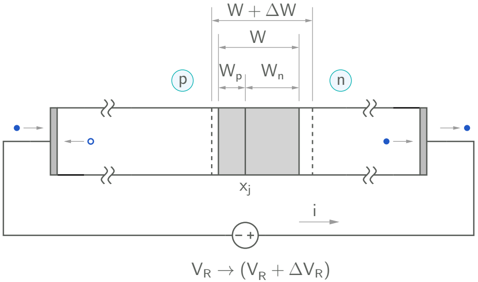

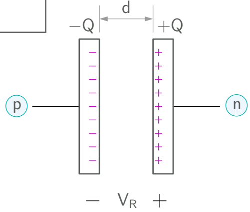

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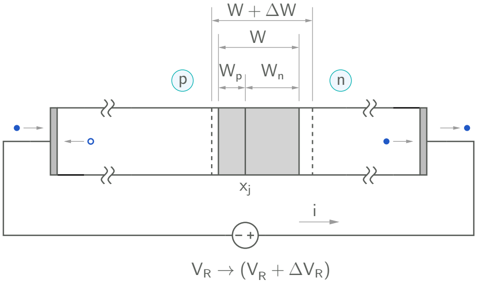

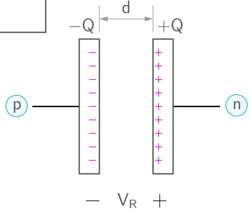

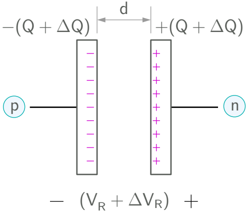

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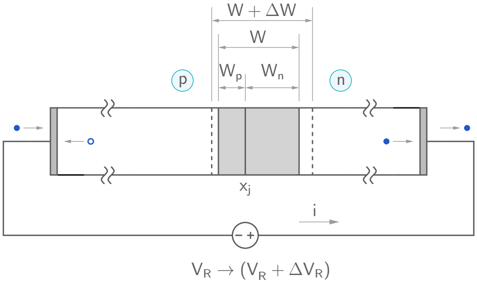

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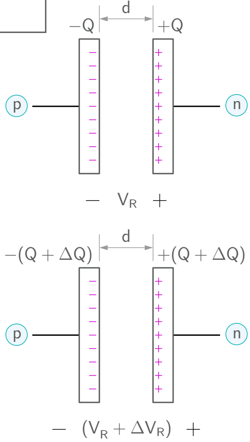
  Note: For simplicity, we have not shown V_{bi} in the figure; the drop across the junction is actually $V_{\text{bi}} + V_R$, as seen before.



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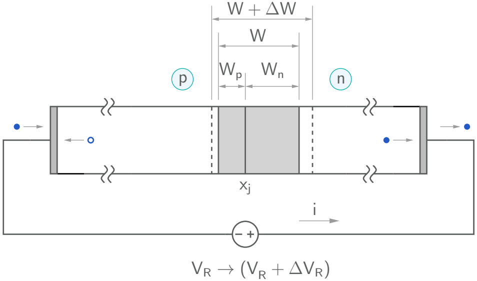

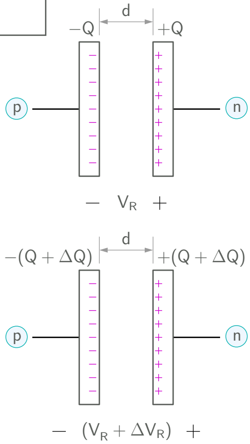

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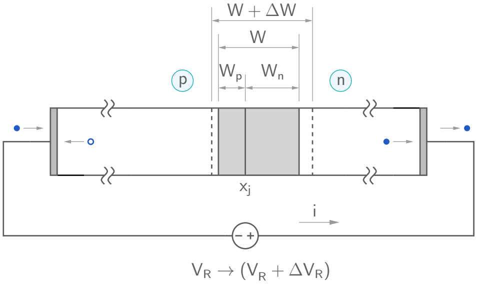

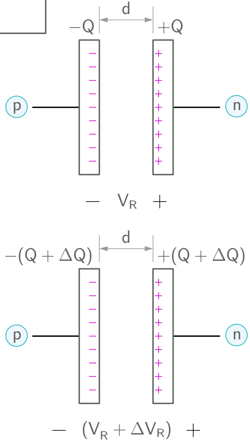

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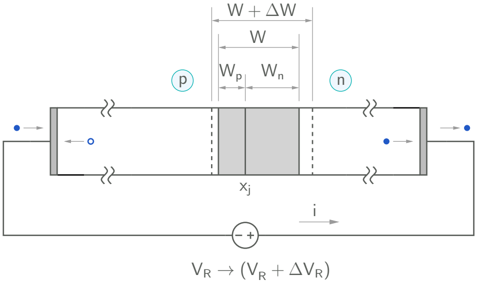

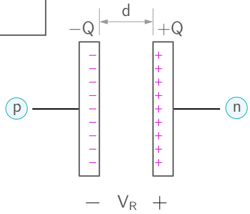

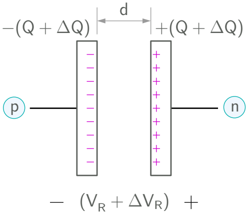

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* For a reverse-biased *pn* junction, $C = \dfrac{\Delta Q}{\Delta V_R}$.

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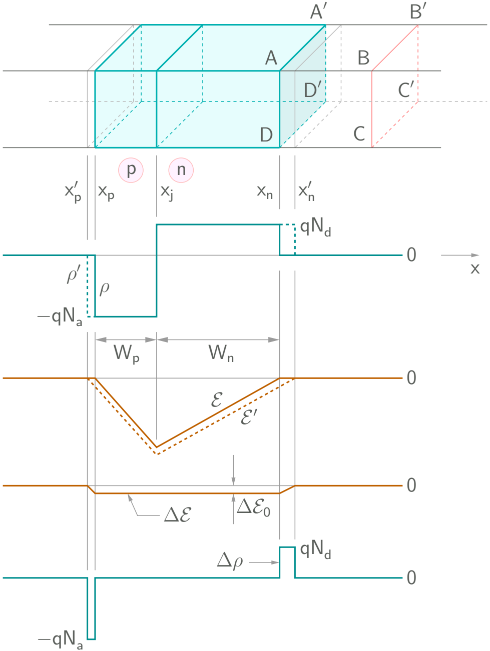
$$V_{\text{bi}} + V_R = -\int_{x_p}^{x_n} \mathcal{E}(x)\,dx$$



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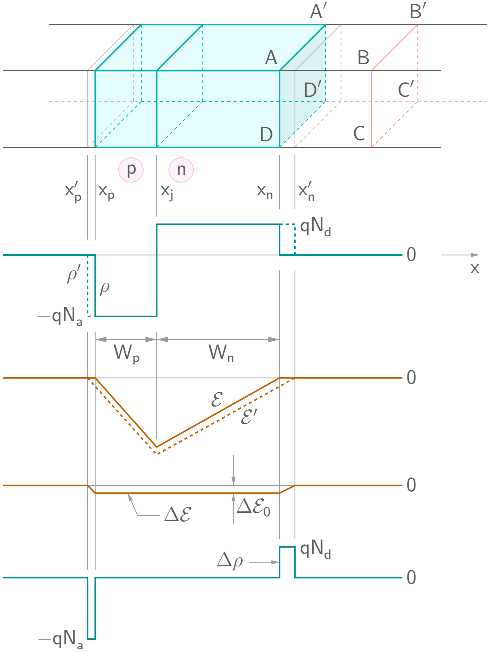

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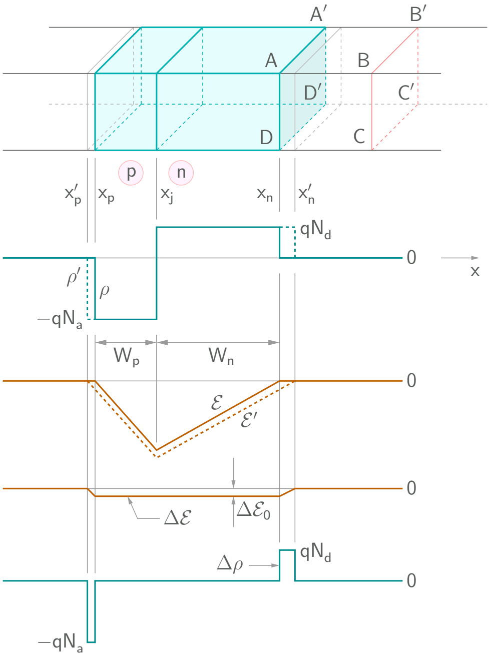

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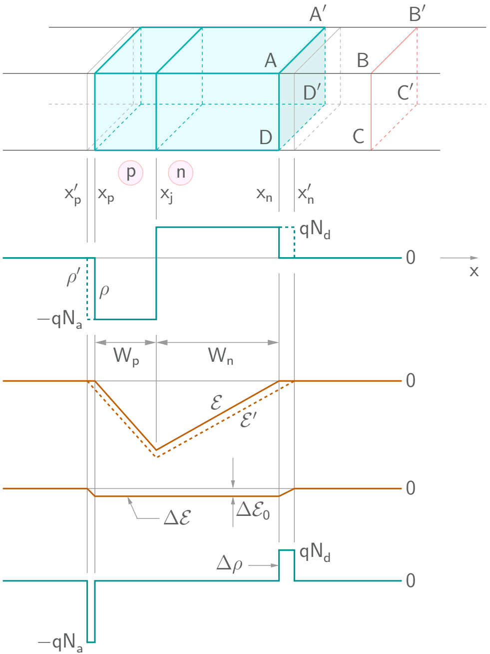

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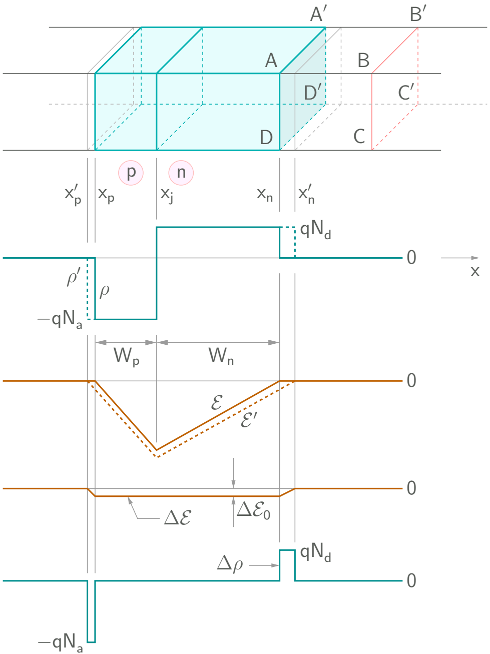

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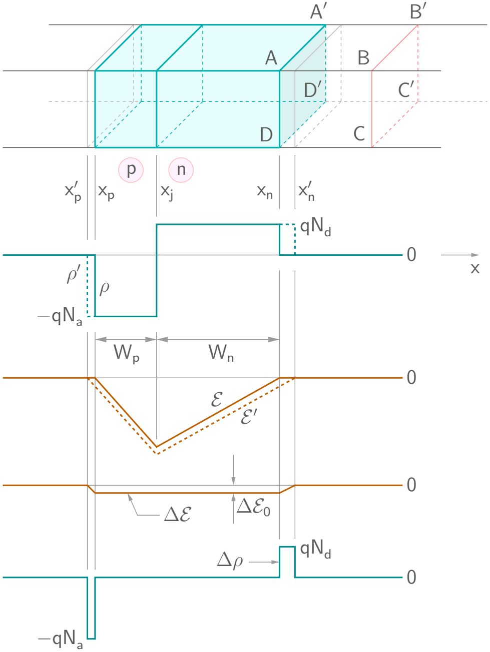
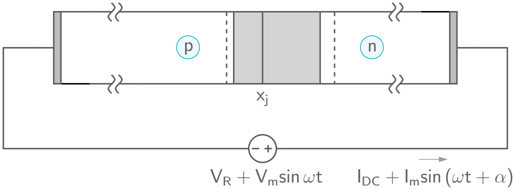
C_J is called the "junction capacitance" or "depletion layer capacitance."



Example



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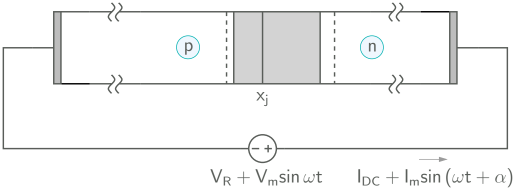

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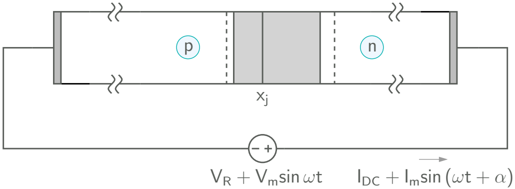

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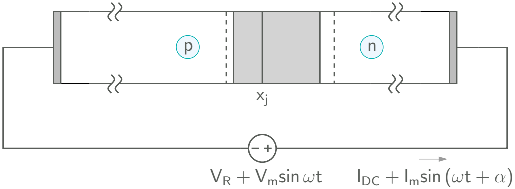

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Capacitance $C_J = \dfrac{A\,\epsilon_s}{W} = \dfrac{0.01\,\text{cm}^2 \times 11.7 \times 8.85 \times 10^{-14}\,\text{F/cm}}{0.464 \times 10^{-4}\,\text{cm}}$

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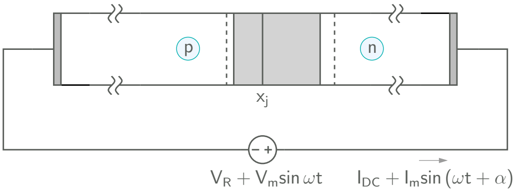

For an abrupt, uniformly doped silicon *pn* junction, with $N_a = 10^{17}\,\text{cm}^{-3}$ and $N_d = 2\times 10^{16}\,\text{cm}^{-3}$, and area $= 0.01\,\text{cm}^2$, calculate the capacitance (i.e., the differential capacitance) for an applied reverse bias of $V_R = 2\,\text{V}$ ($T = 300\,\text{K}$).

Solution: The built-in voltage is

$$V_{\text{bi}} = \frac{kT}{q}\log\left(\frac{N_aN_d}{n_i^2}\right) = 0.77\,\text{V}\,.$$

The depletion width for $V_R = 2\,\text{V}$ is $W = \sqrt{\dfrac{2\epsilon}{q}\,\dfrac{N_a+N_d}{N_aN_d}\,(V_{\text{bi}}+V_R)} = 0.464\,\mu\text{m}$.

$$\text{Capacitance } C_J = \frac{A\,\epsilon_s}{W} = \frac{0.01\,\text{cm}^2 \times 11.7\times 8.85\times 10^{-14}\,\text{F/cm}}{0.464\times 10^{-4}\,\text{cm}}$$

$$= 2.23\times 10^{-10}\ \text{F}$$

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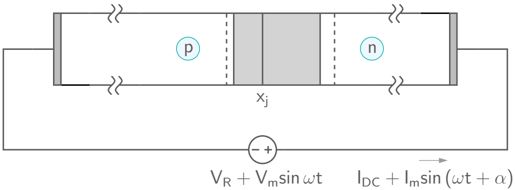

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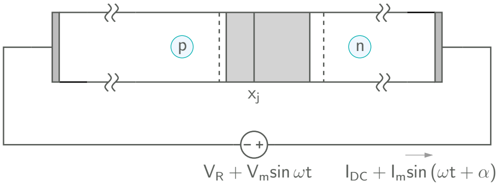

For a silicon n^+p junction, $N_a = 10^{16}\,\text{cm}^{-3}$, and area $= 0.01\,\text{cm}^2$. Plot C_J versus V_a for $-5\,\text{V} < V_a < -0.1\,\text{V}$. Also, plot $1/C_J^2$ versus V_a. What information can one obtain from the second plot? Take $V_{bi} \approx 0.9\,\text{V}$. ($T = 300\,\text{K}$)

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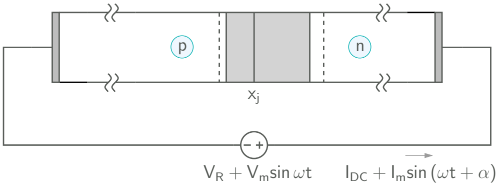

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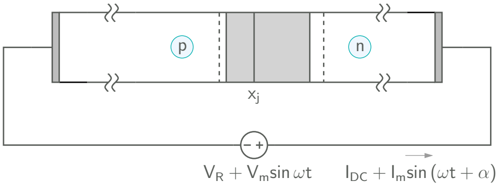

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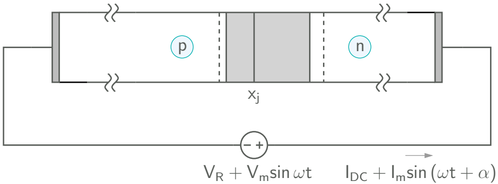

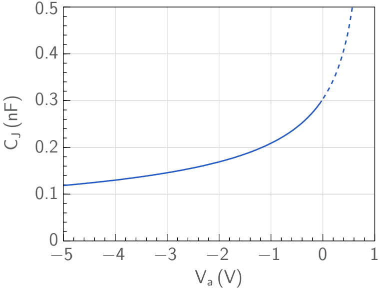

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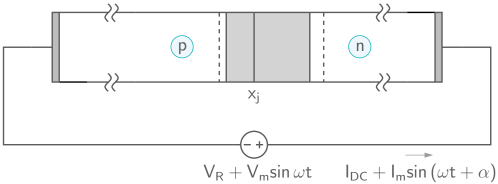

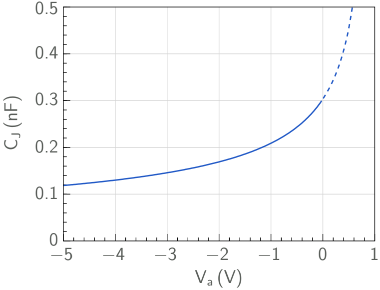

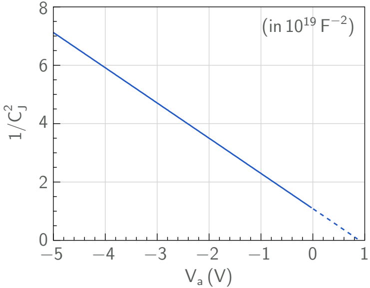

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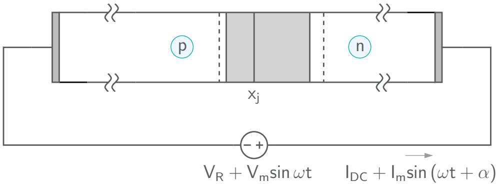

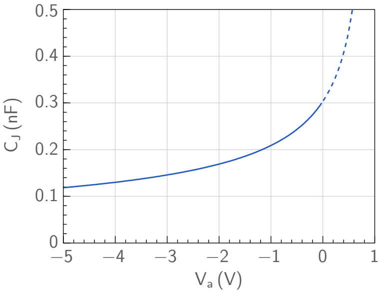

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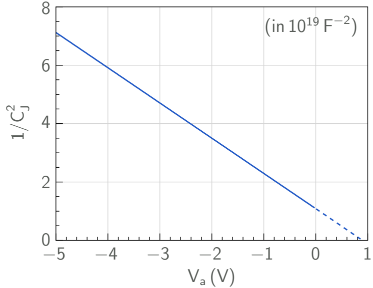

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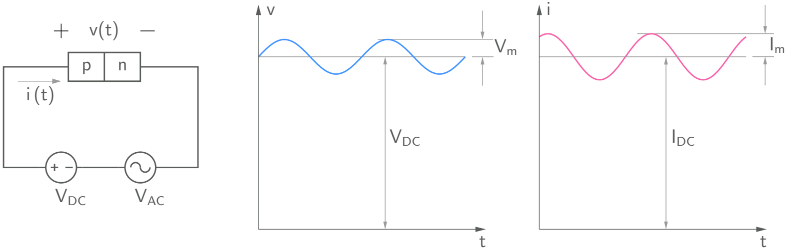
$\rightarrow 1/C_J^2$ versus V_a: Slope gives N_a; x-intercept gives V_{bi}.

What is meant by "small-signal" condition?



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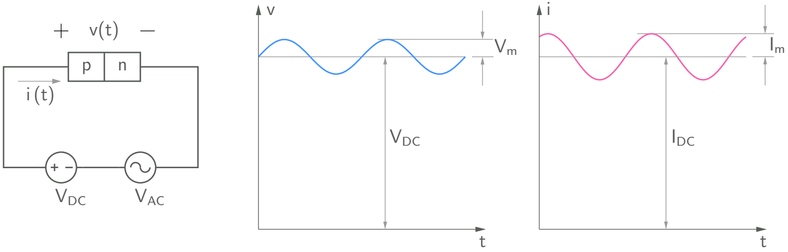

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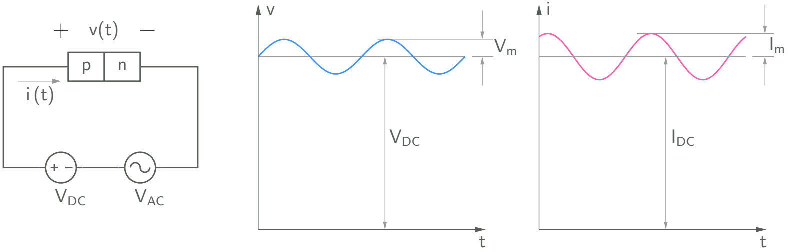

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With $V_a(t) = -(V_R + V_m \sin\omega t)$, $i(t) = \dfrac{dQ}{dt} = \dfrac{dQ}{dV_a}\,\dfrac{dV_a}{dt} = C_J(V_a) \times (-\omega V_m \cos\omega t)$.

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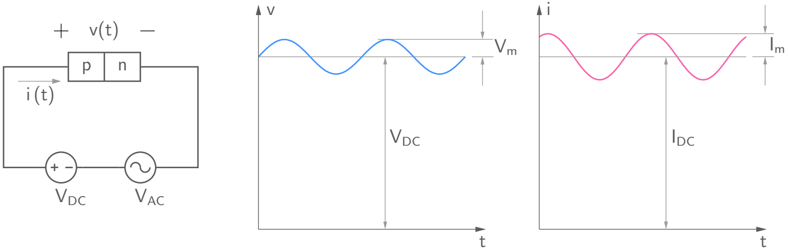

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→ $i(t)$ is sinusoidal if C_J can be treated as a constant.

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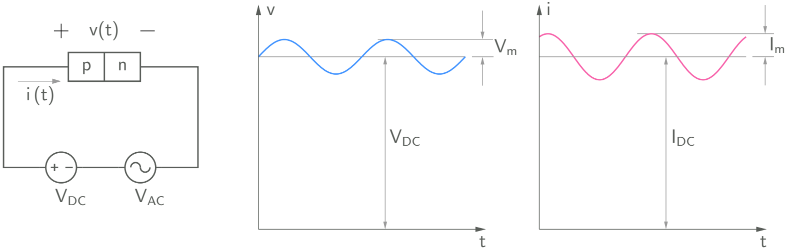

$$v_a(t) = -(V_R + V_m \sin \omega t) \rightarrow -(V_R + V_m) < v_a < -(V_R - V_m).$$

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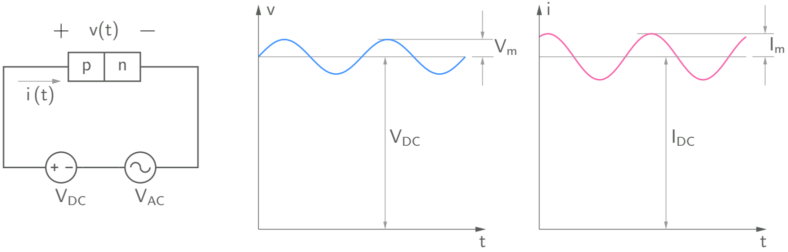

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Consider one of these two extreme values,

$$C_J^{\text{max}} = \frac{K}{\sqrt{V_{\text{bi}} + V_R - V_m}} = \frac{K}{\sqrt{V_{\text{bi}} + V_R}} \times \frac{1}{\sqrt{1 - \dfrac{V_m}{V_{\text{bi}} + V_R}}} \approx \frac{K}{\sqrt{V_{\text{bi}} + V_R}} \left(1 + \frac{1}{2}\,\frac{V_m}{V_{\text{bi}} + V_R}\right).$$

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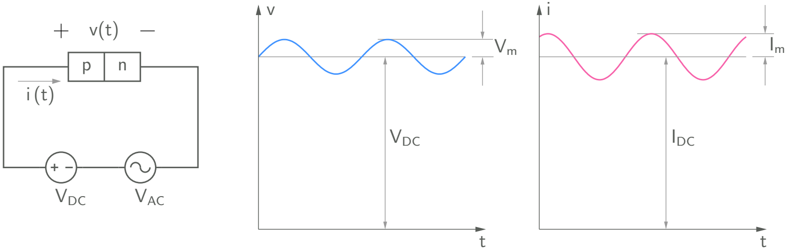

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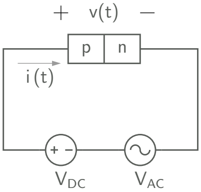

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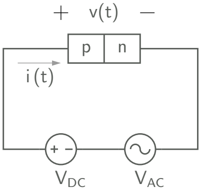

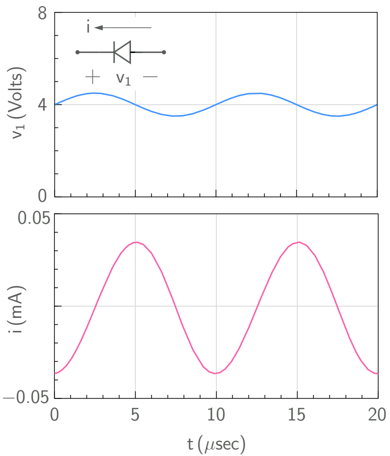

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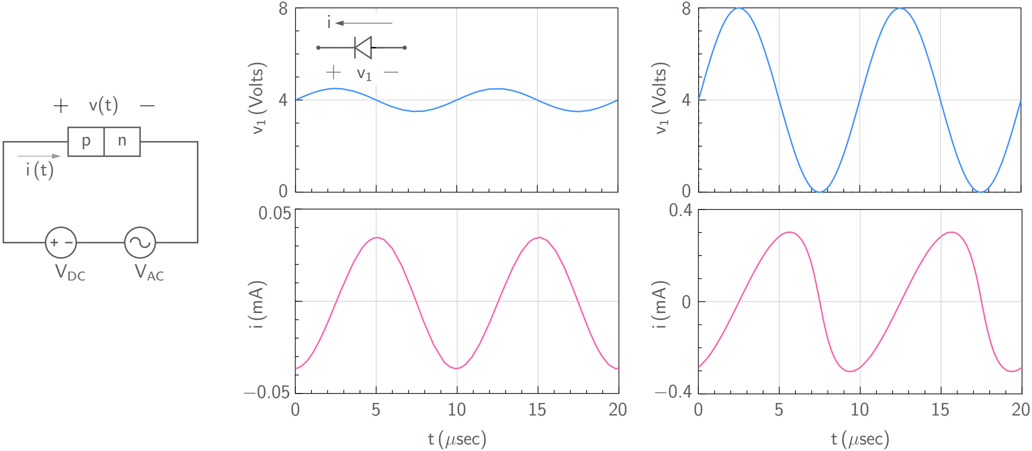

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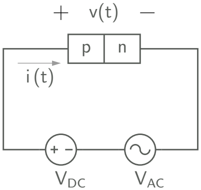
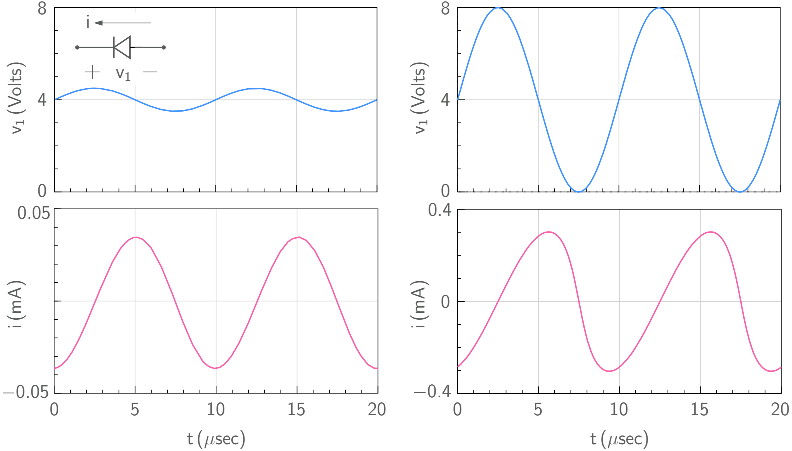
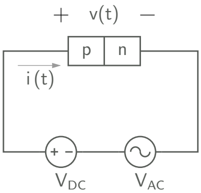
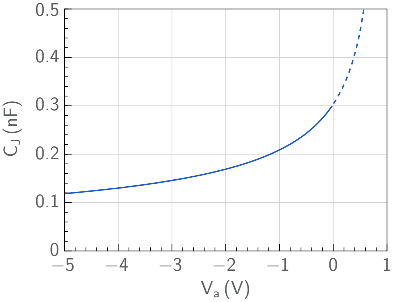


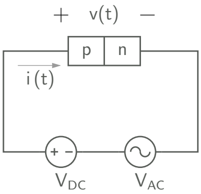


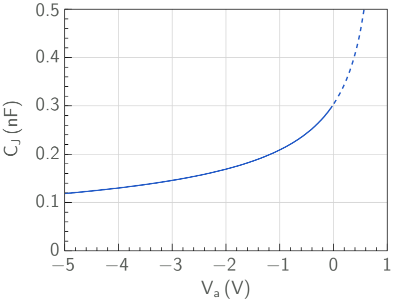
* Small-signal condition: $\dfrac{V_m}{2\,(V_{\text{bi}} + V_R)} \ll 1.$
* If the small-signal condition is not satisfied, $i(t)$ shows distortion.

Small-signal behaviour: reverse bias









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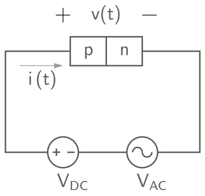

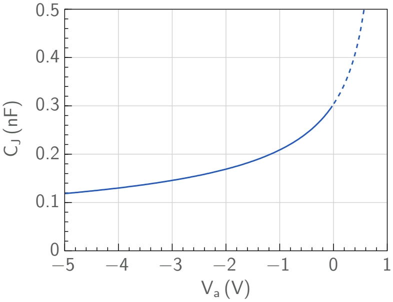

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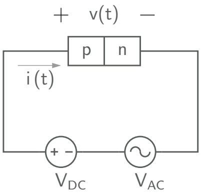
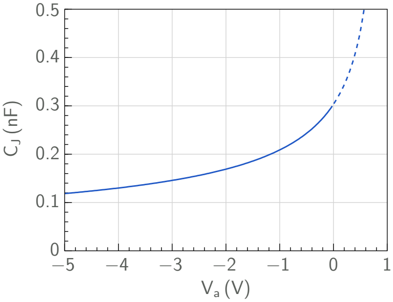
* Specially designed diodes called "varactors" (variable reactors) are used in applications such as voltage-variable tuning, mixing, detection, etc.





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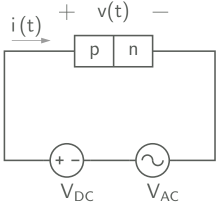
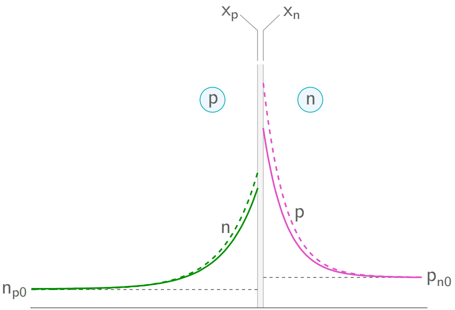
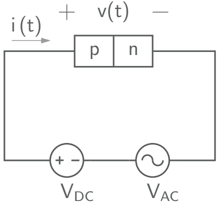
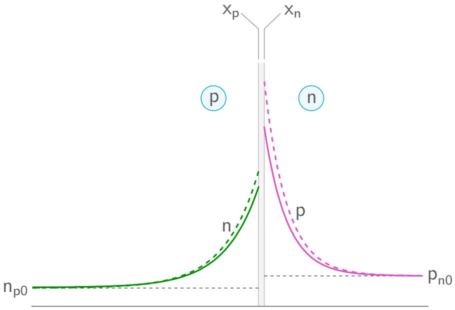
* Specially designed diodes called "varactors" (variable reactors) are used in applications such as voltage-variable tuning, mixing, detection, etc.
* In these devices, the doping density profiles are designed so as to get a large capacitance change for a small change in reverse bias.


i (t)
+ v(t) −
p
n
+ −
V_{DC}
V_{AC}


x_p
x_n
p
n
n
p
n_{p0}
p_{n0}





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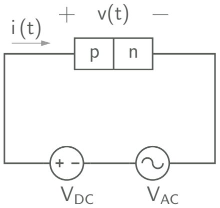

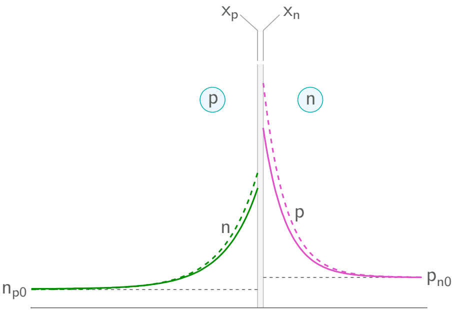

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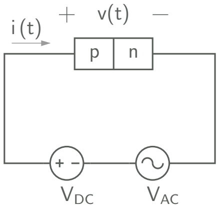
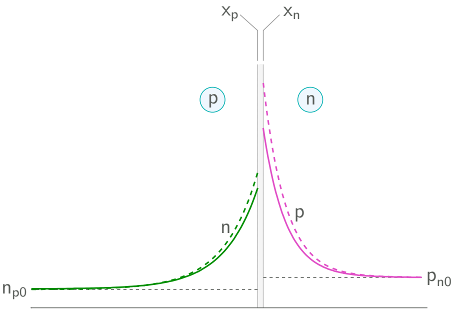
* If the applied voltage is increased from V_a to $(V_a + \Delta V_a)$, the carrier densities would also increase.

pn junction under forward bias: small-signal model





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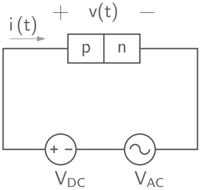
* If the applied voltage is increased from V_a to $(V_a + \Delta V_a)$, the carrier densities would also increase.
* At low frequencies, the minority carrier profiles change in synchronisation with $V_a(t)$.
  $\rightarrow$ The Shockley equation can be used, with $V_a \rightarrow v_a(t)$, $I \rightarrow i(t)$,
  i.e., $i(t) = I_s\left[e^{v_a(t)/V_T} - 1\right] \approx I_s\, e^{v_a(t)/V_T}$.

pn junction under forward bias: small-signal model



$$i(t) = I_s \exp\left(\frac{V_{DC} + V_m \sin \omega t}{V_T}\right) = \left[I_s \exp\left(\frac{V_{DC}}{V_T}\right)\right] \exp\left(\frac{V_m \sin \omega t}{V_T}\right) = I_{DC} \exp\left(\frac{V_m \sin \omega t}{V_T}\right).$$

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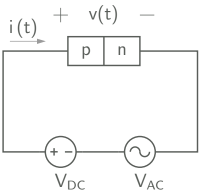

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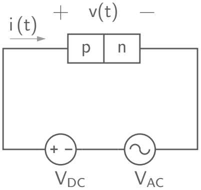

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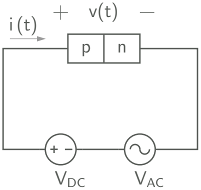

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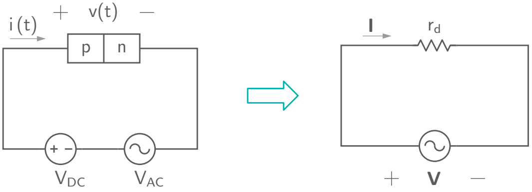
pn junction under forward bias: small-signal model



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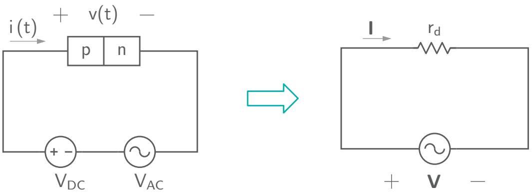

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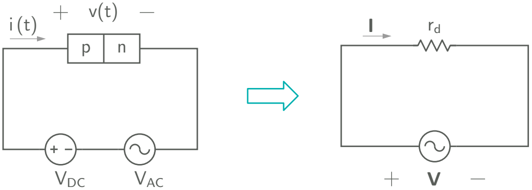
Example: $T = 300\,\text{K}$, $I_{DC} = 1\,\text{mA} \rightarrow r_d = \frac{25.9\,\text{mV}}{1\,\text{mA}} = 25.9\,\Omega$.



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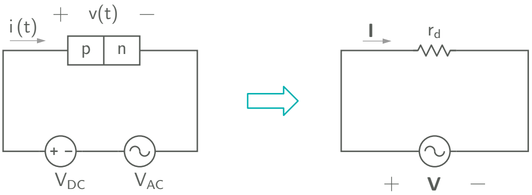
r_d is small, compared to typical resistance values used in electronics ($\sim$ kΩ).

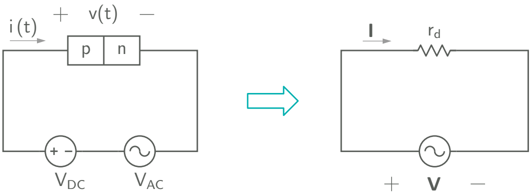


For a silicon *pn* diode, $I_s = 10^{-13}\,A$. Consider $V_a(t) = 0.6\,\text{V} + V_m \sin \omega t$. Assume that the frequency is low enough so that the minority carrier profiles can follow $V_a(t)$.

Plot the diode current $i(t)$ using (a) the Shockley equation, (b) the low-frequency small-signal model.

Consider two values of V_m: 2 mV and 10 mV ($T = 300$ K).

Example



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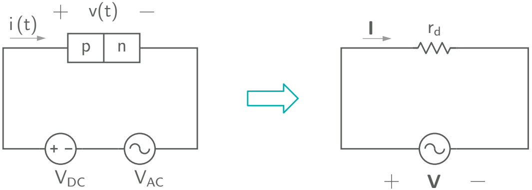
Example



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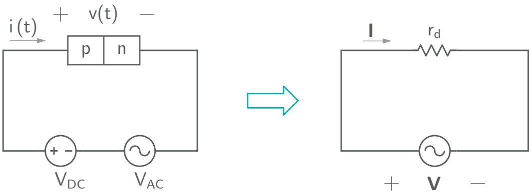

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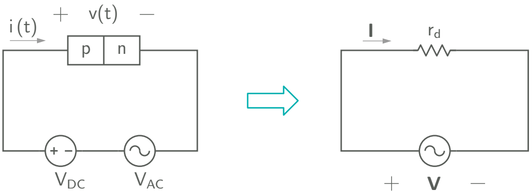

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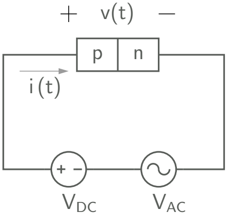
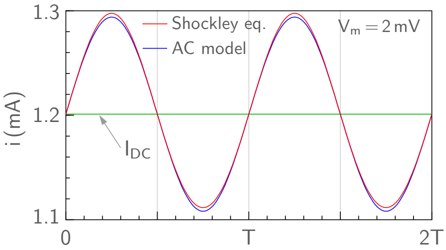
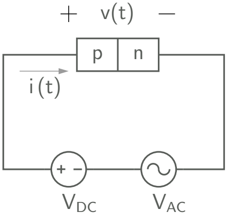
$$i(t) = I_{DC} + i_{ac}.$$

Example

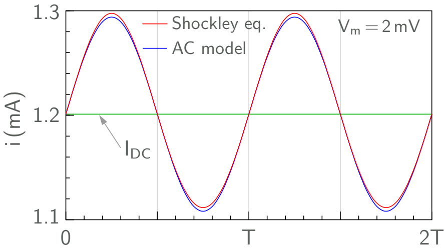

+ v(t) −
p
n
i (t)
+ −
V_{DC}
V_{AC}


1.3
1.2
1.1
i (mA)
0
T
2T
Shockley eq.
AC model
$V_m = 2\,mV$
I_{DC}

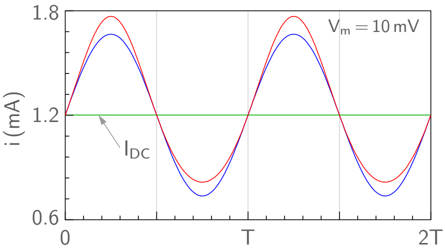
Example



+ v(t) −
p
n
i (t)
V_{DC}
V_{AC}

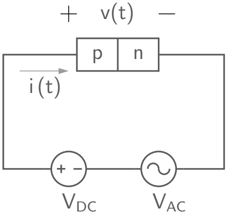


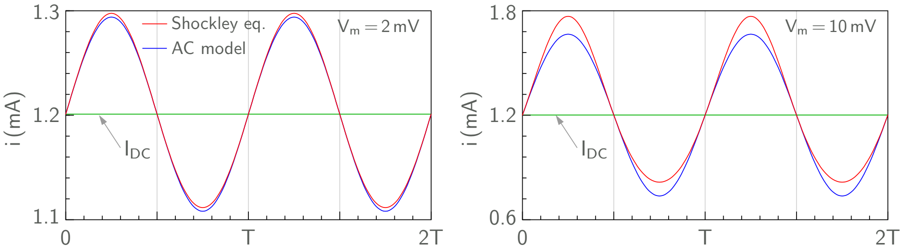
1.3
1.2
1.1
i (mA)
Shockley eq.
AC model
$V_m = 2\,mV$
I_{DC}
0
T
2T



1.8
1.2
0.6
i (mA)
$V_m = 10\,mV$
I_{DC}
0
T
2T

Example

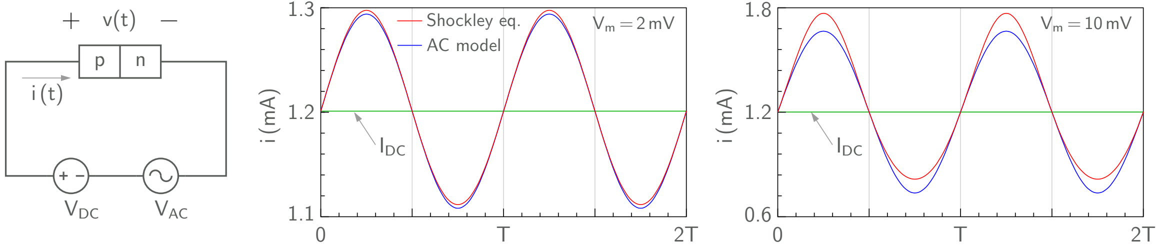




If V_m is not small compared to V_T,

* The diode current waveform is significantly distorted.

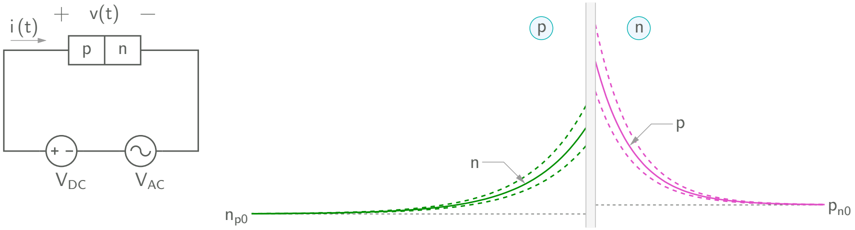
Example



If V_m is not small compared to V_T,

* The diode current waveform is significantly distorted.
* The small-signal model is not accurate.

High-frequency small-signal model (forward bias)



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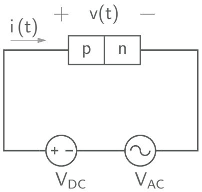

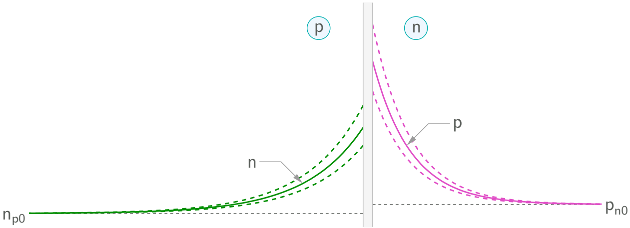

* At high frequencies, the carrier profiles cannot follow changes in the applied voltage, and the minority-carrier continuity equation needs to be solved to obtain
  $p(x, t)$ on the n-side and
  $n(x, t)$ on the p-side.

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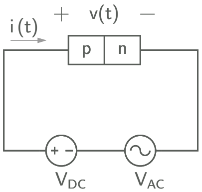

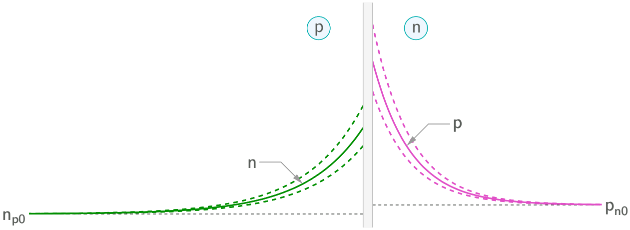

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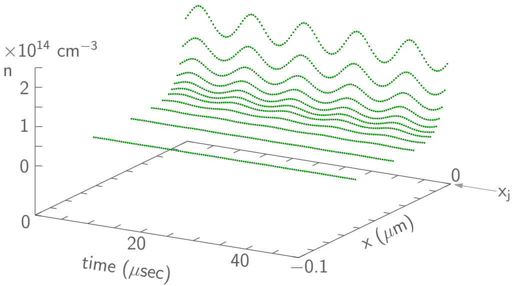

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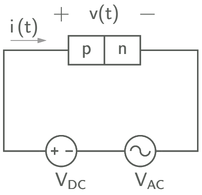

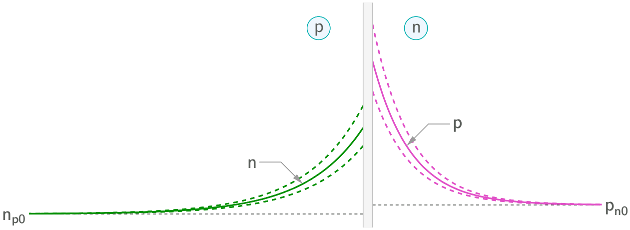

* At high frequencies, the carrier profiles cannot follow changes in the applied voltage, and the minority-carrier continuity equation needs to be solved to obtain
  $p(x,t)$ on the n-side and
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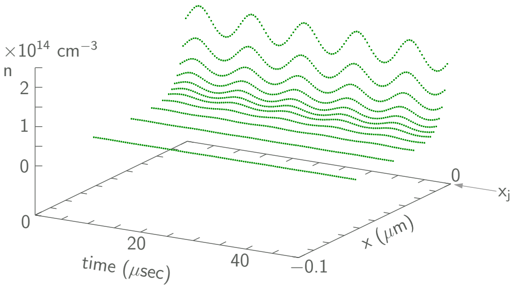
* Using the above solution, the small-signal model can be derived.[1]



[1] M. Shur, *Physics of Semiconductor Devices*. New Delhi: Prentice-Hall India, 1990.

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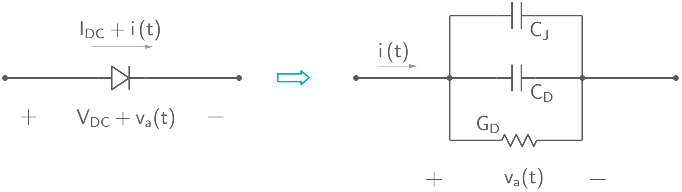

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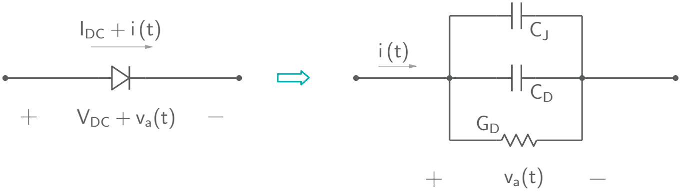
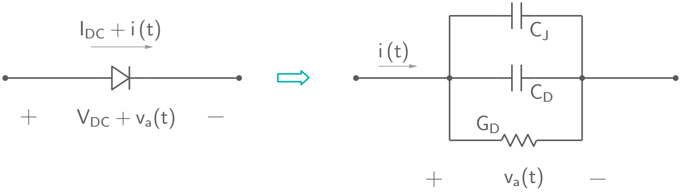


* C_J is the depletion capacitance.

High-frequency small-signal model (forward bias)



* C_J is the depletion capacitance.
* G_D and C_D arise from diffusion of minority carriers.

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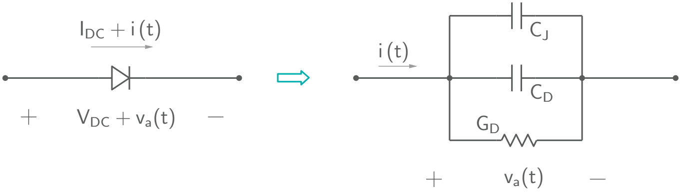
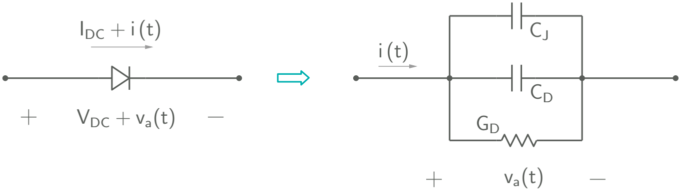


* C_J is the depletion capacitance.
* G_D and C_D arise from diffusion of minority carriers.
* For a p^+n junction,

$$G_D = \frac{G_0}{\sqrt{2}}\left(\sqrt{1+(\omega\tau_p)^2}+1\right)^{1/2}, \quad C_D = \frac{G_0}{\omega\sqrt{2}}\left(\sqrt{1+(\omega\tau_p)^2}-1\right)^{1/2},$$

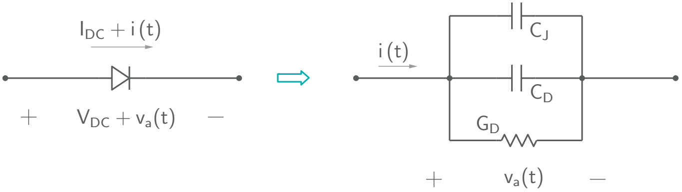
where $G_0 = \dfrac{I_s}{V_T}\exp\left(\dfrac{V_{DC}}{V_T}\right)$ is the low-frequency conductance seen earlier.

Example



For a silicon p^+n diode at 300 K, $V_{bi} = 0.72$ V, $I_s = 1 \times 10^{-13}$ A, and the junction capacitance $C_J = 340$ pF at $V_{DC} = 0$ V. Assume $\tau_p = 1.5\,\mu s$.

Example



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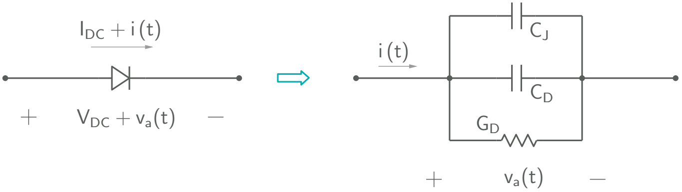
(a) For $f = 1$ kHz, plot C_J and C_D versus V_{DC} for $0\,\text{V} < V_{DC} < 0.65$ V. Also, show $(C_J + C_D)$ on the same plot.

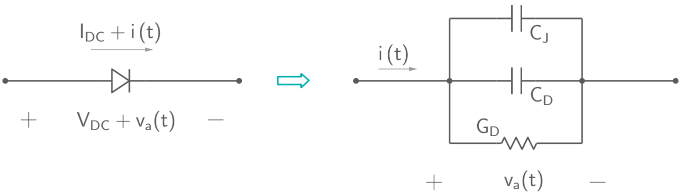
Example



For a silicon p^+n diode at 300 K, $V_{bi} = 0.72$ V, $I_s = 1 \times 10^{-13}$ A, and the junction capacitance $C_J = 340$ pF at $V_{DC} = 0$ V. Assume $\tau_p = 1.5\,\mu$s.

(a) For $f = 1$ kHz, plot C_J and C_D versus V_{DC} for $0\,\text{V} < V_{DC} < 0.65$ V. Also, show $(C_J + C_D)$ on the same plot.

(b) For $f = 1$ kHz, plot G_D versus V_{DC} for $0\,\text{V} < V_{DC} < 0.65$ V.



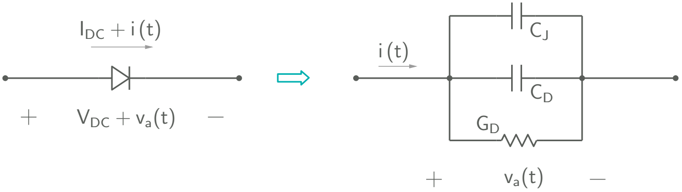
For a silicon p^+n diode at 300 K, $V_{\mathrm{bi}} = 0.72$ V, $I_s = 1 \times 10^{-13}$ A, and the junction capacitance $C_J = 340$ pF at $V_{DC} = 0$ V. Assume $\tau_p = 1.5\ \mu$s.

(a) For $f = 1$ kHz, plot C_J and C_D versus V_{DC} for $0\,\mathrm{V} < V_{DC} < 0.65$ V. Also, show $(C_J + C_D)$ on the same plot.

(b) For $f = 1$ kHz, plot G_D versus V_{DC} for $0\,\mathrm{V} < V_{DC} < 0.65$ V.

(c) Find G_{D0} and C_{D0}, the values of G_D and C_D, respectively, as $\omega \to 0$.

Example



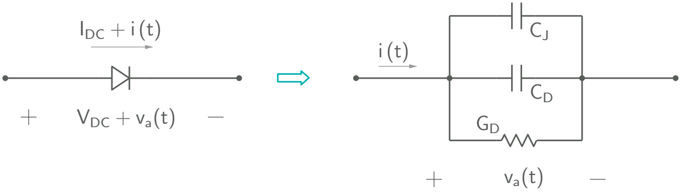
For a silicon p^+n diode at 300 K, $V_{bi} = 0.72$ V, $I_s = 1 \times 10^{-13}$ A, and the junction capacitance $C_J = 340$ pF at $V_{DC} = 0$ V. Assume $\tau_p = 1.5$ µs.

(a) For $f = 1$ kHz, plot C_J and C_D versus V_{DC} for $0\,\text{V} < V_{DC} < 0.65\,\text{V}$. Also, show $(C_J + C_D)$ on the same plot.

(b) For $f = 1$ kHz, plot G_D versus V_{DC} for $0\,\text{V} < V_{DC} < 0.65\,\text{V}$.

(c) Find G_{D0} and C_{D0}, the values of G_D and C_D, respectively, as $\omega \rightarrow 0$.

(d) For $V_{DC} = 0.6$ V, plot G_D/G_{D0} and C_D/C_{D0} versus f for $10\,\text{Hz} < f < 1\,\text{MHz}$.



For a silicon p^+n diode at 300 K, $V_{bi} = 0.72$ V, $I_s = 1 \times 10^{-13}$ A, and the junction capacitance $C_J = 340$ pF at $V_{DC} = 0$ V. Assume $\tau_p = 1.5$ µs.

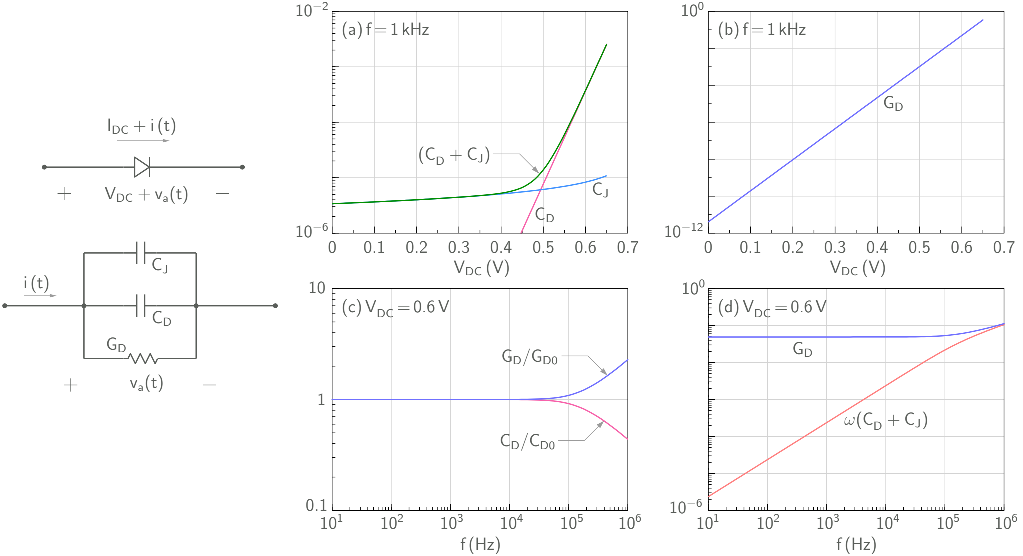
(a) For $f = 1$ kHz, plot C_J and C_D versus V_{DC} for $0\,\text{V} < V_{DC} < 0.65$ V. Also, show $(C_J + C_D)$ on the same plot.

(b) For $f = 1$ kHz, plot G_D versus V_{DC} for $0\,\text{V} < V_{DC} < 0.65$ V.

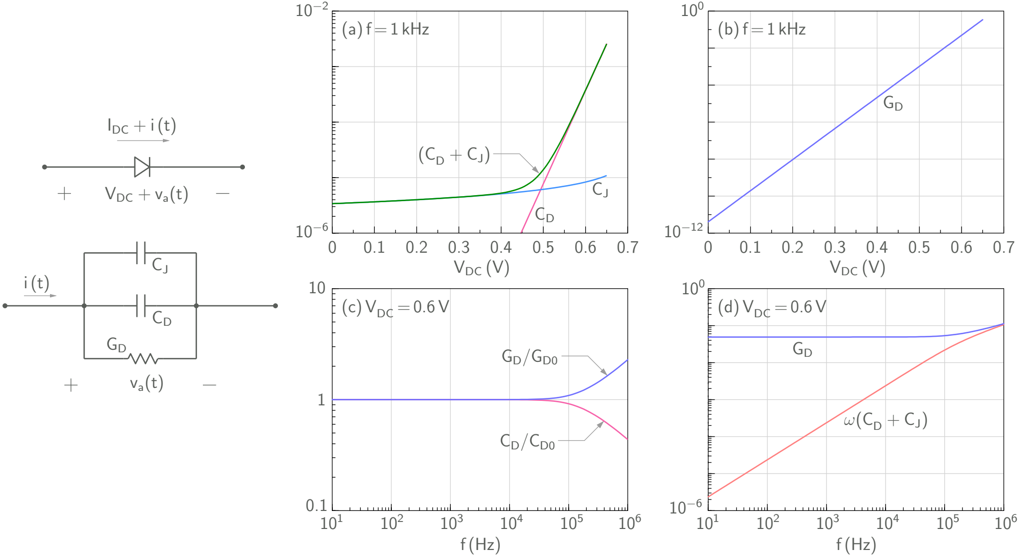
(c) Find G_{D0} and C_{D0}, the values of G_D and C_D, respectively, as $\omega \rightarrow 0$.

(d) For $V_{DC} = 0.6$ V, plot G_D/G_{D0} and C_D/C_{D0} versus f for $10\,\text{Hz} < f < 1$ MHz.

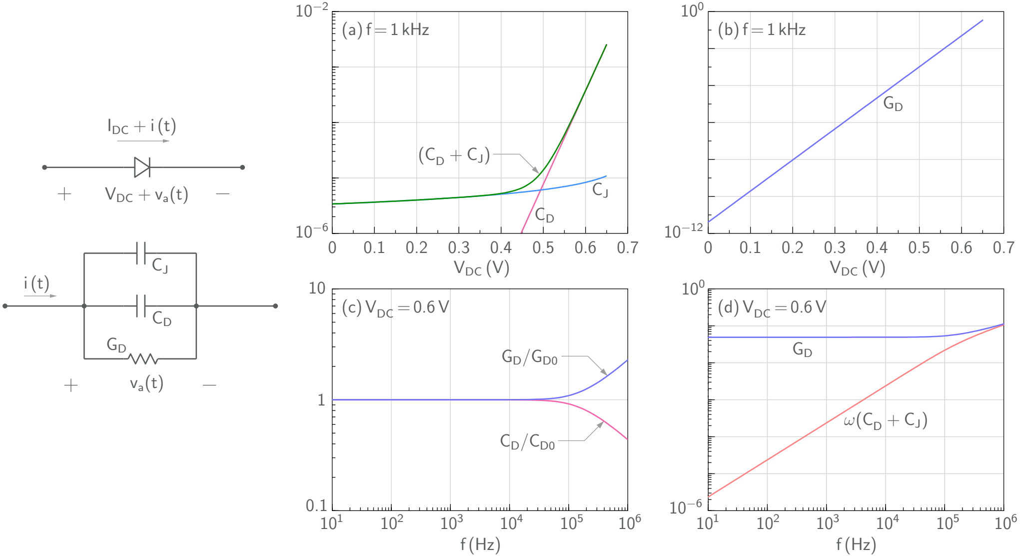
(e) For $V_{DC} = 0.6$ V, plot G_D and $\omega(C_J + C_D)$ versus frequency for $10\,\text{Hz} < f < 1$ MHz.



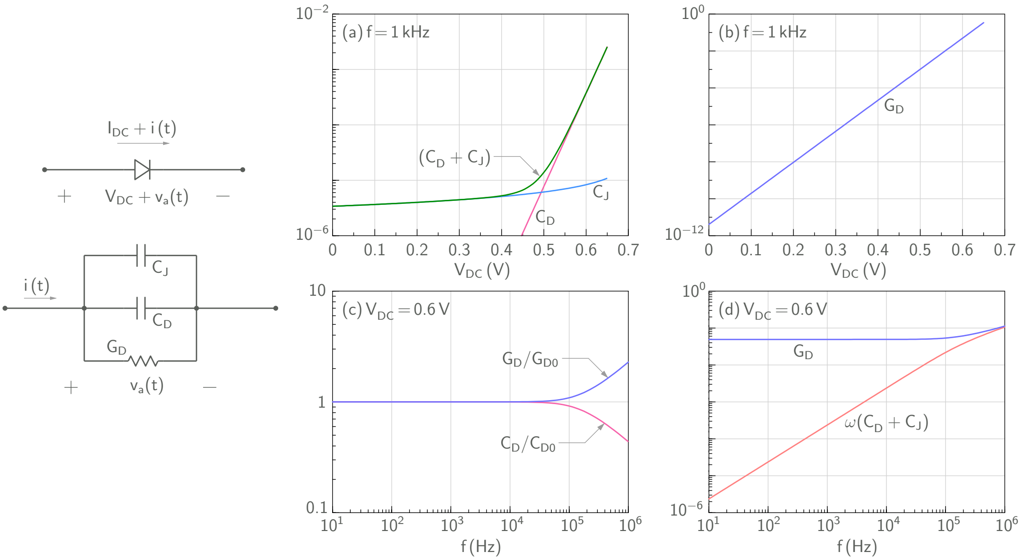
I_DC + i (t)
+ V_DC + v_a(t) −
C_J
i (t)
C_D
G_D
+ v_a(t) −
(a) f = 1 kHz
(C_D + C_J)
C_J
C_D
V_DC (V)
(b) f = 1 kHz
G_D
V_DC (V)
(c) V_DC = 0.6 V
G_D/G_D0
C_D/C_D0
f (Hz)
(d) V_DC = 0.6 V
G_D
ω(C_D + C_J)
f (Hz)



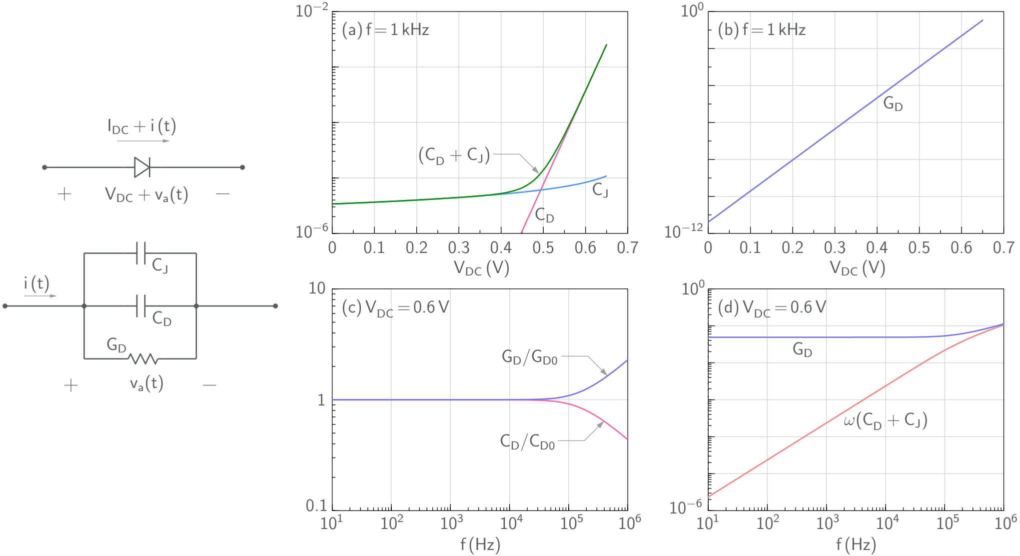
$$* \;\; C_J = \frac{A\epsilon_s}{W} = \frac{K}{\sqrt{V_{\rm bi} - V_{DC}}} \rightarrow \frac{C_J(V_{DC})}{C_J(0\,{\rm V})} = \sqrt{\frac{V_{\rm bi}}{V_{\rm bi} - V_{DC}}} \rightarrow C_J \uparrow \text{ as } V_{DC} \uparrow.$$



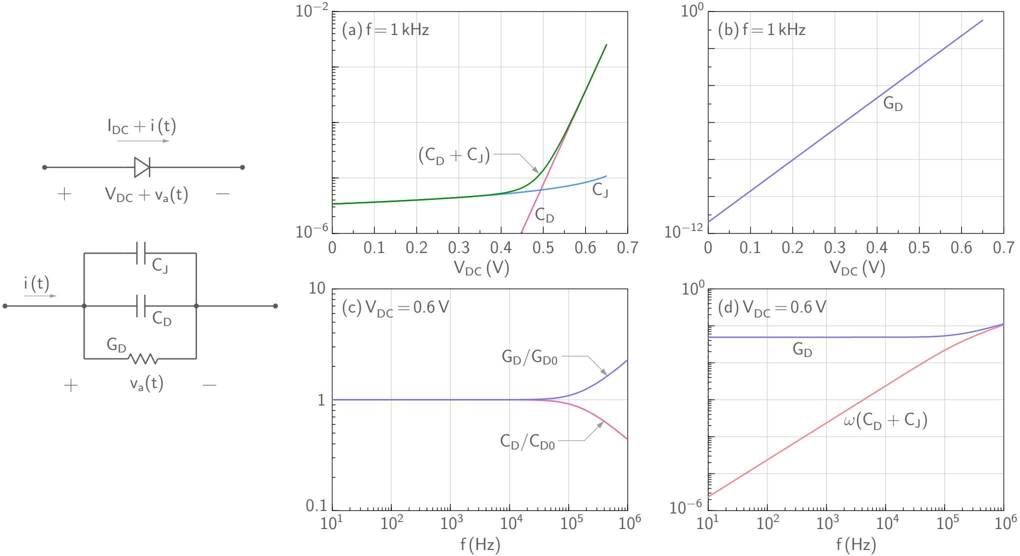
$I_{DC} + i(t)$
$V_{DC} + v_a(t)$
$i(t)$
C_J
C_D
G_D
$v_a(t)$
(a) f = 1 kHz
$(C_D + C_J)$
C_J
C_D
V_{DC} (V)
(b) f = 1 kHz
G_D
V_{DC} (V)
(c) V_{DC} = 0.6 V
G_D/G_{D0}
C_D/C_{D0}
f (Hz)
(d) V_{DC} = 0.6 V
G_D
$\omega(C_D + C_J)$
f (Hz)



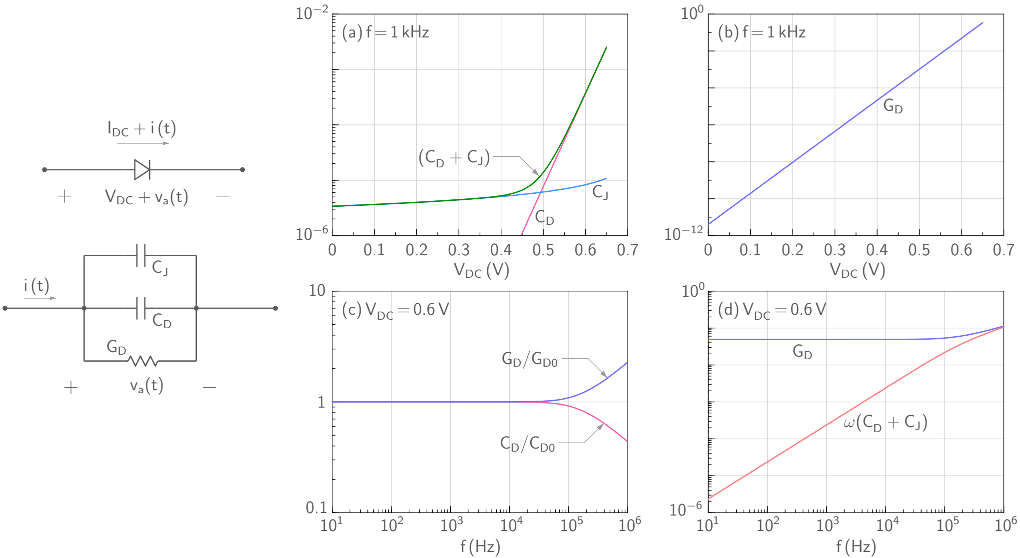
* $C_D = \dfrac{G_0}{\omega\sqrt{2}}\left(\sqrt{1+(\omega\tau_p)^2}-1\right)^{1/2}, \; G_0 = \dfrac{I_s}{V_T}\exp\left(\dfrac{V_{DC}}{V_T}\right) = \dfrac{I_{DC}}{V_T} \rightarrow C_D \uparrow$ as $V_{DC} \uparrow$



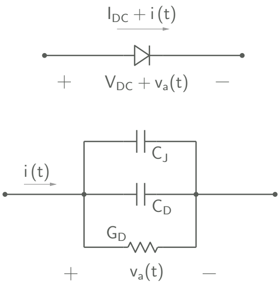
$I_{DC} + i(t)$
$+$ $V_{DC} + v_a(t)$ $-$
$i(t)$
C_J
C_D
G_D
$+$ $v_a(t)$ $-$
(a) f = 1 kHz
$(C_D + C_J)$
C_J
C_D
V_{DC} (V)
(b) f = 1 kHz
G_D
V_{DC} (V)
(c) $V_{DC} = 0.6$ V
G_D/G_{D0}
C_D/C_{D0}
f (Hz)
(d) $V_{DC} = 0.6$ V
G_D
$\omega(C_D + C_J)$
f (Hz)

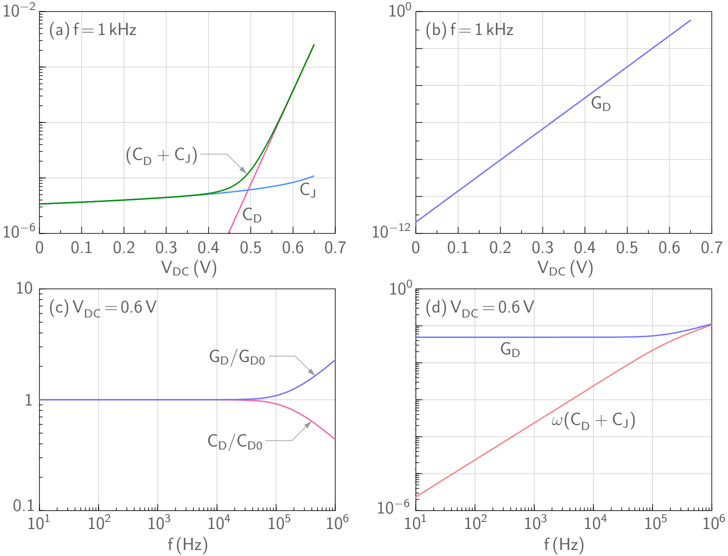


* C_J dominates at low V_{DC}, C_D dominates at high V_{DC}.



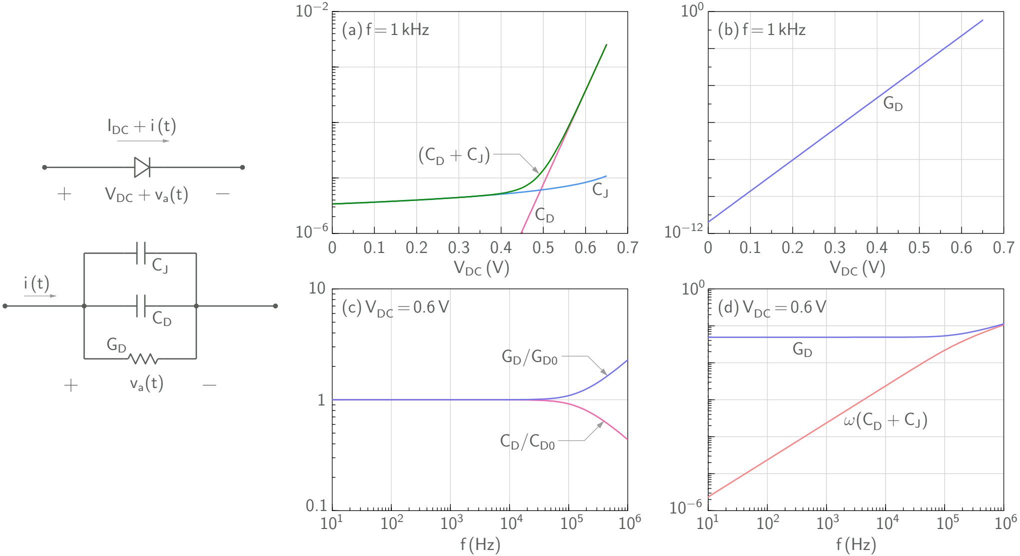
I_DC + i(t)
+ V_DC + v_a(t) −
C_J
i(t)
C_D
G_D
+ v_a(t) −
(a) f = 1 kHz
(C_D + C_J)
C_J
C_D
V_DC (V)
(b) f = 1 kHz
G_D
V_DC (V)
(c) V_DC = 0.6 V
G_D/G_D0
C_D/C_D0
f (Hz)
(d) V_DC = 0.6 V
G_D
ω(C_D + C_J)
f (Hz)



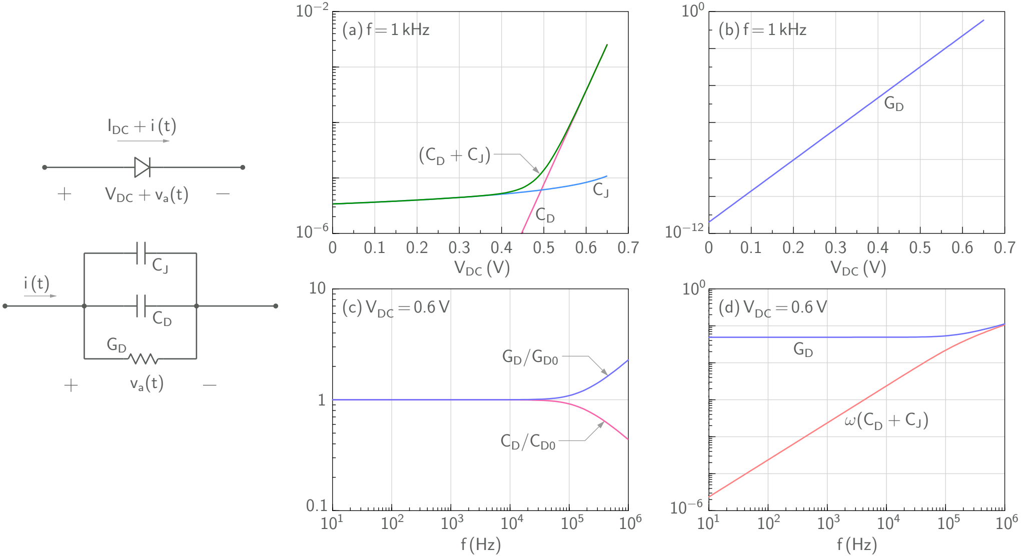


* $G_D = \dfrac{G_0}{\sqrt{2}} \left(\sqrt{1 + (\omega\tau_p)^2} + 1 \right)^{1/2}, \quad C_D = \dfrac{G_0}{\omega\sqrt{2}} \left(\sqrt{1 + (\omega\tau_p)^2} - 1 \right)^{1/2}, \quad G_0 = \dfrac{I_s}{V_T} \exp\left(\dfrac{V_{DC}}{V_T} \right).$

$\rightarrow$ Both G_D and C_D increase exponentially with V_{DC}.

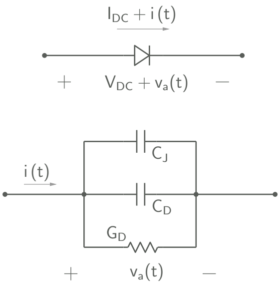


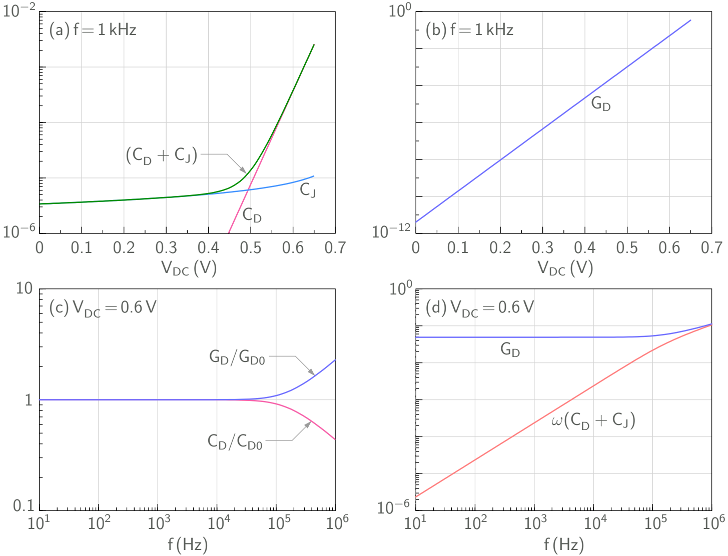
IDC + i (t)
+ VDC + va(t) −
i (t)
CJ
CD
GD
+ va(t) −
(a) f = 1 kHz
(CD + CJ)
CJ
CD
VDC (V)
(b) f = 1 kHz
GD
VDC (V)
(c) VDC = 0.6 V
GD/GD0
CD/CD0
f (Hz)
(d) VDC = 0.6 V
GD
ω(CD + CJ)
f (Hz)

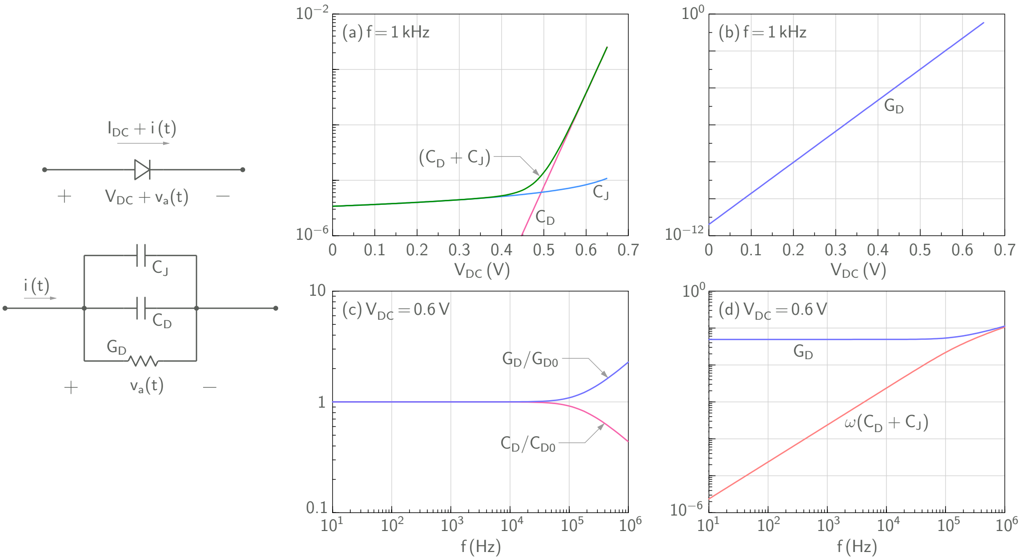


$$* \quad G_D = \frac{G_0}{\sqrt{2}} \left(\sqrt{1 + (\omega\tau_p)^2} + 1 \right)^{1/2}, \quad C_D = \frac{G_0}{\omega\sqrt{2}} \left(\sqrt{1 + (\omega\tau_p)^2} - 1 \right)^{1/2}, \quad G_0 = \frac{I_s}{V_T} \exp\left(\frac{V_{DC}}{V_T}\right).$$

For $\omega\tau_p \ll 1$, $G_D \rightarrow G_0$, $C_D \rightarrow \dfrac{G_0\tau_p}{2}$ $\left(\because \sqrt{1+(\omega\tau_p)^2} \approx 1 + \dfrac{1}{2}(\omega\tau_p)^2\right)$.

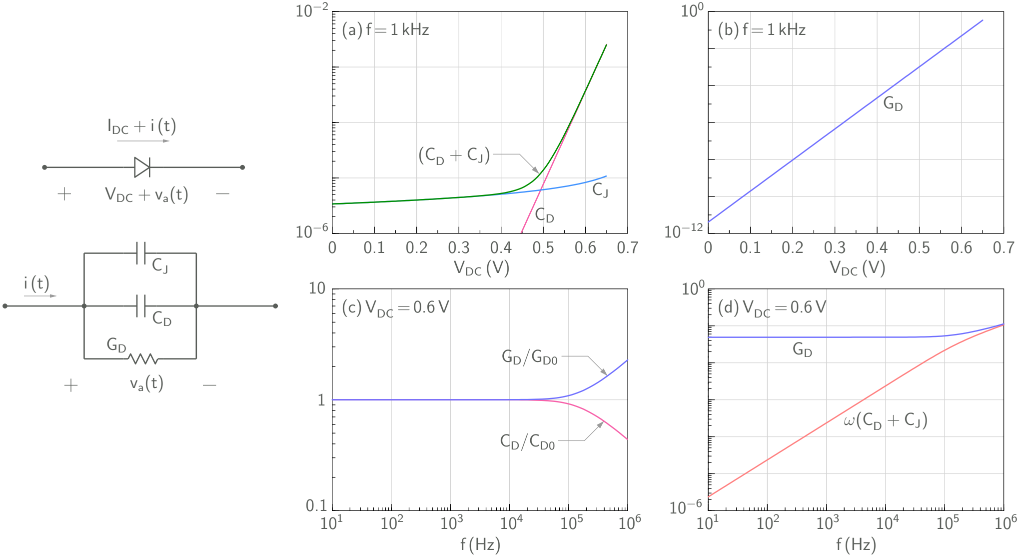

$I_{DC} + i\,(t)$
$+$
$V_{DC} + v_a(t)$
$-$
C_J
$i\,(t)$
C_D
G_D
$+$
$v_a(t)$
$-$


(a) f = 1 kHz
10^{-2}
10^{-6}
$(C_D + C_J)$
C_J
C_D
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
V_{DC} (V)
(b) f = 1 kHz
10^{0}
10^{-12}
G_D
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
V_{DC} (V)
(c) $V_{DC} = 0.6$ V
10
1
0.1
G_D/G_{D0}
C_D/C_{D0}
10^1 10^2 10^3 10^4 10^5 10^6
f (Hz)
(d) $V_{DC} = 0.6$ V
10^{0}
10^{-6}
G_D
$\omega(C_D + C_J)$
10^1 10^2 10^3 10^4 10^5 10^6
f (Hz)

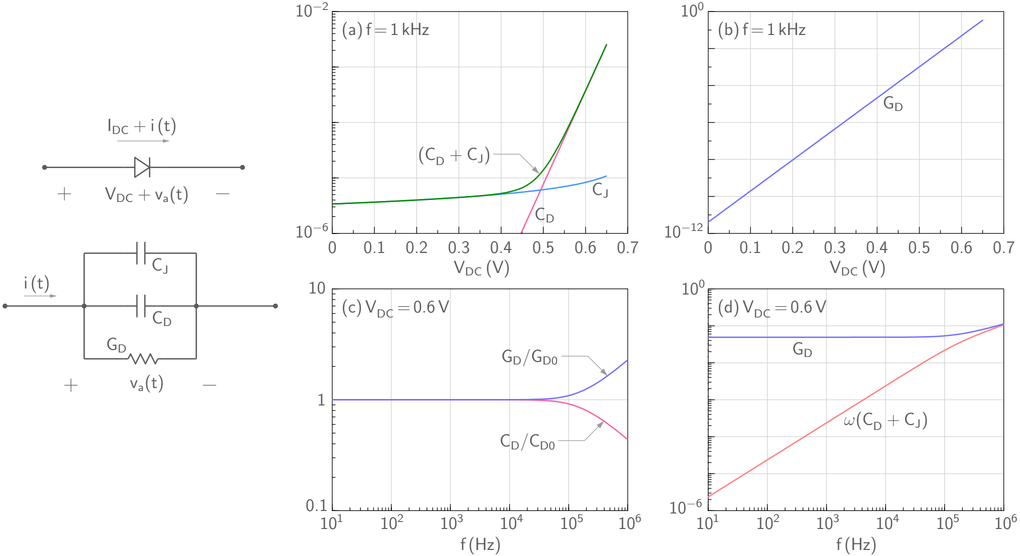


$$* \quad G_D = \frac{G_0}{\sqrt{2}}\left(\sqrt{1+(\omega\tau_p)^2}+1\right)^{1/2}, \quad C_D = \frac{G_0}{\omega\sqrt{2}}\left(\sqrt{1+(\omega\tau_p)^2}-1\right)^{1/2}, \quad G_0 = \frac{I_s}{V_T}\exp\left(\frac{V_{DC}}{V_T}\right).$$

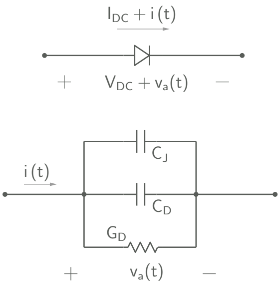
At high frequencies, $G_D \propto \sqrt{\omega}$, $C_D \propto \dfrac{1}{\sqrt{\omega}}$.

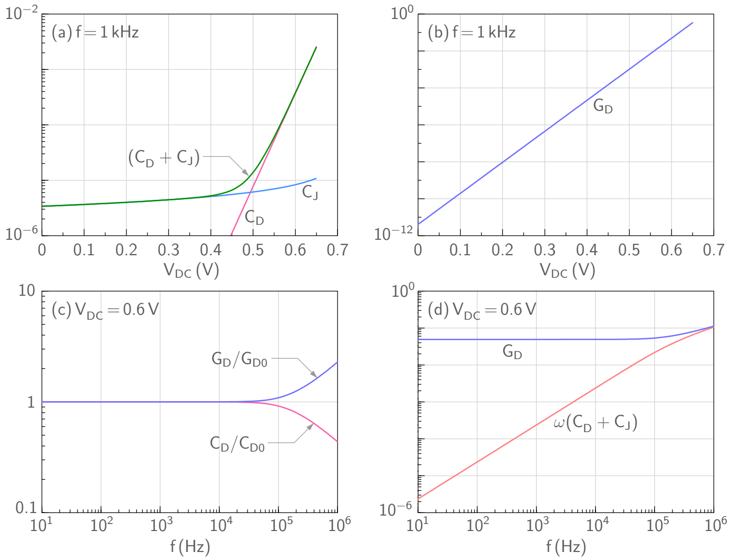


$I_{DC} + i(t)$
$+$ $V_{DC} + v_a(t)$ $-$
$i(t)$
C_J
C_D
G_D
$+$ $v_a(t)$ $-$
(a) f = 1 kHz
$(C_D + C_J)$
C_J
C_D
V_{DC} (V)
(b) f = 1 kHz
G_D
V_{DC} (V)
(c) $V_{DC} = 0.6$ V
G_D/G_{D0}
C_D/C_{D0}
f (Hz)
(d) $V_{DC} = 0.6$ V
G_D
$\omega(C_D + C_J)$
f (Hz)



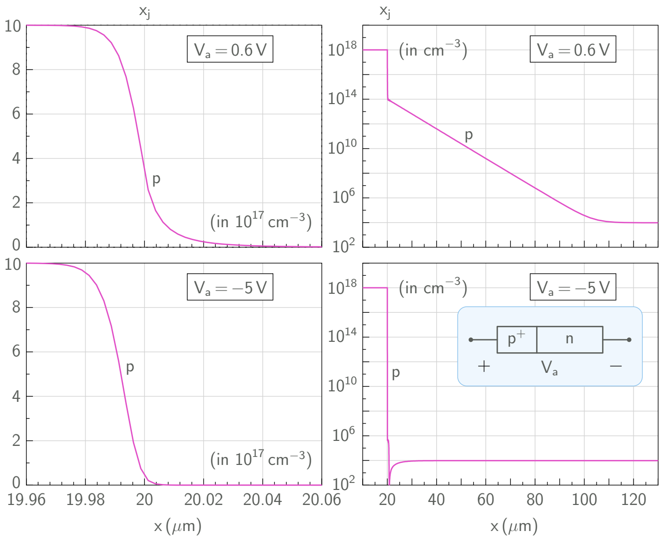
* For $V_D = 0.6\,\text{V}$, the conductance G_D dominates for this device.





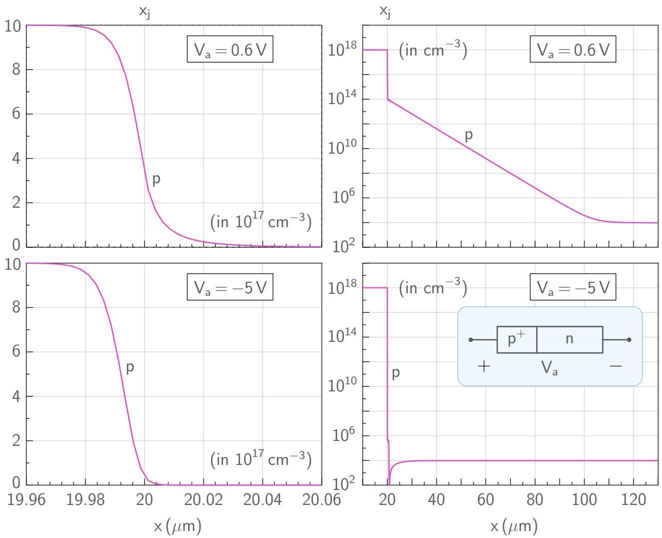
* For $V_D = 0.6\,\text{V}$, the conductance G_D dominates for this device.

Home work: Show that (a) For $\omega\tau_p \ll 1$, $\dfrac{G_D}{\omega C_D} = \dfrac{2}{\omega\tau_p}$, (b) For $\omega\tau_p \gg 1$, $\dfrac{G_D}{\omega C_D} \rightarrow 1$.

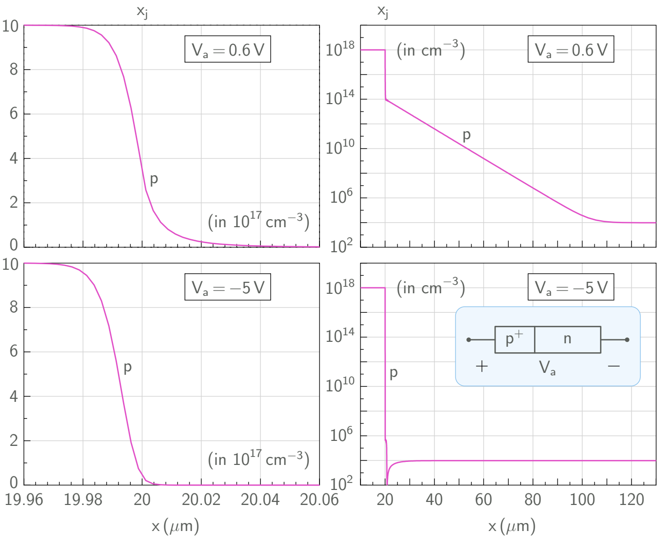


x_j
10
8
6
4
2
0
V_a = 0.6 V
p
(in 10^17 cm^−3)
x_j
10^18
10^14
10^10
10^6
10^2
(in cm^−3)
V_a = 0.6 V
p
V_a = −5 V
p
(in 10^17 cm^−3)
(in cm^−3)
V_a = −5 V
p
p+
n
+
V_a
−
19.96
19.98
20
20.02
20.04
20.06
x (μm)
20
40
60
80
100
120
x (μm)

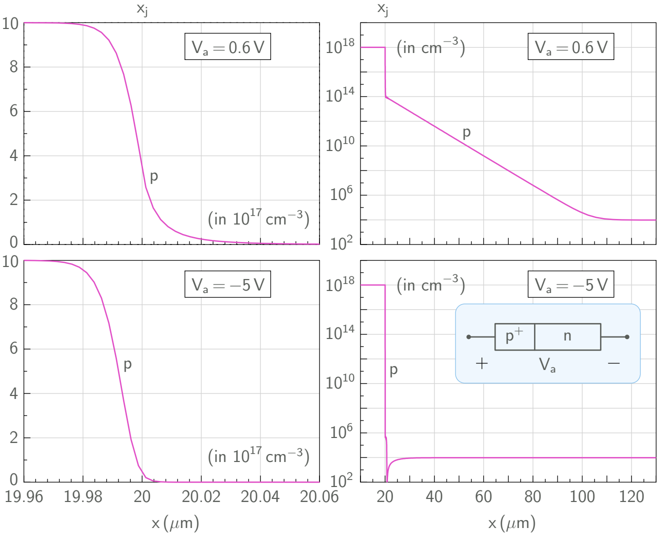
* There are major differences between forward and reverse bias:



* There are major differences between forward and reverse bias:
  - The depletion region is wider in reverse bias.

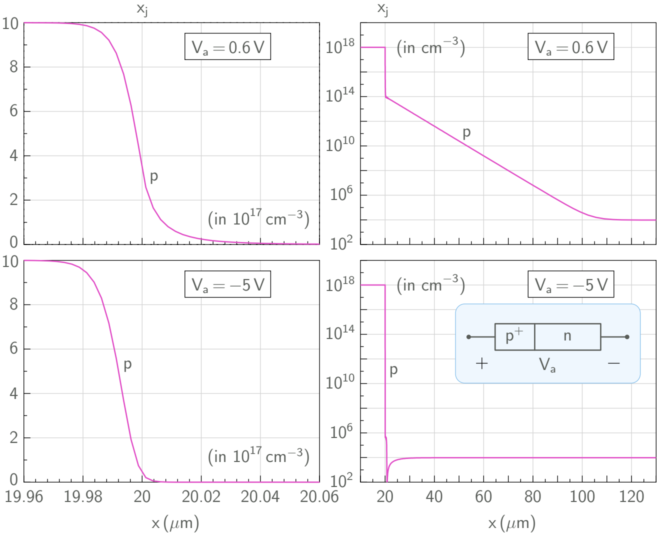


* There are major differences between forward and reverse bias:
  - The depletion region is wider in reverse bias.
  - The hole density at x_n is much larger in forward bias.

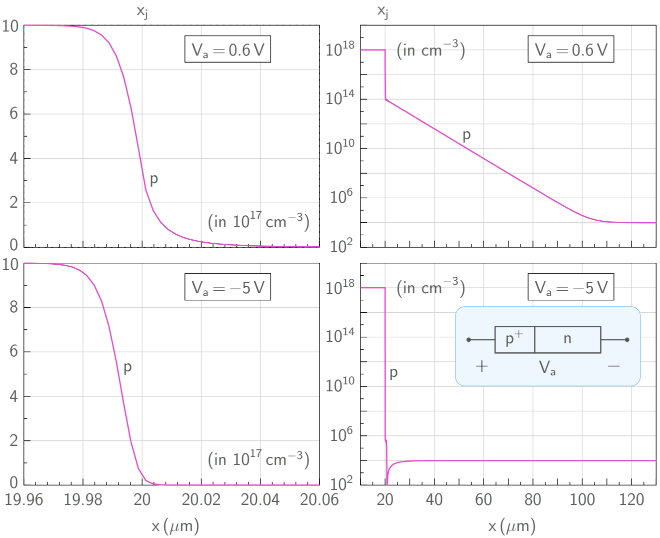


pn junction diode: large-signal behaviour

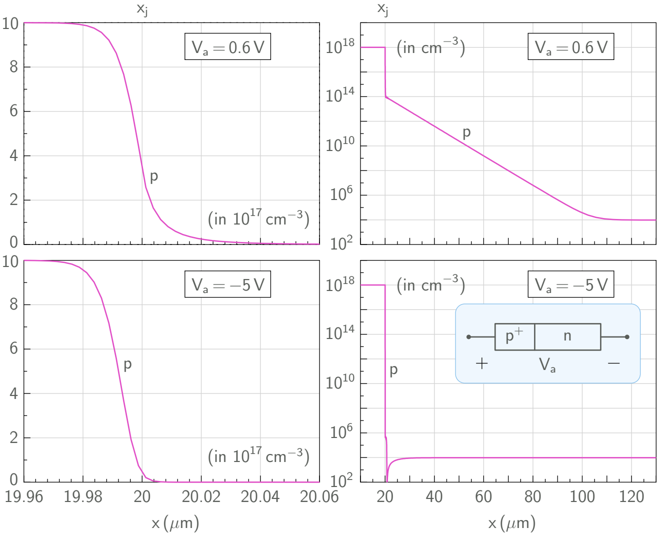
* There are major differences between forward and reverse bias:
  - The depletion region is wider in reverse bias.
  - The hole density at x_n is much larger in forward bias.
  - The total minority carrier charge $q\int_{x_n}^{\infty} p(x)\,dx$ is also much larger in forward bias.



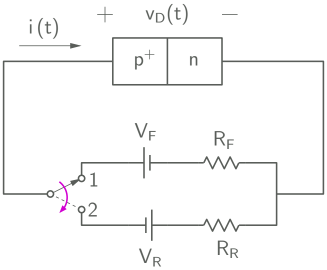
* There are major differences between forward and reverse bias:
  - The depletion region is wider in reverse bias.
  - The hole density at x_n is much larger in forward bias.
  - The total minority carrier charge $q\int_{x_n}^{\infty} p(x)\,dx$ is also much larger in forward bias.
* For the diode to change from f.b. to r.b. (or *vice versa*), large changes within the device are required. These transients are called "large-signal" transients.



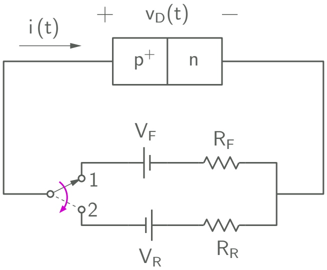
* There are major differences between forward and reverse bias:
  - The depletion region is wider in reverse bias.
  - The hole density at x_n is much larger in forward bias.
  - The total minority carrier charge $q\int_{x_n}^{\infty} p(x)\,dx$ is also much larger in forward bias.
* For the diode to change from f.b. to r.b. (or *vice versa*), large changes within the device are required. These transients are called "large-signal" transients.
* We will consider a representative circuit and look at the turn-on and turn-off transients.

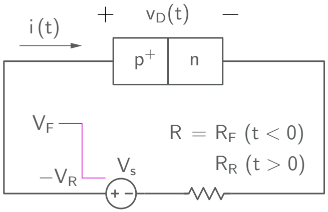


pn junction diode: large-signal behaviour



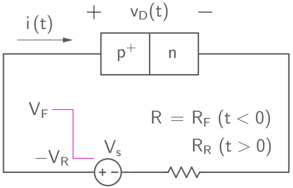
* Turn-off: switch changes from position 1 to 2.
* Turn-on: switch changes from position 2 to 1.



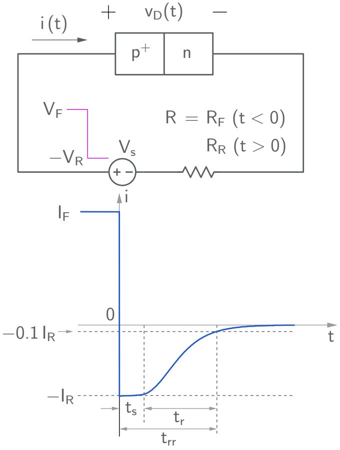


* Turn-off: switch changes from position 1 to 2.
* Turn-on: switch changes from position 2 to 1.

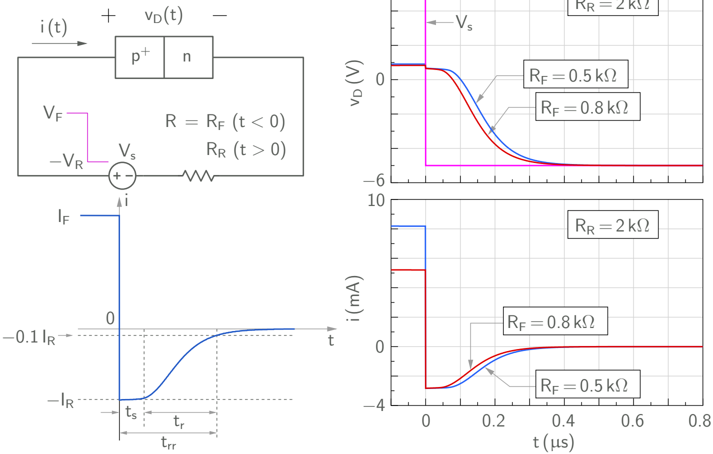
Turn-off transient



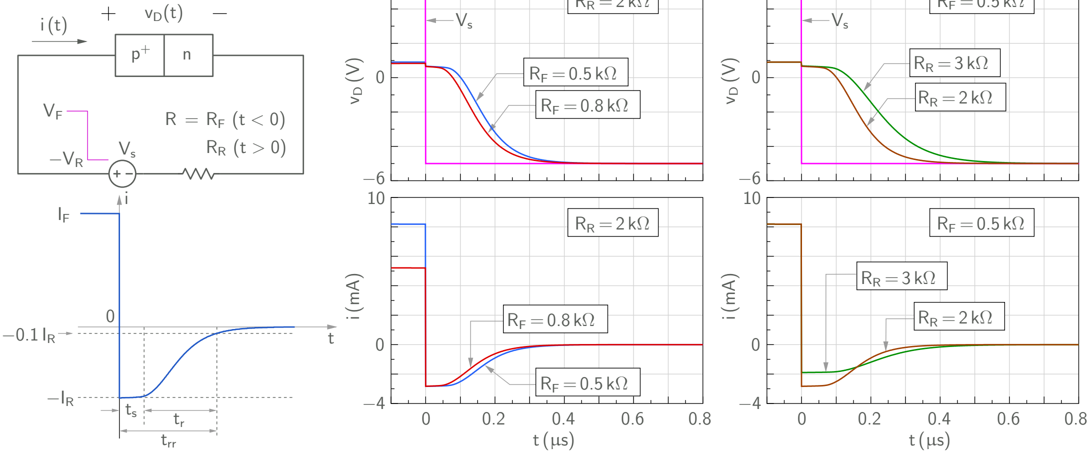
Turn-off transient



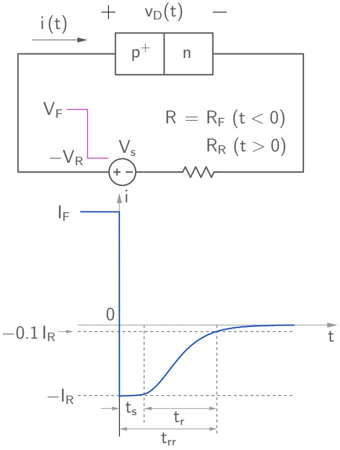
Turn-off transient

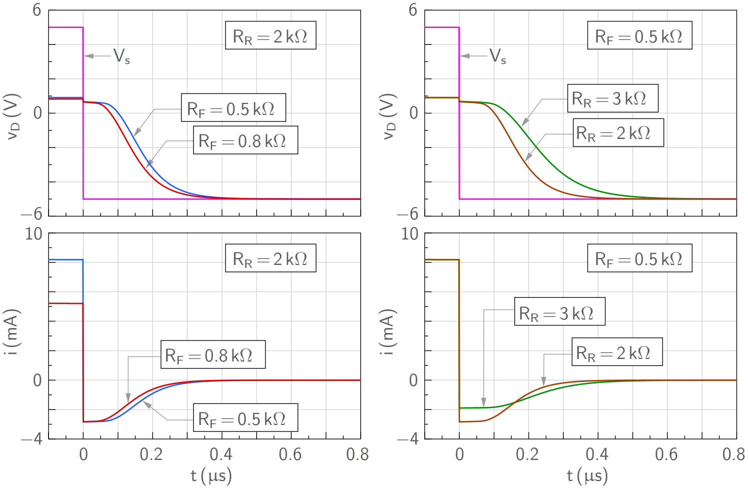


Turn-off transient



Turn-off transient





$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$

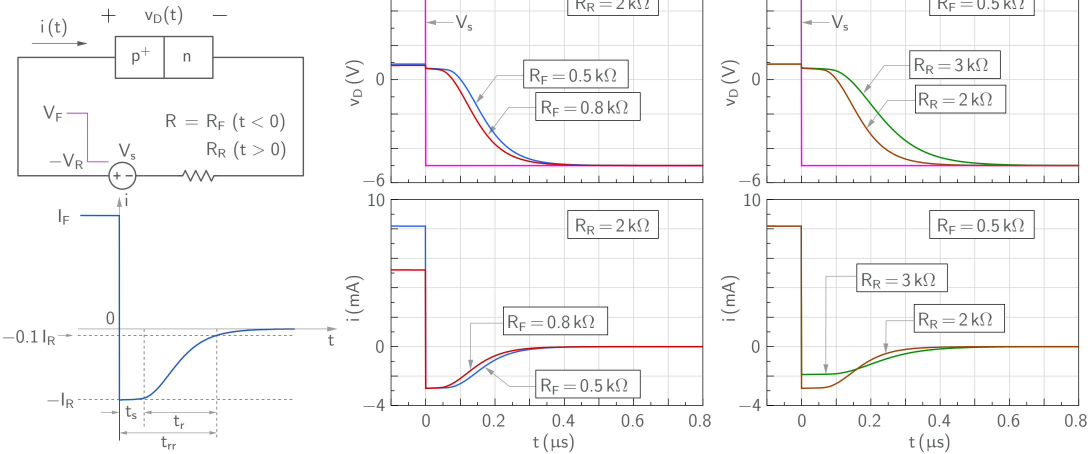
$\mu_p = 500\ \text{cm}^2/\text{V-s}$

$\tau_p = 0.1\ \mu\text{s}$

$A = 0.001\ \text{cm}^2$

$T = 300\ \text{K}$

Turn-off transient



* There is an interval t_s, known as the "storage time" or "storage delay time" during which the diode current is approximately constant ($-I_R$), and the diode voltage continues to be positive.

$N_d = 5 \times 10^{16}$ cm^{-3}
$\mu_p = 500$ cm^2/V-s
$\tau_p = 0.1\ \mu$s
$A = 0.001$ cm^2
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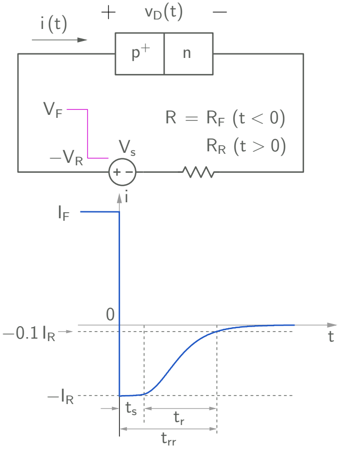
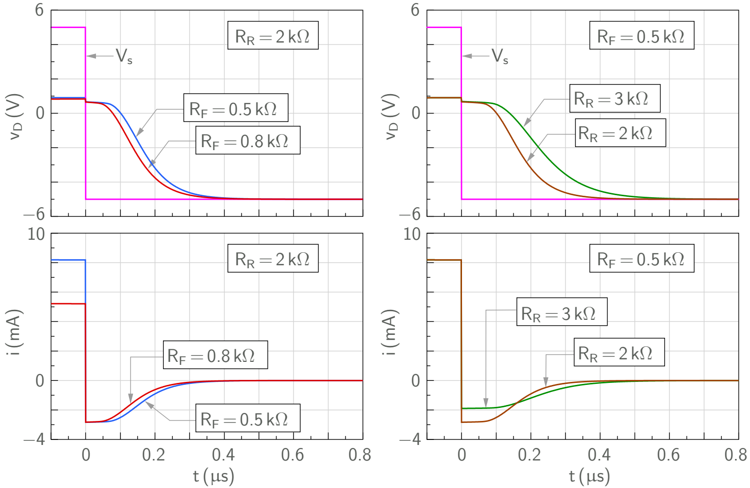
Turn-off transient



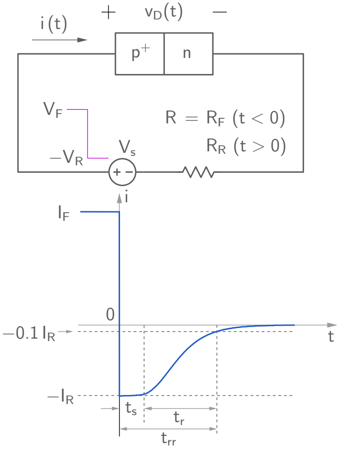


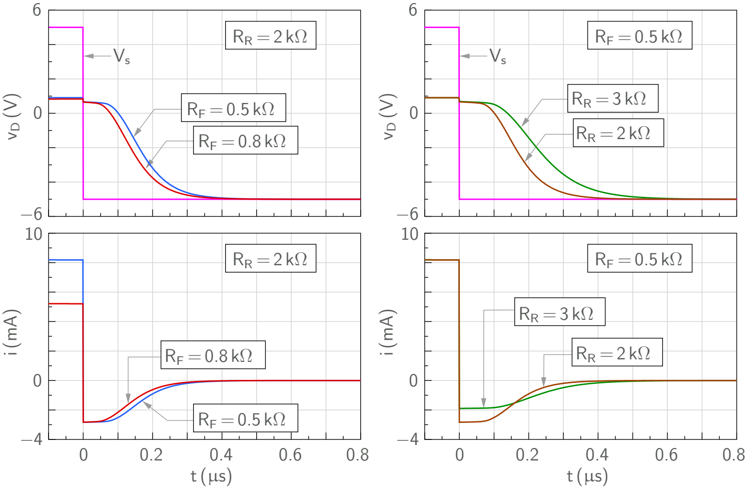
* There is an interval t_s, known as the "storage time" or "storage delay time" during which the diode current is approximately constant ($-I_R$), and the diode voltage continues to be positive.
* After the t_s phase, the diode current and voltage start approaching their final values. The "reverse recovery time" t_r is defined as the time required for the current to decrease (in magnitude) from I_R to 0.1 I_R.

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu s$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$

M. B. Patil, IIT Bombay

Turn-off transient





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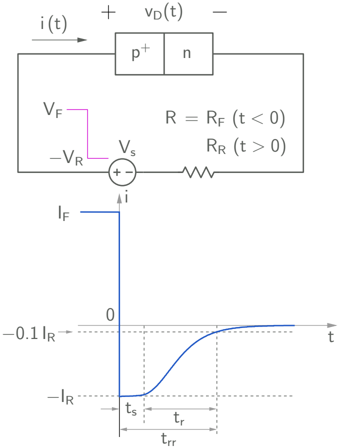
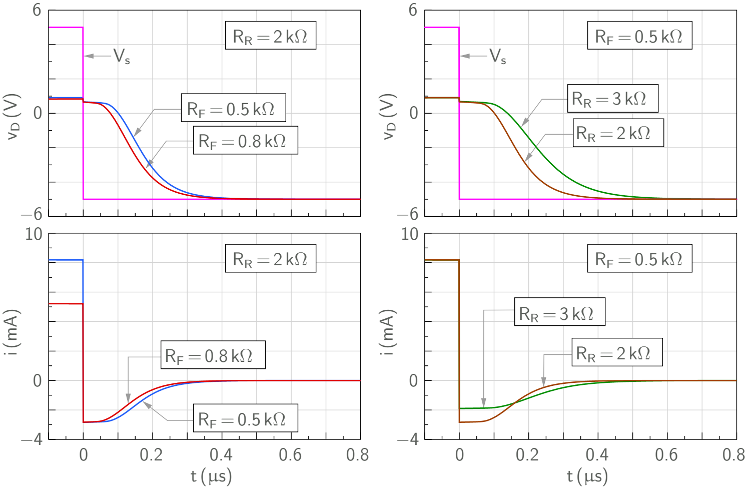
Turn-off transient





* The total time $t_{rr} = t_s + t_r$ that the diode takes to "recover" to the reverse-bias steady state condition is called the "reverse recovery" time.

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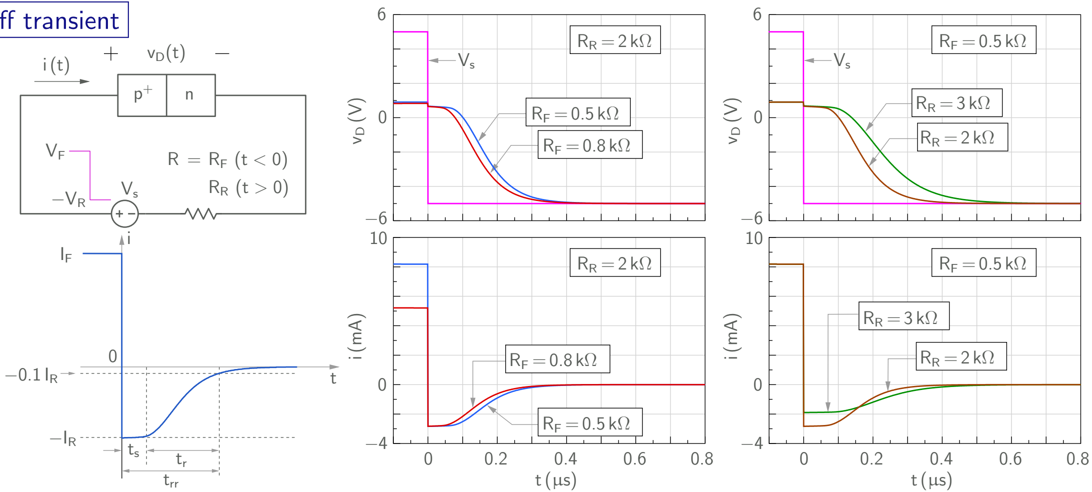
Turn-off transient



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* If R_F is reduced, t_s increases.

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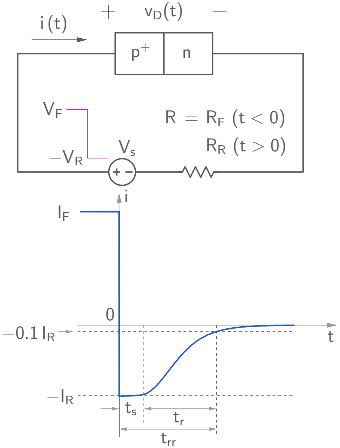
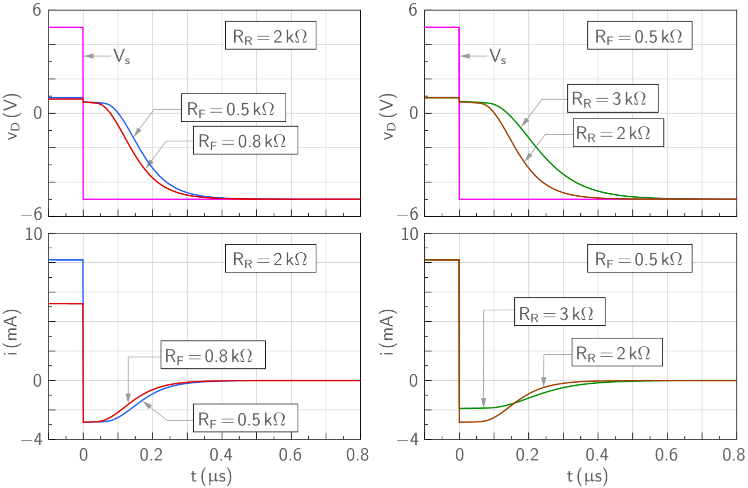
Turn-off transient





* The total time $t_{rr} = t_s + t_r$ that the diode takes to "recover" to the reverse-bias steady state condition is called the "reverse recovery" time.
* If R_F is reduced, t_s increases.
* If R_R is reduced, t_s decreases.

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
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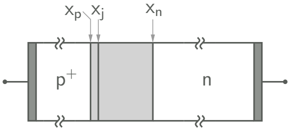
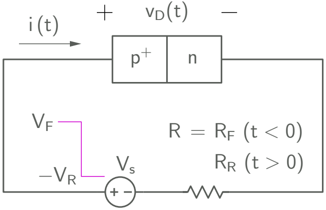
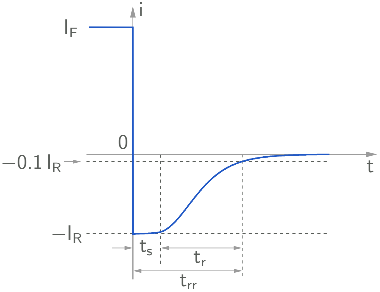
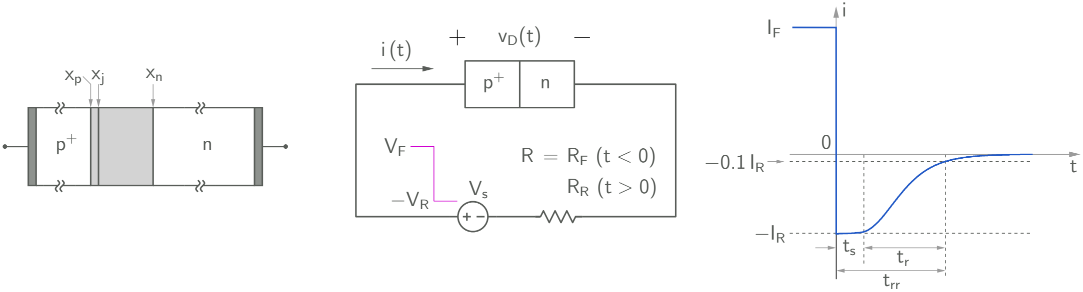
Turn-off transient: charge control approach



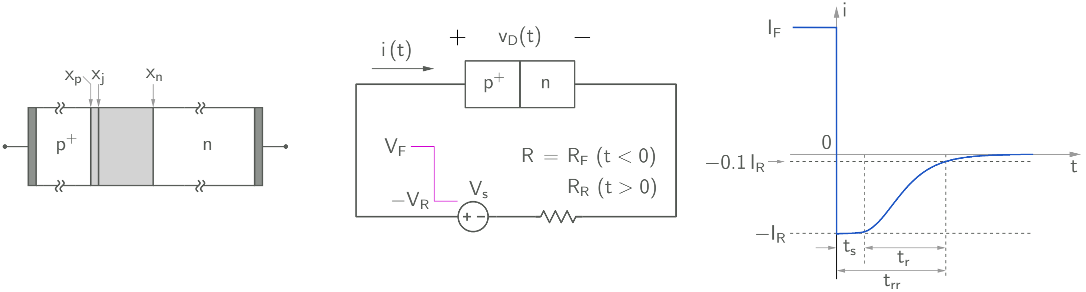






Continuity equation for holes in the neutral n region ($x > x_n$):

$$\frac{\partial p}{dt} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - (R - G), \quad (R - G) = \frac{p - p_{n0}}{\tau_p}.$$



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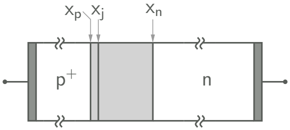
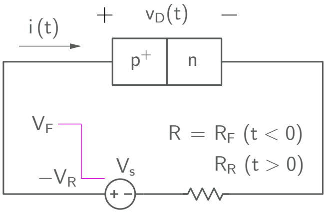
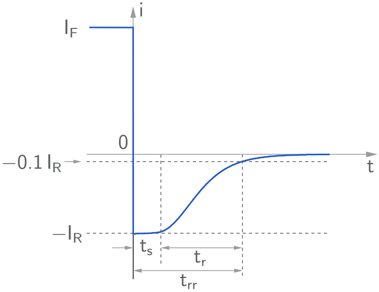
In terms of the excess hole density, $\Delta p(x) = p(x) - p_{n0}$,

$$q\,\frac{\partial \Delta p}{\partial t} = -\frac{\partial J_p}{\partial x} - q\,\frac{\Delta p}{\tau_p}$$

Turn-off transient: charge control approach







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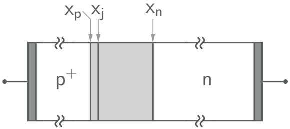
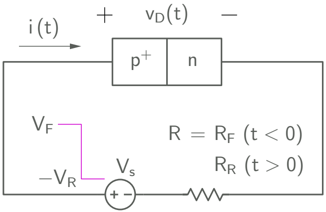
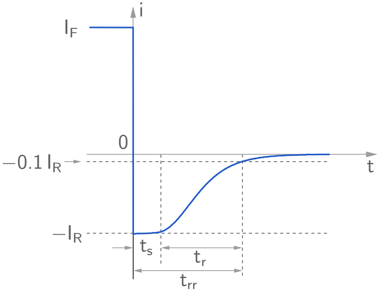
$$q\,\frac{\partial \Delta p}{\partial t} = -\frac{\partial J_p}{\partial x} - q\,\frac{\Delta p}{\tau_p} \;\rightarrow\; qA\,\frac{\partial}{\partial t}\int_{x_n}^{\infty} \Delta p\,dx = -A\int_{x_n}^{\infty}\frac{\partial J_p}{\partial x}\,dx - \frac{qA}{\tau_p}\int_{x_n}^{\infty}\Delta p\,dx.$$

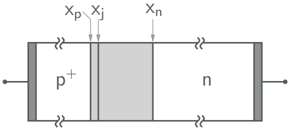
Turn-off transient: charge control approach

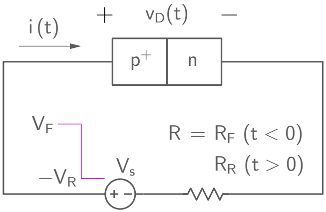


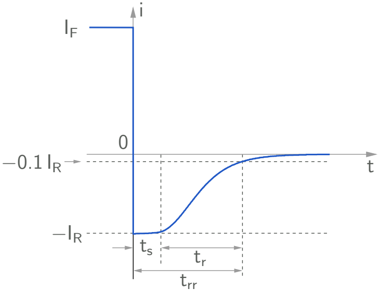




$$qA\,\frac{\partial}{\partial t}\int_{x_n}^{\infty}\Delta p\,dx = -A\int_{x_n}^{\infty}\frac{\partial J_p}{\partial x}\,dx - \frac{qA}{\tau_p}\int_{x_n}^{\infty}\Delta p\,dx.$$







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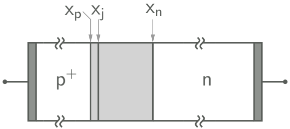
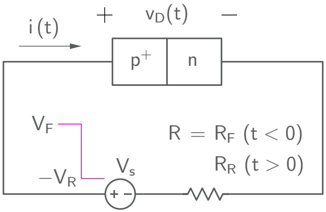
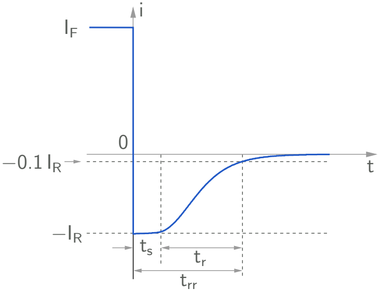
The first term on the right is $A\,[J_p(x_n) - J_p(\infty)] = A\,J_p^{\rm diff}(x_n) = I_p^{\rm diff}(x_n)$.

Turn-off transient: charge control approach







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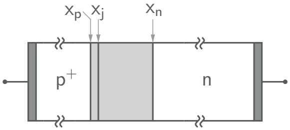
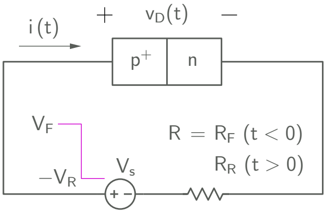
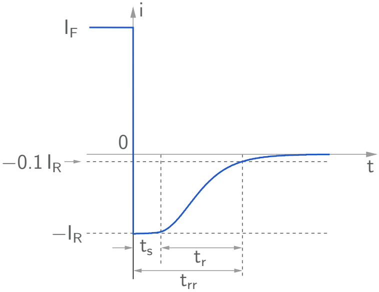
The quantity $qA\displaystyle\int_{x_n}^{\infty}\Delta p dx$ is the "excess hole charge" Q_p in the neutral n region.







$$qA \frac{\partial}{\partial t} \int_{x_n}^{\infty} \Delta p \, dx = -A \int_{x_n}^{\infty} \frac{\partial J_p}{\partial x} \, dx - \frac{qA}{\tau_p} \int_{x_n}^{\infty} \Delta p \, dx.$$

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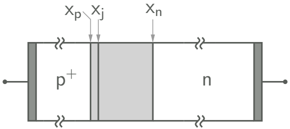
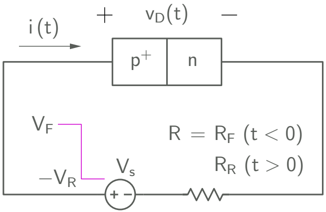
The quantity $qA \displaystyle\int_{x_n}^{\infty} \Delta p dx$ is the "excess hole charge" Q_p in the neutral n region.

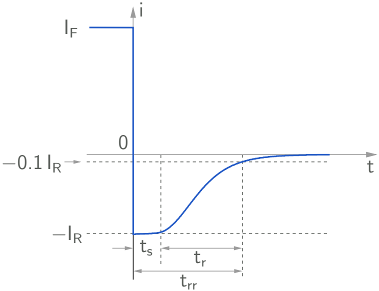
We can rewrite the continuity equation as

$$\frac{\partial Q_p}{\partial t} = I_p^{\text{diff}}(x_n) - \frac{Q_p}{\tau_p}.$$

Turn-off transient: charge control approach







$$qA\,\frac{\partial}{\partial t}\int_{x_n}^{\infty}\Delta p\,dx = -A\int_{x_n}^{\infty}\frac{\partial J_p}{\partial x}\,dx - \frac{qA}{\tau_p}\int_{x_n}^{\infty}\Delta p\,dx.$$

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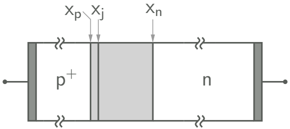
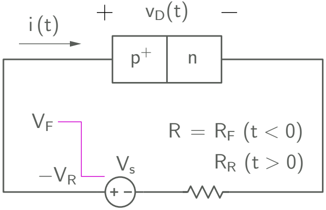
We can rewrite the continuity equation as

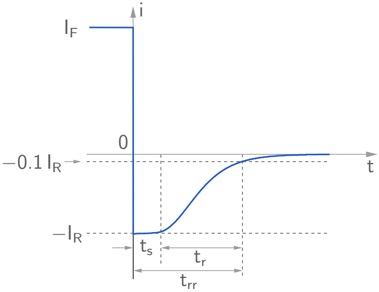
$$\frac{\partial Q_p}{\partial t} = I_p^{\text{diff}}(x_n) - \frac{Q_p}{\tau_p}.$$

We can think of this equation as the continuity equation for the total excess hole charge in the neutral n region.

Turn-off transient: charge control approach

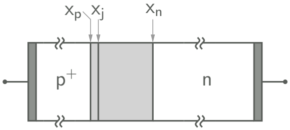


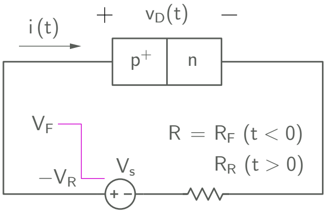


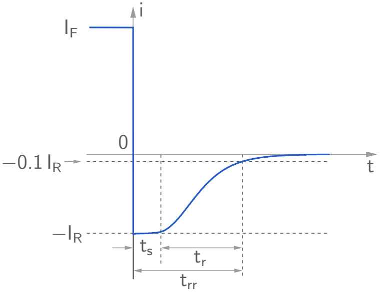


$$\frac{\partial Q_p}{\partial t} = I_p^{\text{diff}}(x_n) - \frac{Q_p}{\tau_p}.$$

Turn-off transient: charge control approach



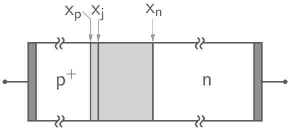


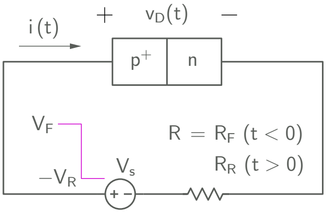


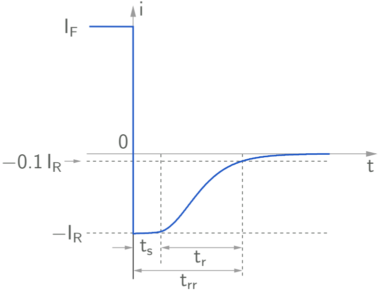
$$\frac{\partial Q_p}{\partial t} = I_p^{\text{diff}}(x_n) - \frac{Q_p}{\tau_p}.$$

For a p^+n junction, $J \approx J_p^{\text{diff}}(x_n) \rightarrow I \approx I_p^{\text{diff}}(x_n)$.

Turn-off transient: charge control approach







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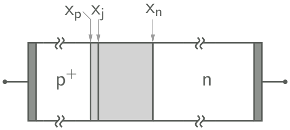
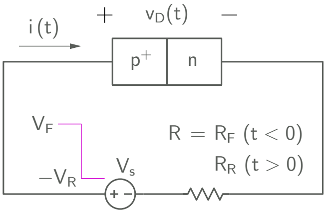
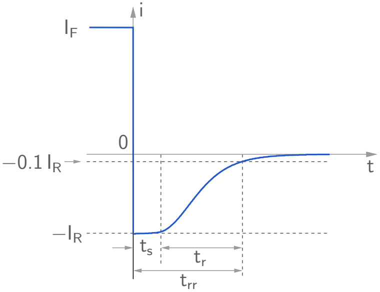
Turn-off transient: charge control approach







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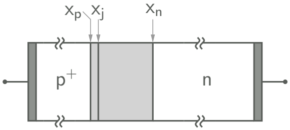
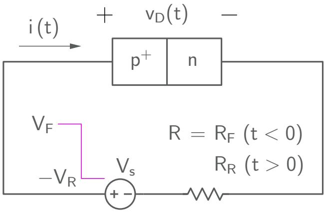
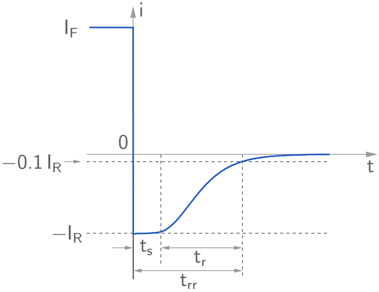
$$\rightarrow \frac{\partial Q_p}{\partial t} = I - \frac{Q_p}{\tau_p}.$$

At $t = 0^-$, we have a steady-state situation, with $I \equiv I_F = \dfrac{V_F - V_{\text{on}}}{R_F}$, where V_{on} is the voltage drop across the diode when conducting. ($V_{\text{on}} \approx 0.7\,\text{V}$ for a typical low-power silicon diode.)







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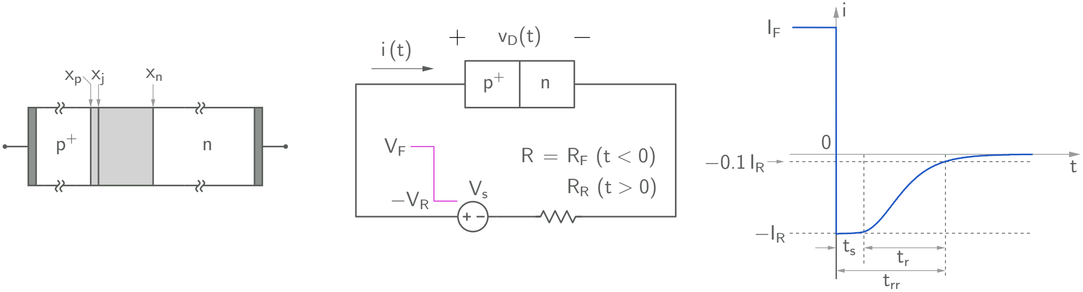
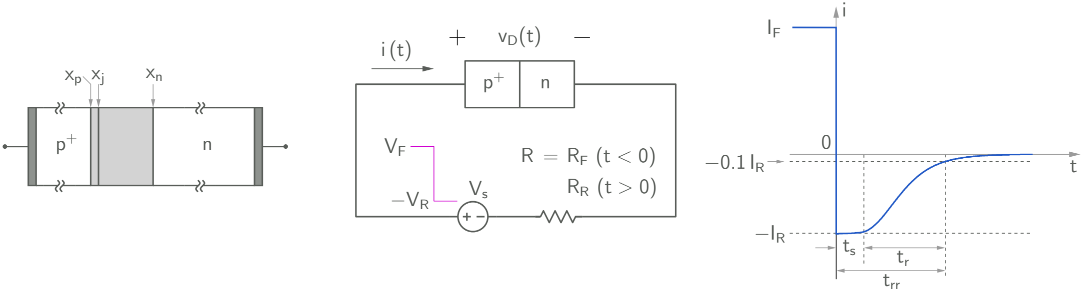
$\rightarrow 0 = I_F - \dfrac{Q_p(0^-)}{\tau_p} \;\rightarrow\; Q_p(0^-) = I_F\tau_p$ is the excess hole charge in the neutral n region at $t = 0^-$.



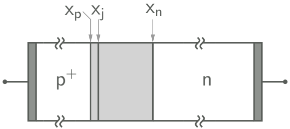
After the transient is over ($t > t_{rr}$), we have a steady-state situation again, with $I \approx 0\,\mathrm{A}$, $V \approx -V_R$.

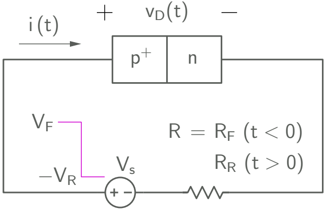
Turn-off transient: charge control approach

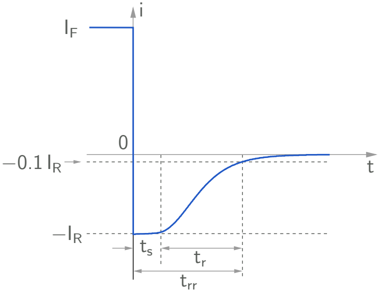


After the transient is over ($t > t_{rr}$), we have a steady-state situation again, with $I \approx 0\,\text{A}$, $V \approx -V_R$.

$$\frac{\partial Q_p}{\partial t} = I - \frac{Q_p}{\tau_p} \;\rightarrow\; 0 = 0 - \frac{Q_p(\infty)}{\tau_p} \;\rightarrow\; Q_p(\infty) = 0.$$







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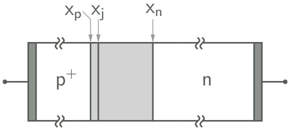
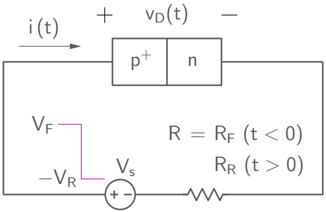
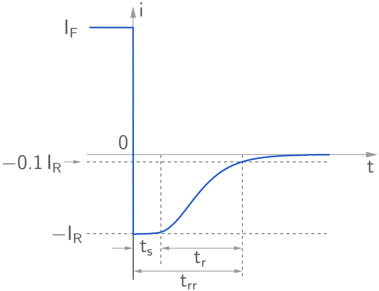
Starting from $Q_p(0^-) = I_F \tau_p$ at $t = 0^-$, the excess hole charge must become nearly zero at $t = t_{rr}$.

Turn-off transient: charge control approach







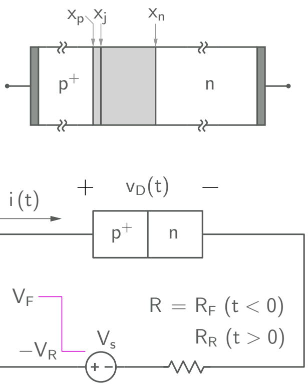
After the transient is over ($t > t_{rr}$), we have a steady-state situation again, with $I \approx 0\,\text{A}$, $V \approx -V_R$.

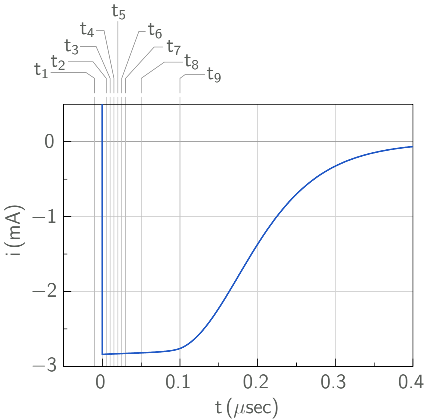
$$\frac{\partial Q_p}{\partial t} = I - \frac{Q_p}{\tau_p} \rightarrow 0 = 0 - \frac{Q_p(\infty)}{\tau_p} \rightarrow Q_p(\infty) = 0.$$

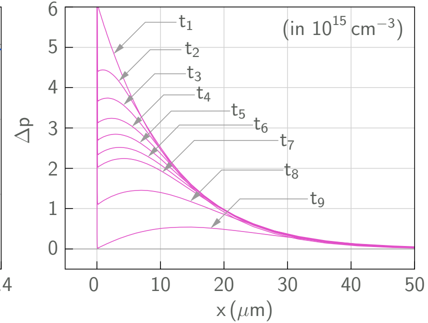
Starting from $Q_p(0^-) = I_F\tau_p$ at $t = 0^-$, the excess hole charge must become nearly zero at $t = t_{rr}$.

How does this happen?

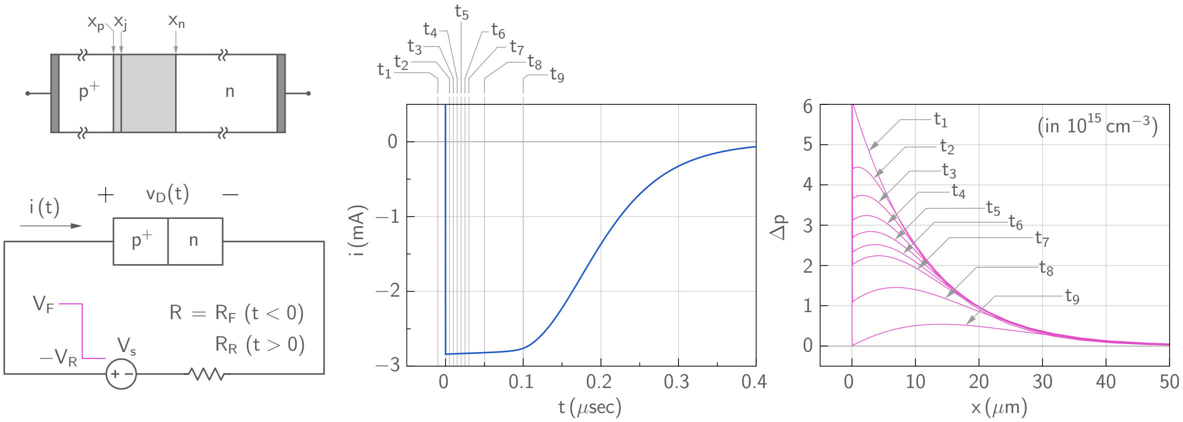
Turn-off transient: charge control approach





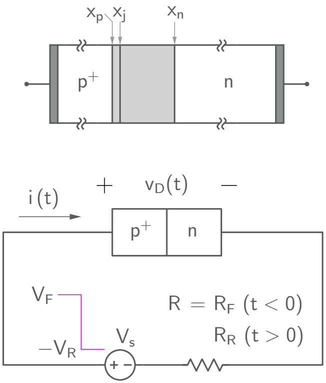


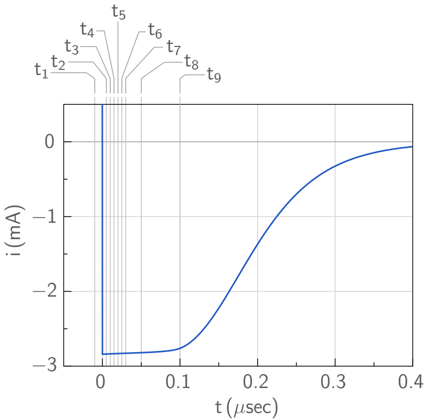
Turn-off transient: charge control approach

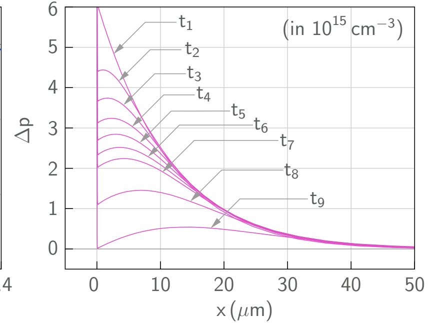


∗ The storage time t_s ($\approx t_9$ in the figure) can be estimated by observing that $I \approx -I_R$ for $0 < t < t_s$.

Turn-off transient: charge control approach



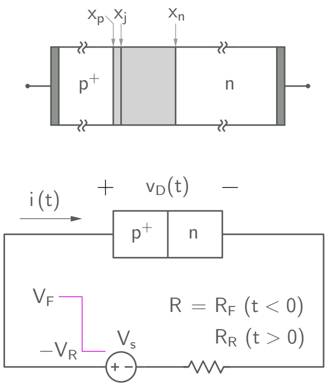


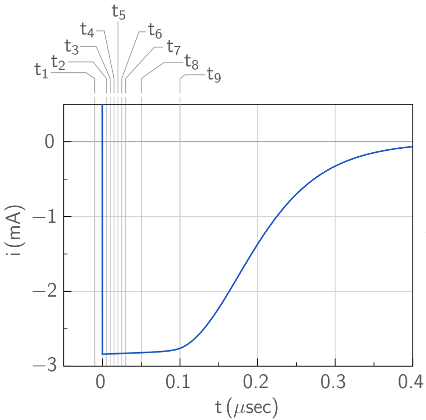


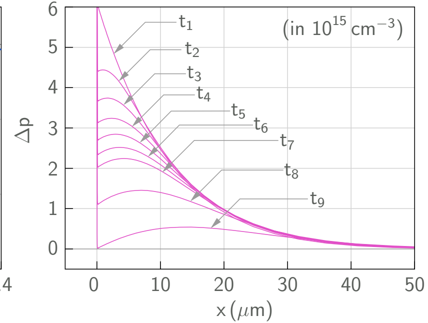
* The storage time t_s ($\approx t_9$ in the figure) can be estimated by observing that $I \approx -I_R$ for $0 < t < t_s$.

* Note that, in the interval $0 < t < t_s$, the slope $\dfrac{dp}{dx}(x_n)$ is positive, corresponding to a negative current.

Turn-off transient: charge control approach

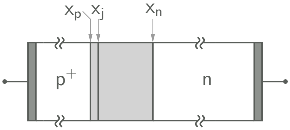


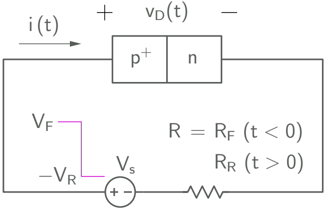


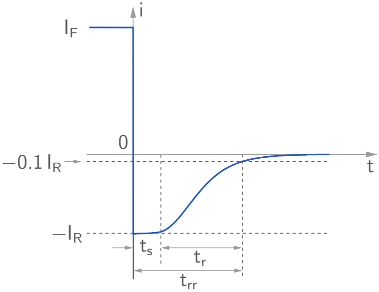


* The storage time t_s ($\approx t_9$ in the figure) can be estimated by observing that $I \approx -I_R$ for $0 < t < t_s$.
* Note that, in the interval $0 < t < t_s$, the slope $\dfrac{dp}{dx}(x_n)$ is positive, corresponding to a negative current.
* By $t = t_s$, the hole charge Q_p in the neutral n region has reduced substantially. As an approximation, we may use $Q_p(t_s) = 0$.

Turn-off transient: charge control approach

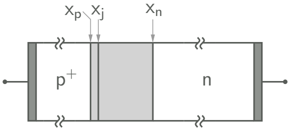


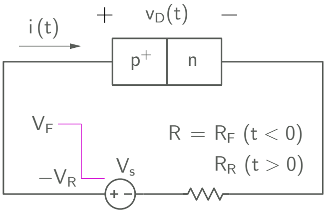


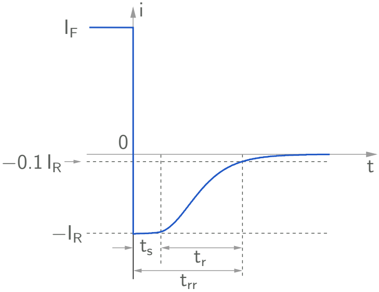


$0 < t < t_s$: $\dfrac{dQ_p}{dt} = -I_R - \dfrac{Q_p}{\tau_p}$, with $Q_p(0^+) = I_F\tau_p$ and $Q_p(t_s) \approx 0$.

Turn-off transient: charge control approach



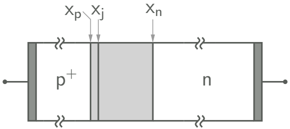


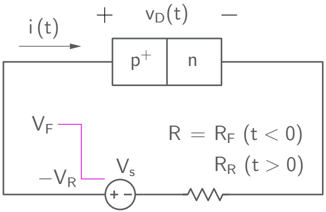


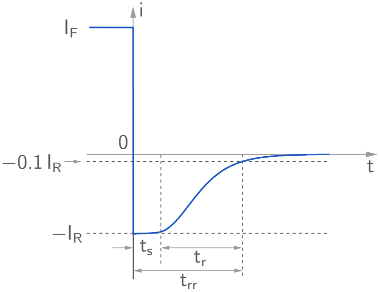
$$0 < t < t_s: \quad \frac{dQ_p}{dt} = -I_R - \frac{Q_p}{\tau_p}, \quad \text{with } Q_p(0^+) = I_F\tau_p \text{ and } \quad Q_p(t_s) \approx 0.$$

$$\rightarrow Q_p(t) = \tau_p\,(I_F + I_R)\,e^{-t/\tau_p} - I_R\tau_p, \quad 0 < t < t_s.$$

Turn-off transient: charge control approach







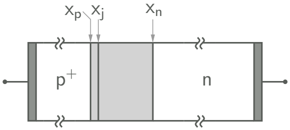
$0 < t < t_s$: $\dfrac{dQ_p}{dt} = -I_R - \dfrac{Q_p}{\tau_p}$, with $Q_p(0^+) = I_F\tau_p$ and $Q_p(t_s) \approx 0$.

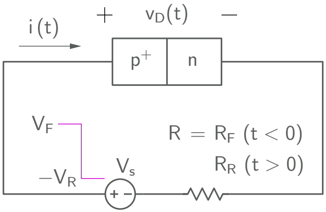
$\rightarrow Q_p(t) = \tau_p\,(I_F + I_R)\,e^{-t/\tau_p} - I_R\tau_p, \quad 0 < t < t_s.$

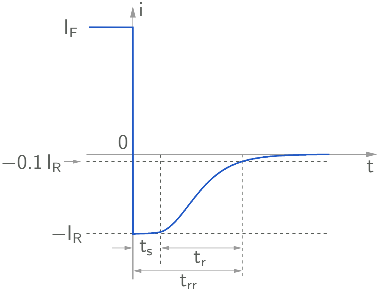
We can now use $Q_p(t_s) \approx 0$ to estimate t_s as

$$t_s = \tau_p \log\left(1 + \frac{I_F}{I_R}\right).$$

Turn-off transient: charge control approach







$0 < t < t_s$: $\dfrac{dQ_p}{dt} = -I_R - \dfrac{Q_p}{\tau_p}$, with $Q_p(0^+) = I_F \tau_p$ and $Q_p(t_s) \approx 0$.

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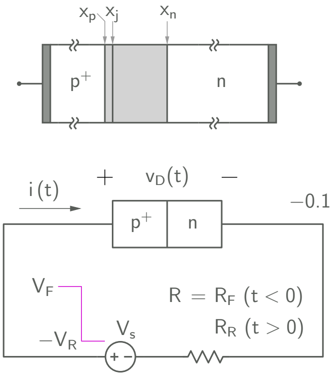
We can now use $Q_p(t_s) \approx 0$ to estimate t_s as

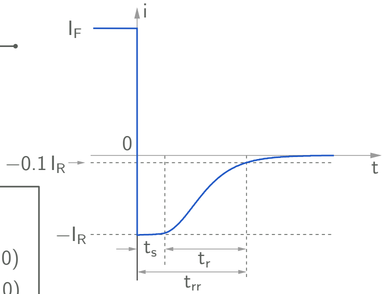
$$t_s = \tau_p \log\left(1 + \frac{I_F}{I_R}\right).$$

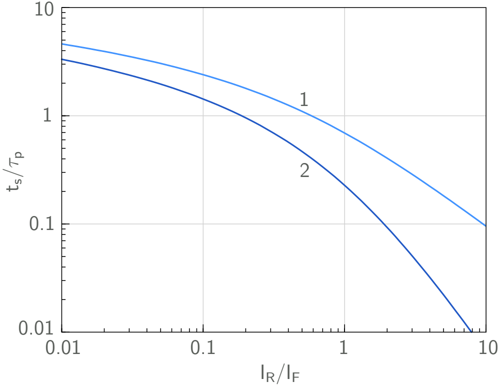
A more accurate analysis yields[2] $\mathrm{erf}\left(\sqrt{\dfrac{t_s}{\tau_p}}\right) = \dfrac{1}{1 + \dfrac{I_R}{I_F}}$.

[2]R.F. Pierret, *Semiconductor Device Fundamentals*. New Delhi: Pearson Education, 1996.

Turn-off transient: charge control approach

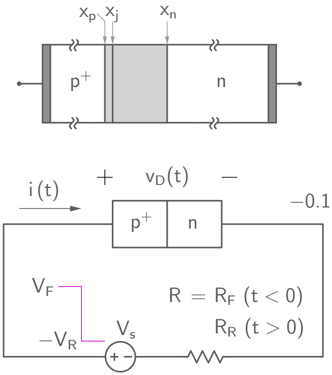


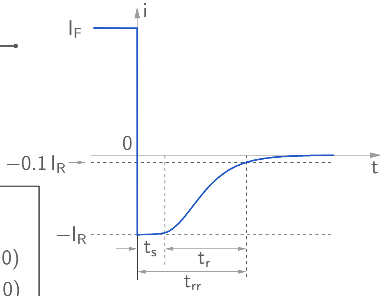


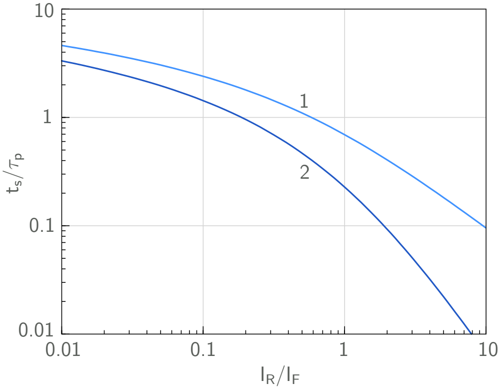


$$(1)\quad t_s = \tau_p \log\left(1 + \frac{I_F}{I_R}\right)$$

$$(2)\quad \mathrm{erf}\left(\sqrt{\frac{t_s}{\tau_p}}\right) = \frac{1}{1 + \dfrac{I_R}{I_F}}$$



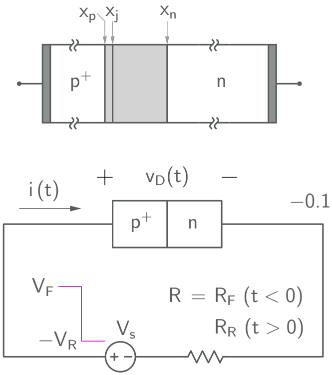


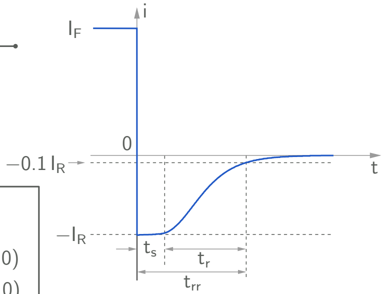


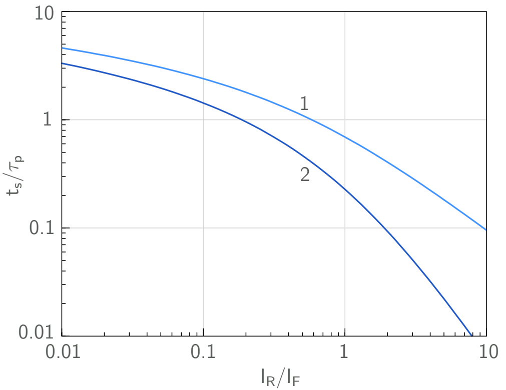
(1) $t_s = \tau_p \log\left(1 + \dfrac{I_F}{I_R}\right)$

(2) $\mathrm{erf}\left(\sqrt{\dfrac{t_s}{\tau_p}}\right) = \dfrac{1}{1 + \dfrac{I_R}{I_F}}$

* If I_F is increased, the initial charge $Q_p(0^-) = I_F\tau_p$ is larger $\rightarrow$ t_s increases.



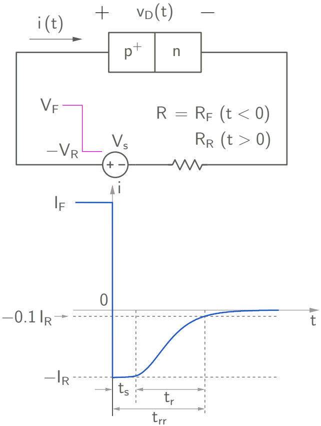




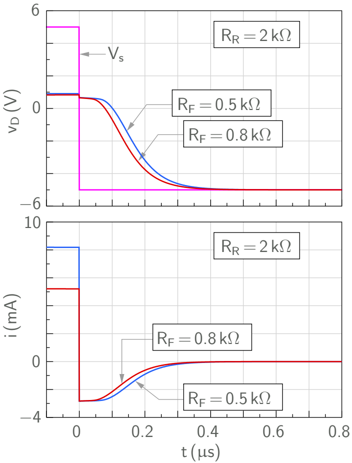
(1) $t_s = \tau_p \log\left(1 + \dfrac{I_F}{I_R}\right)$

(2) $\mathrm{erf}\left(\sqrt{\dfrac{t_s}{\tau_p}}\right) = \dfrac{1}{1 + \dfrac{I_R}{I_F}}$

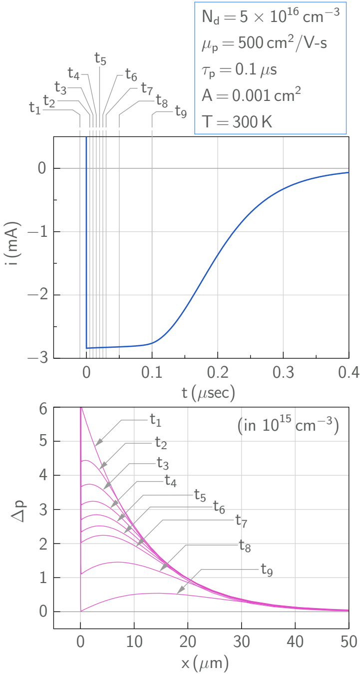
* If I_F is increased, the initial charge $Q_p(0^-) = I_F \tau_p$ is larger
  $\rightarrow t_s$ increases.
* If I_R is increased, the excess charge is removed at a higher rate
  $\rightarrow t_s$ decreases.



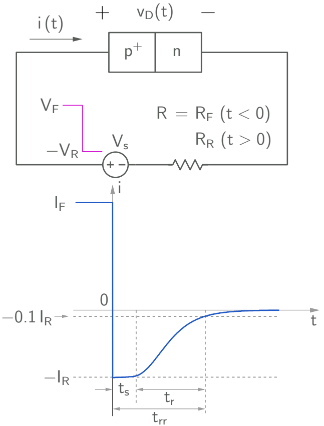
i (t)
+
vD(t)
−
p+
n
VF
−VR
Vs
R = RF (t < 0)
RR (t > 0)
i
IF
0
−0.1 IR
t
−IR
ts
tr
trr

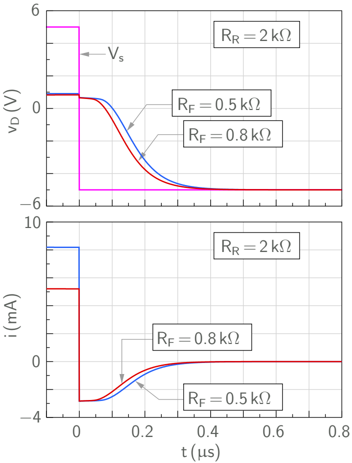


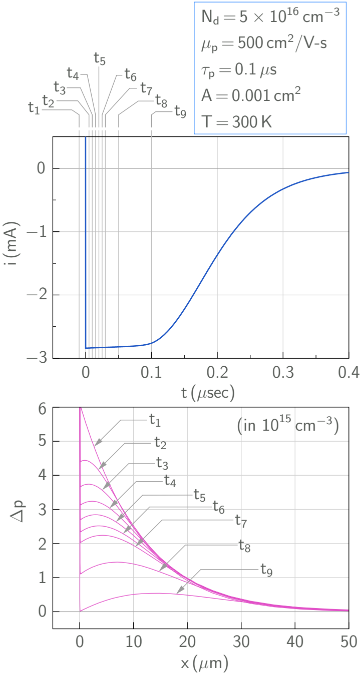
vD (V)
RR = 2 kΩ
Vs
RF = 0.5 kΩ
RF = 0.8 kΩ
i (mA)
RR = 2 kΩ
RF = 0.8 kΩ
RF = 0.5 kΩ
t (μs)



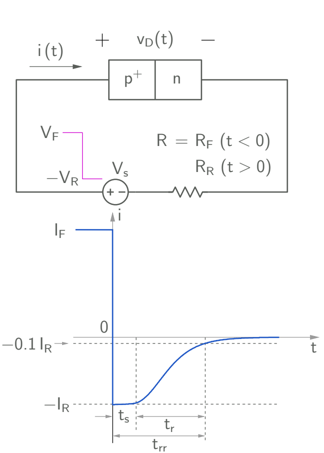
Nd = 5 × 10^16 cm^−3
μp = 500 cm^2/V-s
τp = 0.1 μs
A = 0.001 cm^2
T = 300 K
t1 t2 t3 t4 t5 t6 t7 t8 t9
i (mA)
t (μsec)
Δp
(in 10^15 cm^−3)
x (μm)

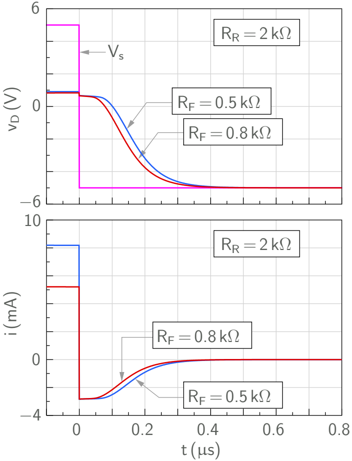


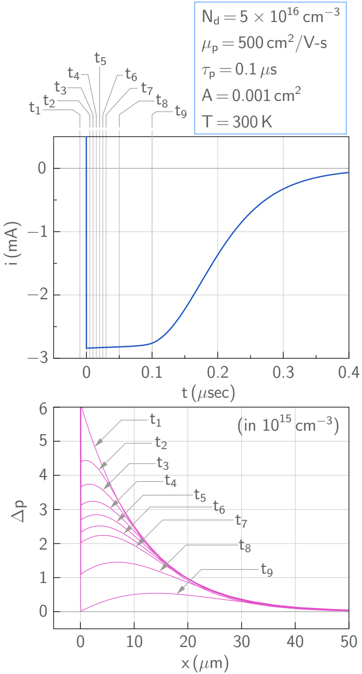




* In the t_s phase, $\Delta p(x_n) > 0$ which is consistent with $v_D > 0\,\text{V}$ during this phase (since $\Delta p(x_n) = p_{n0}\,(e^{v_D/V_T} - 1)$).

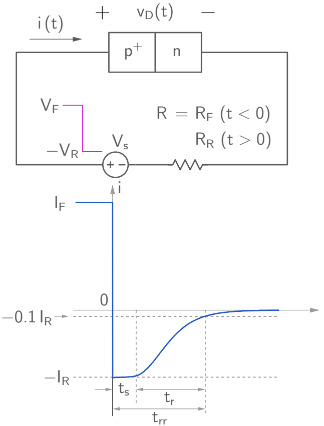


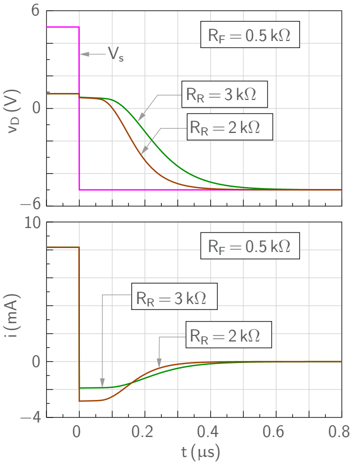


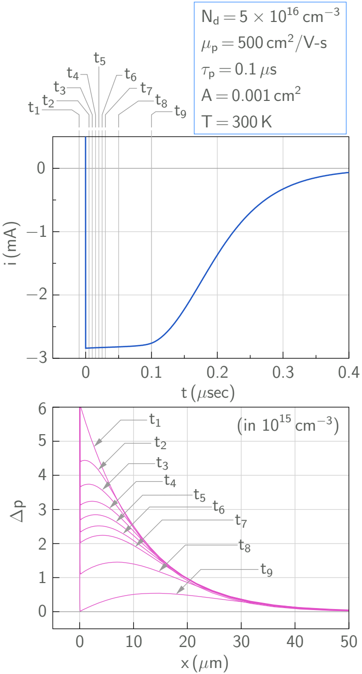


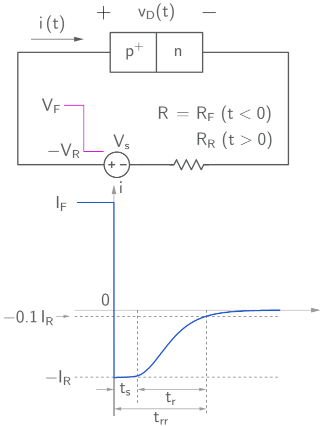
* In the t_s phase, $\Delta p(x_n) > 0$ which is consistent with $v_D > 0\,\mathrm{V}$ during this phase (since $\Delta p(x_n) = p_{n0}\left(e^{v_D/V_T} - 1\right)$).

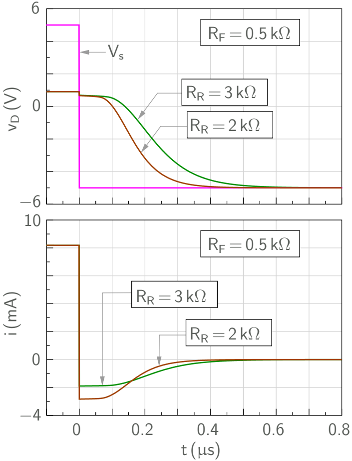
* If R_F is reduced, I_F increases, and the initial excess hole charge $Q_p(0) = I_F\tau_p$ also increases. The increased charge takes a longer time for removal, leading to a larger value of t_s.

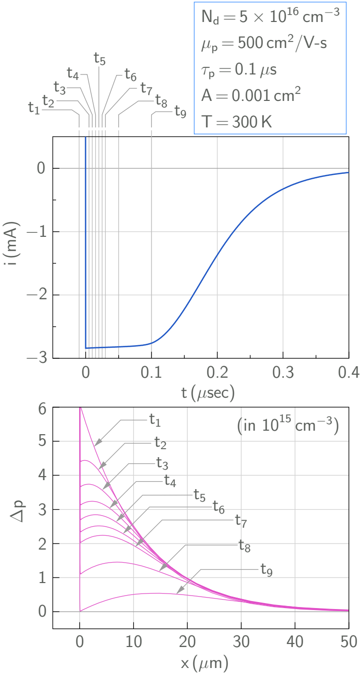

i (t)
+
$v_D(t)$
−
p^+
n
V_F
$-V_R$
V_s
$R = R_F\ (t < 0)$
$R_R\ (t > 0)$
i
I_F
0
$-0.1\, I_R$
$-I_R$
t_s
t_r
t_{rr}


$R_F = 0.5\,\text{k}\Omega$
V_s
$R_R = 3\,\text{k}\Omega$
$R_R = 2\,\text{k}\Omega$
v_D (V)
6
0
−6
$R_F = 0.5\,\text{k}\Omega$
$R_R = 3\,\text{k}\Omega$
$R_R = 2\,\text{k}\Omega$
i (mA)
10
0
−4
0
0.2
0.4
0.6
0.8
t (μs)

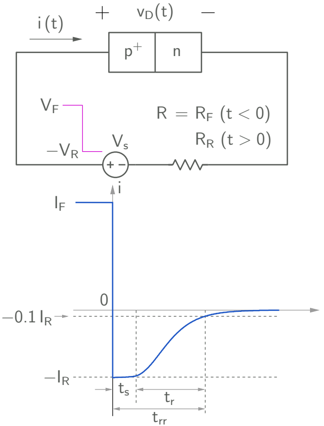

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu\text{s}$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$
t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9
i (mA)
0
−1
−2
−3
0
0.1
0.2
0.3
0.4
t (μsec)
(in 10^{15} cm^{-3})
Δp
6
5
4
3
2
1
0
10
20
30
40
50
x (μm)

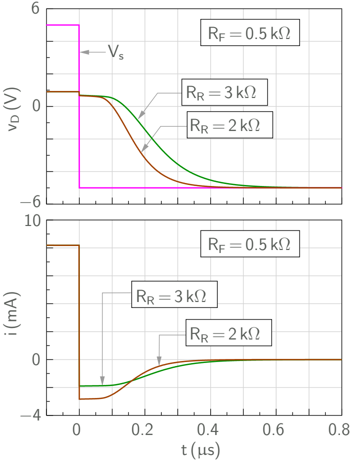


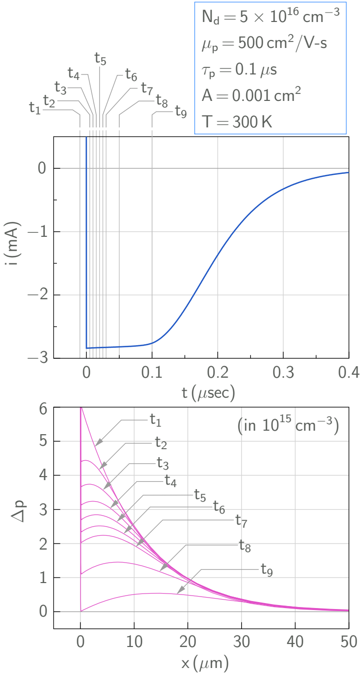




* If R_R is reduced, the reverse current (magnitude) I_R increases. Since I_R is one of the factors responsible for removal of Q_p, a larger value of I_R results in a smaller t_s.

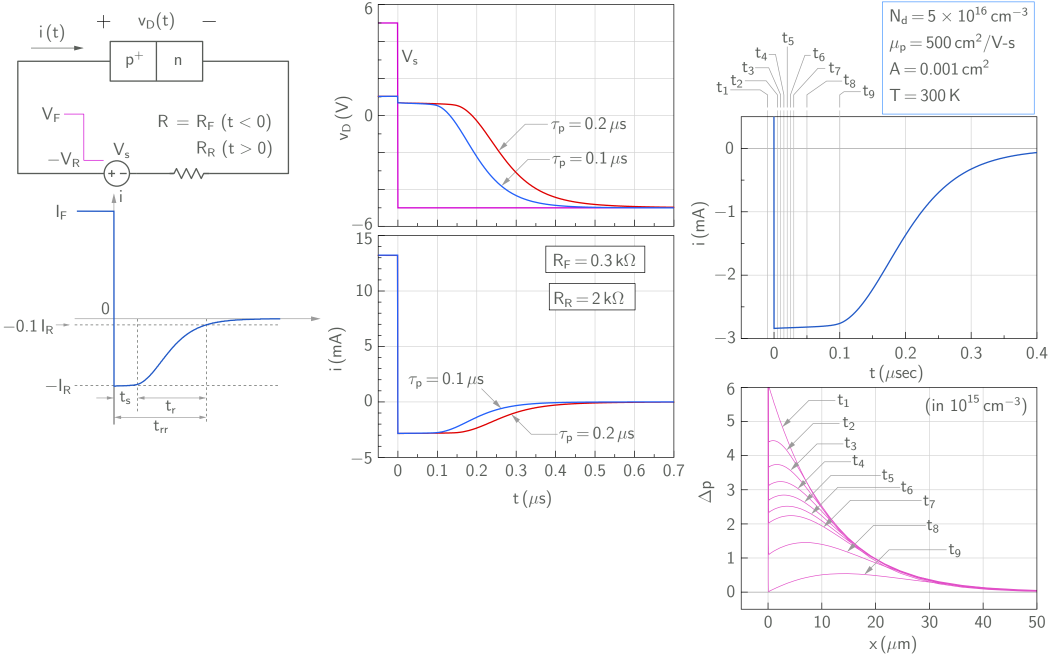






* If R_R is reduced, the reverse current (magnitude) I_R increases. Since I_R is one of the factors responsible for removal of Q_p, a larger value of I_R results in a smaller t_s.

  (The other factor is recombination in the neutral n region.)



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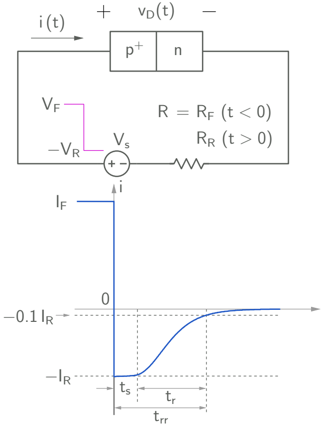
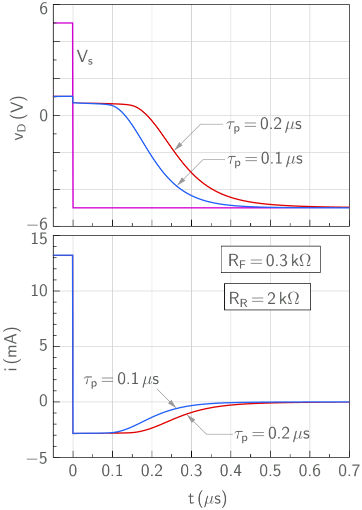
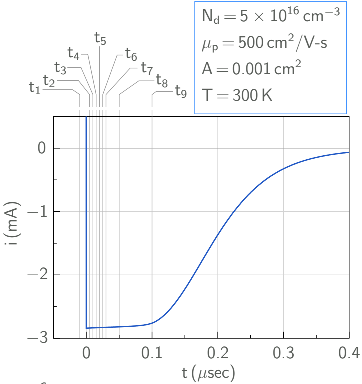
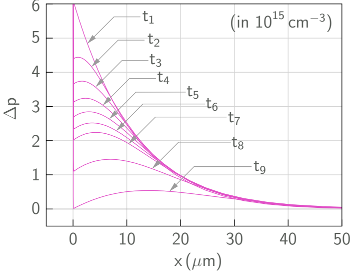
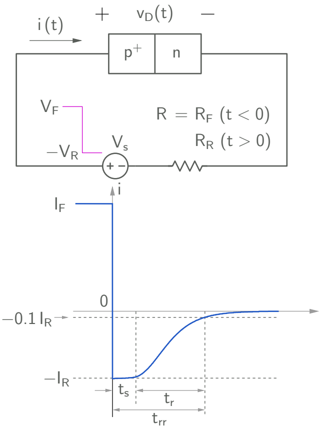


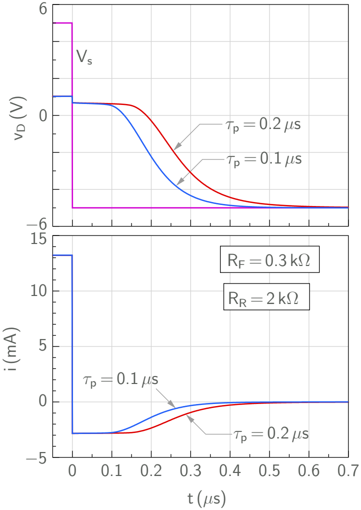


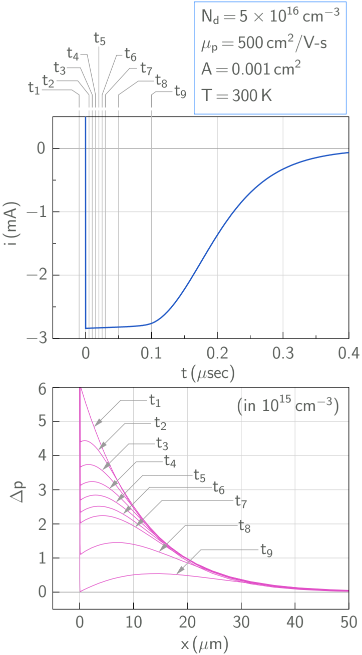




* $t_s \approx \tau_p \log \left(1 + \dfrac{I_F}{I_R} \right)$.





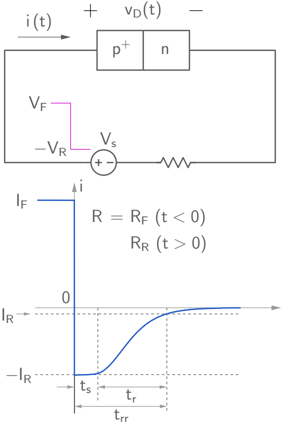


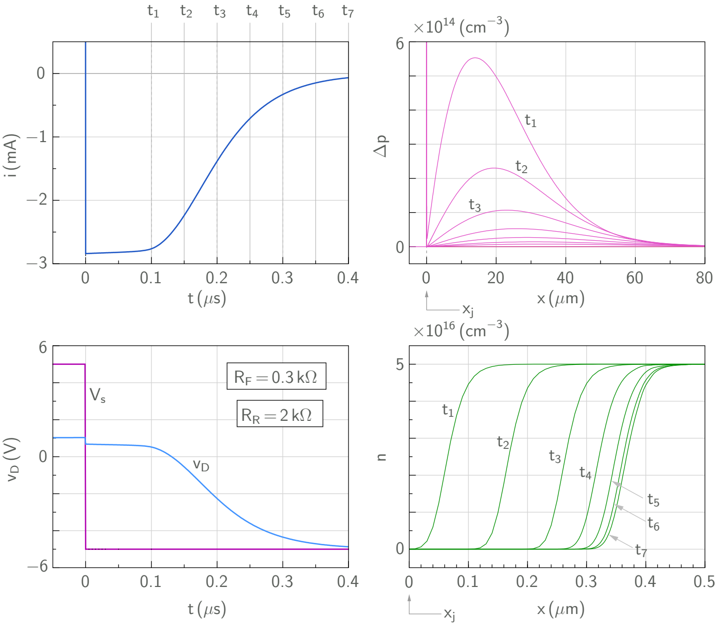
* $t_s \approx \tau_p \log\left(1 + \dfrac{I_F}{I_R}\right)$.

* In practice, a smaller τ_p can be achieved by introducing effective recombination centres (such as gold in silicon) with a trap level in the middle of the energy gap.

Turn-off transient: Part 2

$N_d = 5 \times 10^{16}\,\text{cm}^{-3}$
$\mu_p = 500\,\text{cm}^2/\text{V-s}$
$\tau_p = 0.1\,\mu\text{s}$
$A = 0.001\,\text{cm}^2$
$T = 300\,\text{K}$





Turn-off transient: Part 2

$N_d = 5 \times 10^{16}\ \mathrm{cm}^{-3}$
$\mu_p = 500\ \mathrm{cm}^2/\mathrm{V\text{-}s}$
$\tau_p = 0.1\ \mu s$
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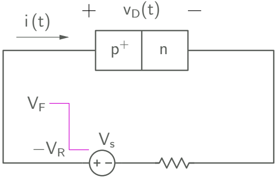
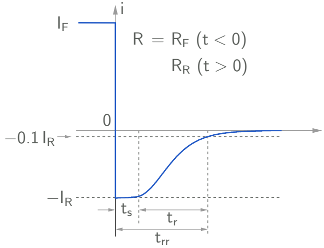
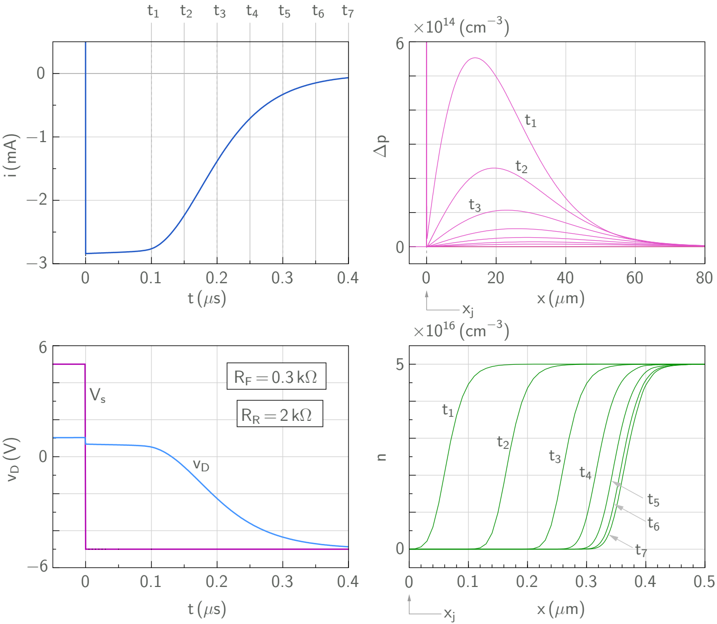


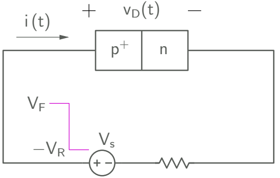


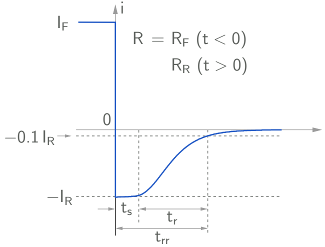


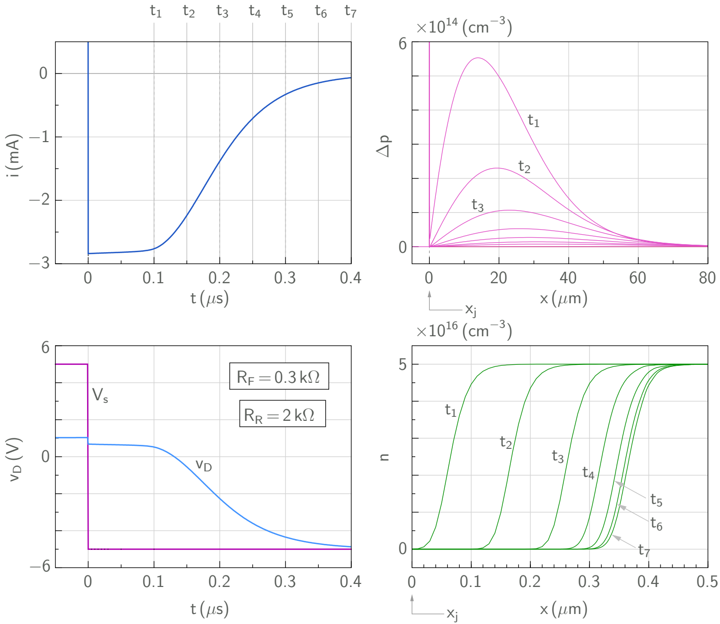
* The second part of the turn-off transient (the interval t_r) is very complex because several changes take place simultaneously.

Turn-off transient: Part 2

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu\text{s}$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$

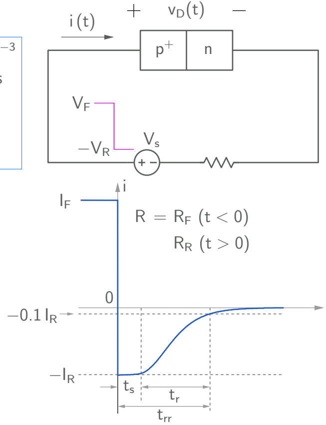


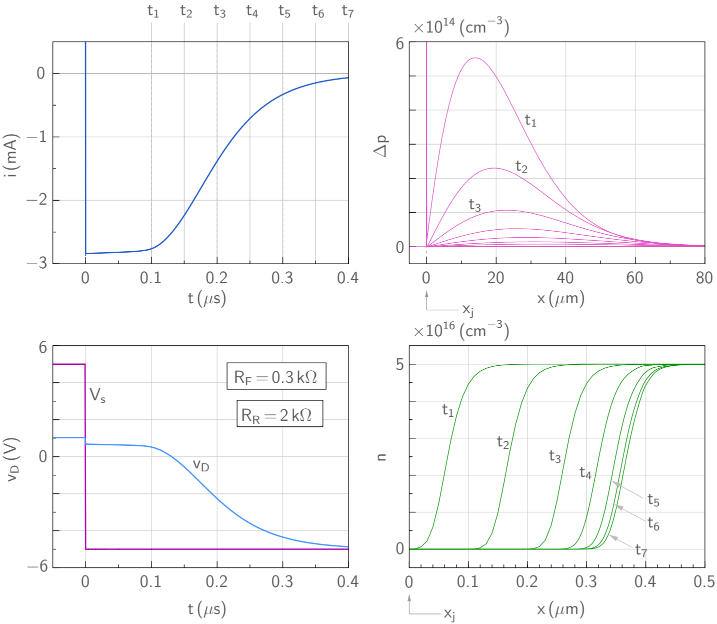




Turn-off transient: Part 2

$N_d = 5 \times 10^{16}\,\text{cm}^{-3}$
$\mu_p = 500\,\text{cm}^2/\text{V-s}$
$\tau_p = 0.1\,\mu\text{s}$
$A = 0.001\,\text{cm}^2$
$T = 300\,\text{K}$

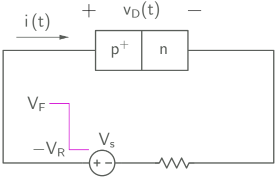


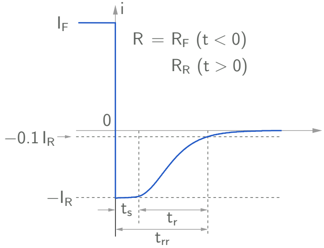


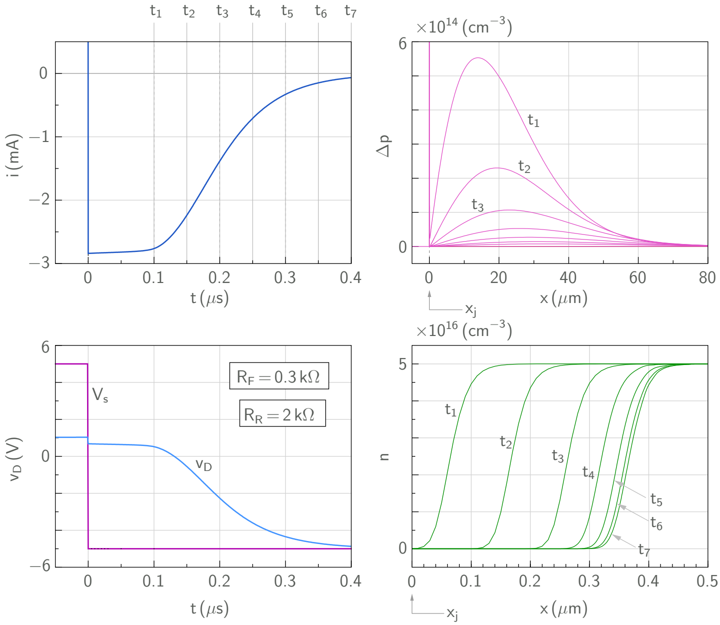
* The slope $\left.\dfrac{dp}{dx}\right|_{x=x_n}$ decreases with time, and therefore the diode current $i(t) \approx I_p^{\text{diff}}(x_n, t)$ decreases (in magnitude).

Turn-off transient: Part 2

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu\text{s}$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$







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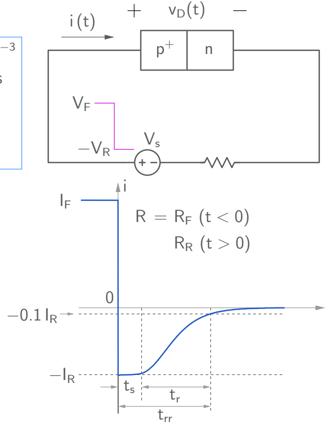
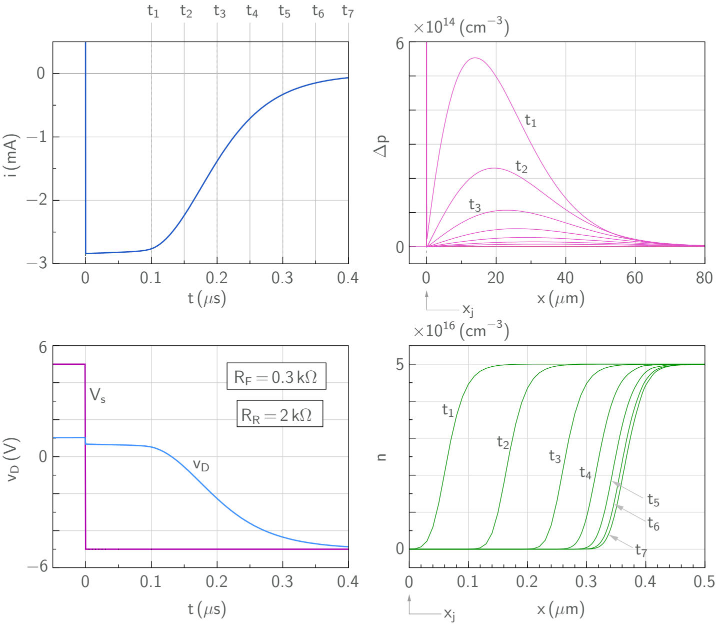
Turn-off transient: Part 2

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu\text{s}$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$





* The depletion region expands as the reverse voltage across the diode builds up. The expansion of the depletion region is indicated by the movement of the majority carriers away from the junction.

Turn-on transient

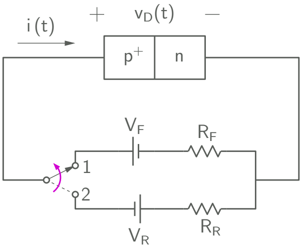
$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$

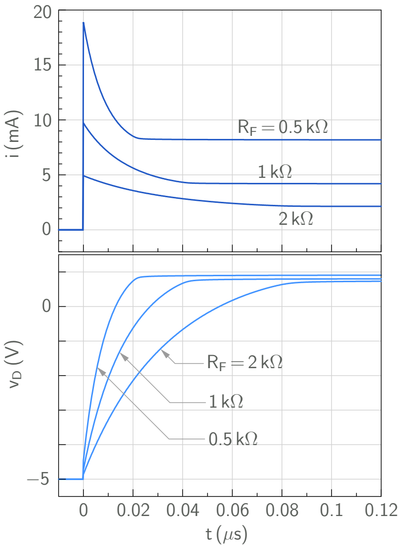
$\mu_p = 500\ \text{cm}^2/\text{V-s}$

$\tau_p = 0.1\ \mu\text{s}$

$A = 0.001\ \text{cm}^2$

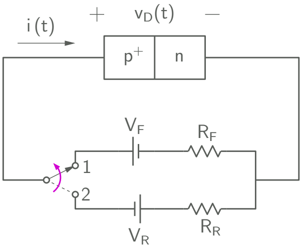
$T = 300\ \text{K}$

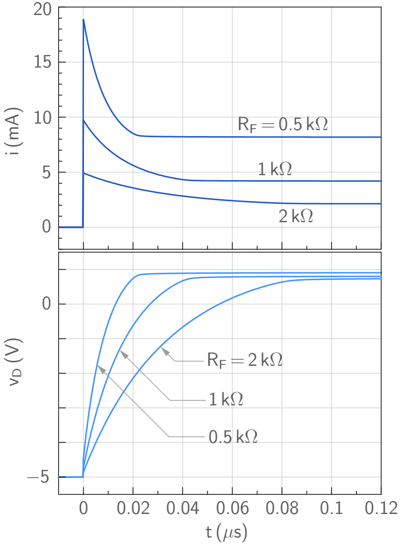




Turn-on transient

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu\text{s}$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$



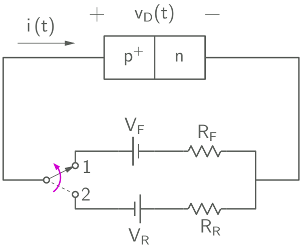


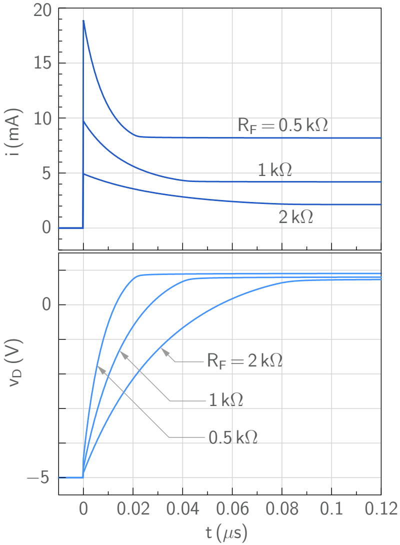
During turn-on,

- The diode current must change from nearly zero to a significant forward current $I_D \approx \dfrac{V_F - V_{on}}{R_F}$.

Turn-on transient

$N_d = 5 \times 10^{16}\,\text{cm}^{-3}$
$\mu_p = 500\,\text{cm}^2/\text{V-s}$
$\tau_p = 0.1\,\mu\text{s}$
$A = 0.001\,\text{cm}^2$
$T = 300\,\text{K}$



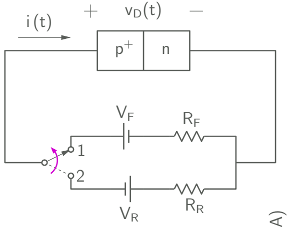


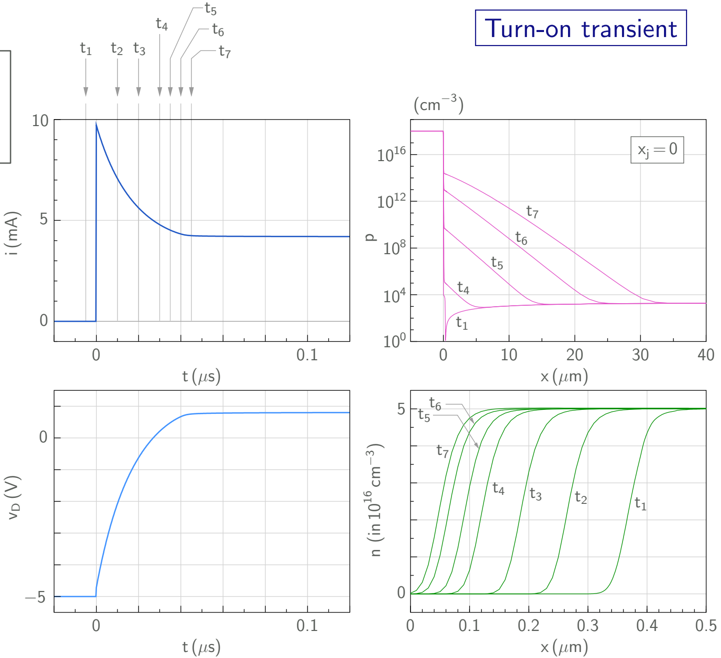
During turn-on,

- The diode current must change from nearly zero to a significant forward current $I_D \approx \dfrac{V_F - V_{on}}{R_F}$.
- The diode voltage must change from $-V_R$ to the steady-state forward bias value corresponding to the steady-state forward current.

Turn-on transient

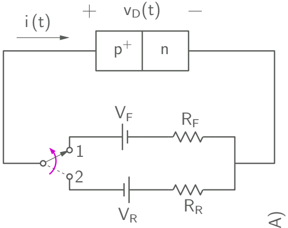
$N_d = 5 \times 10^{16}\ \mathrm{cm}^{-3}$
$\mu_p = 500\ \mathrm{cm}^2/\mathrm{V\text{-}s}$
$\tau_p = 0.1\ \mu\mathrm{s}$
$A = 0.001\ \mathrm{cm}^2$
$T = 300\ \mathrm{K}$
$R_F = 1\ \mathrm{k}\Omega$
$V_F = 5\ \mathrm{V}$
$V_R = -5\ \mathrm{V}$



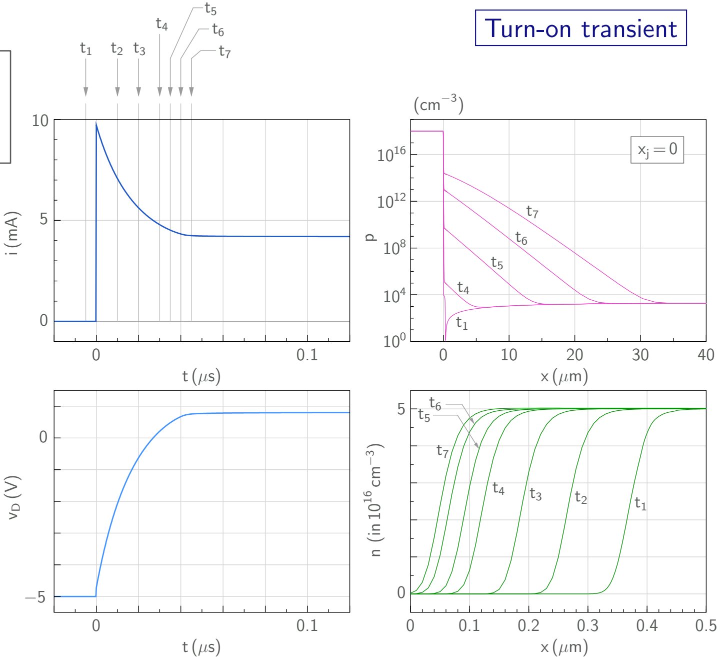


Turn-on transient

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu\text{s}$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$
$R_F = 1\ \text{k}\Omega$
$V_F = 5\ \text{V}$
$V_R = -5\ \text{V}$

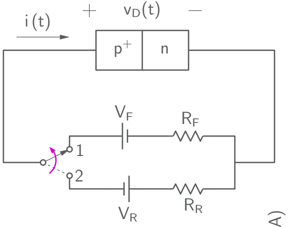


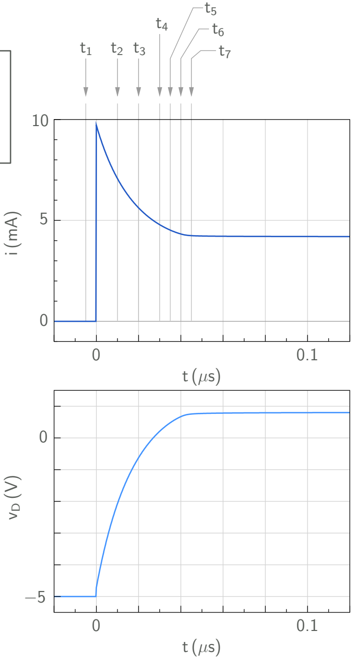
* Phase 1: The depletion region shrinks. During this phase, the minority carrier profile does not change significantly.

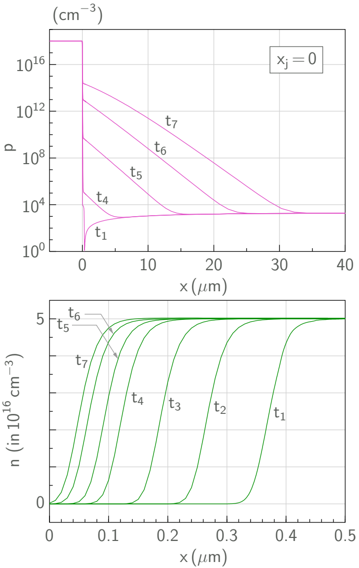


Turn-on transient

$N_d = 5 \times 10^{16}\ \text{cm}^{-3}$
$\mu_p = 500\ \text{cm}^2/\text{V-s}$
$\tau_p = 0.1\ \mu\text{s}$
$A = 0.001\ \text{cm}^2$
$T = 300\ \text{K}$
$R_F = 1\ \text{k}\Omega$
$V_F = 5\ \text{V}$
$V_R = -5\ \text{V}$



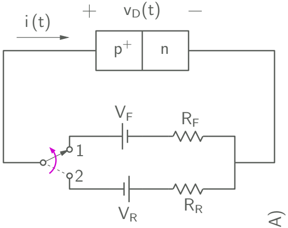


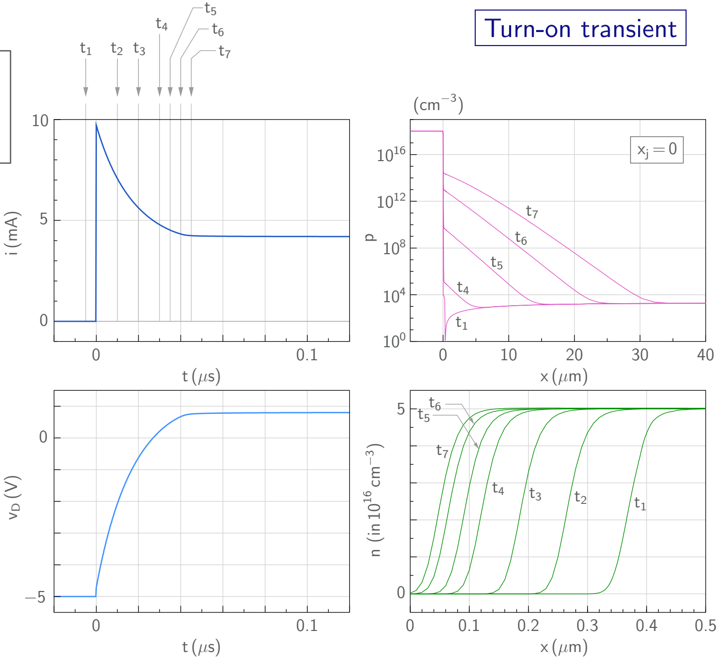


* Phase 1: The depletion region shrinks. During this phase, the minority carrier profile does not change significantly.
* Phase 2: Some time between t_3 and t_4, the *pn* junction enters the forward bias regime. Beyond this point, the minority carrier charge builds up relatively quickly.

Turn-on transient

$N_d = 5 \times 10^{16}\ \mathrm{cm}^{-3}$
$\mu_p = 500\ \mathrm{cm}^2/\mathrm{V\text{-}s}$
$\tau_p = 0.1\ \mu s$
$A = 0.001\ \mathrm{cm}^2$
$T = 300\ K$
$R_F = 1\ k\Omega$
$V_F = 5\ V$
$V_R = -5\ V$





* Phase 1: The depletion region shrinks. During this phase, the minority carrier profile does not change significantly.
* Phase 2: Some time between t_3 and t_4, the *pn* junction enters the forward bias regime. Beyond this point, the minority carrier charge builds up relatively quickly.
* In practice, the turn-off transient is a matter of greater concern since it is longer than the turn-on transient.