SEMICONDUCTOR DEVICES

Carrier Transport: Part 1

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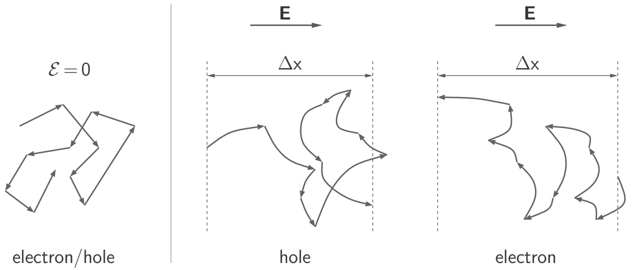
- This background is useful even in non-equilibrium situations because there are regions in a semiconductor device which are almost in equilibrium.
- We now go one step further and develop an understanding of current flow and carrier dynamics in a semiconductor, an essential ingredient in any semiconductor device.

Drift current



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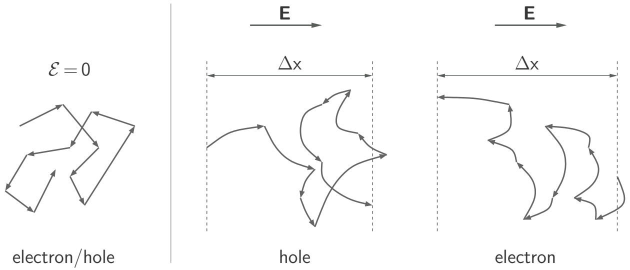

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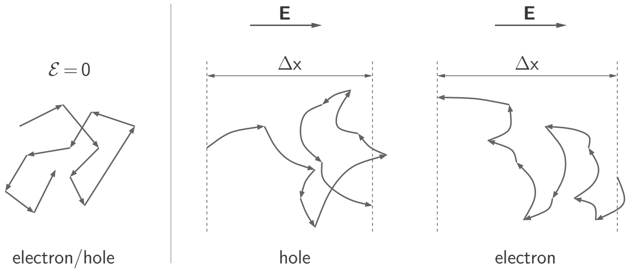

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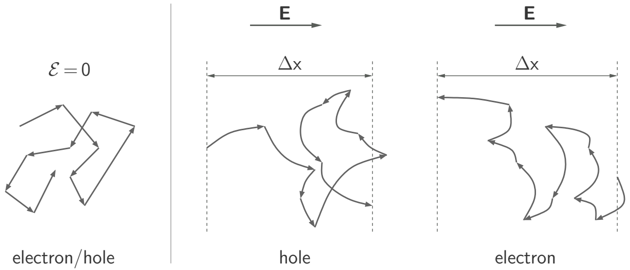

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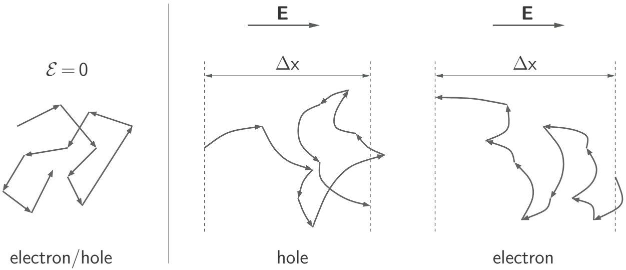

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- In a semiconductor, electrons and holes are continuously moving with large instantaneous velocities. However, their trajectories are interrupted because of “scattering events.”
- With zero electric field, the average displacement of a carrier is zero.
- In the presence of an electric field, a carrier undergoes a net change Δx in its position over a time interval Δt. $\dfrac{\Delta x}{\Delta t}$ is called the “drift velocity” of the carrier.

Scattering of carriers

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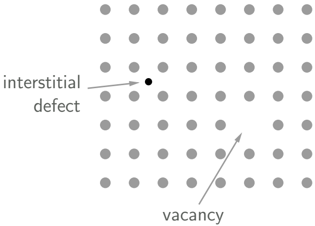
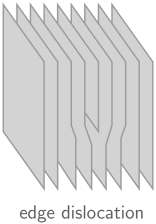
- Defects: A semiconductor crystal may have defects, i.e., departures from its periodic structure. These deviations cause a change in the periodic lattice potential and therefore lead to scattering.





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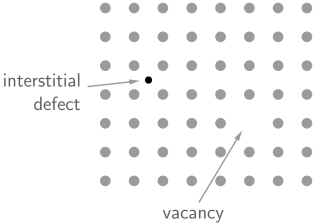
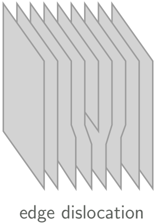
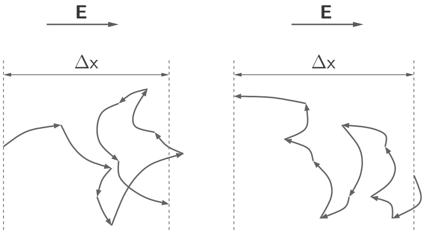




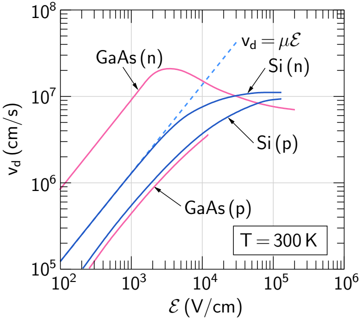
- Note that the atoms of the semiconductor crystal do not cause scattering. They are already accounted for in computing the band structure of the semiconductor.

Drift current



hole

electron



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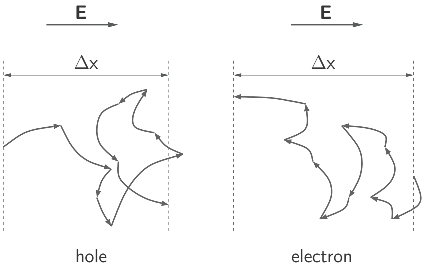

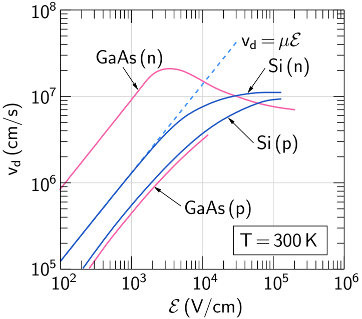

∗ At low fields (up to a few kV/cm), the drift velocity v_d varies linearly with the electric field $\mathcal{E}$. (Note that a 10 x change in $\mathcal{E}$ causes a 10 x change in v_d.)

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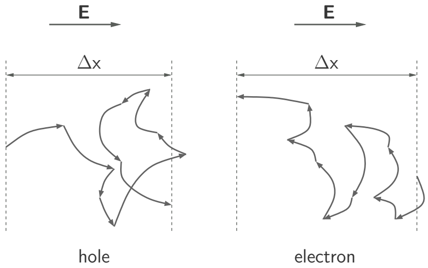

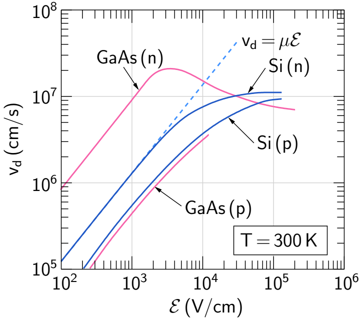

- $*$ At low fields (up to a few kV/cm), the drift velocity v_d varies linearly with the electric field $\mathcal{E}$. (Note that a 10 x change in $\mathcal{E}$ causes a 10 x change in v_d.)
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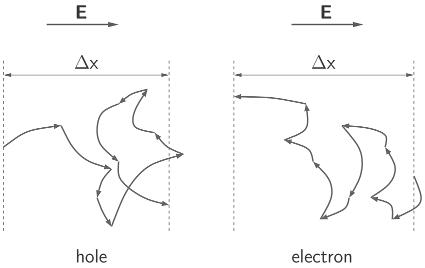

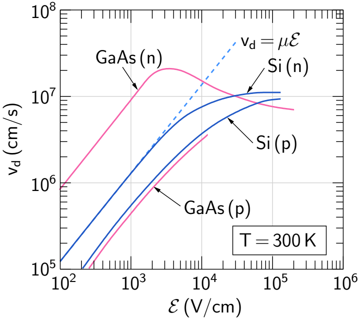

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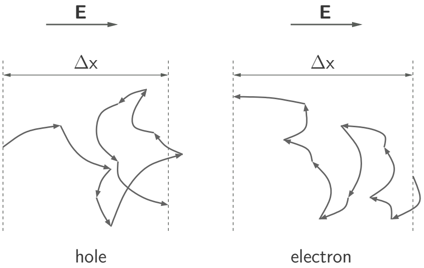

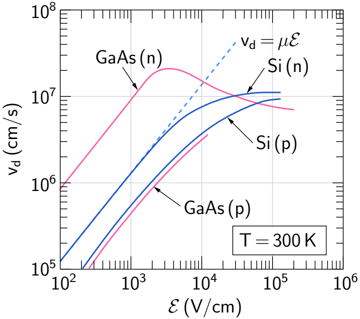

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- $\mu = \dfrac{q\tau}{m^*}$, where m^* is the effective mass and τ is the momentum relaxation time, i.e., the average time interval between successive scattering events (typically 10^{-14} to 10^{-12} sec, i.e., 0.01 ps to 1 ps).

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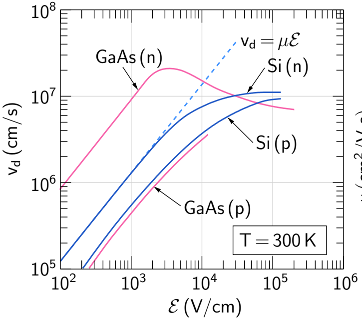

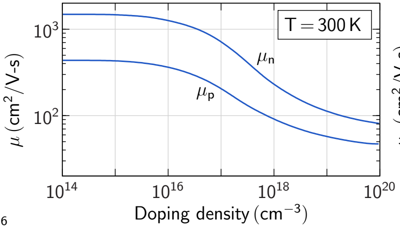

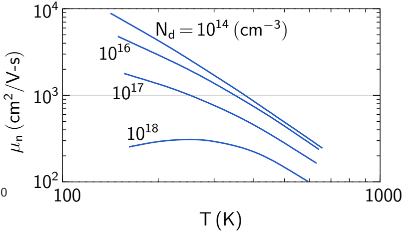

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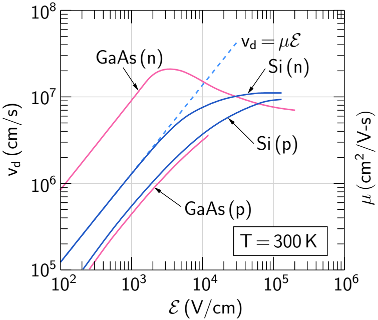

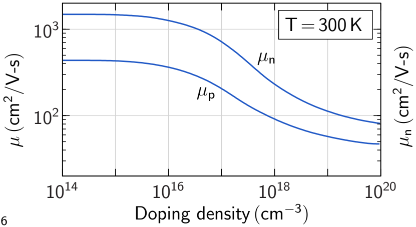

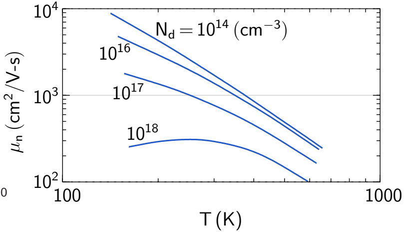

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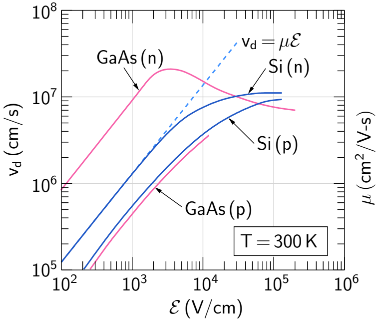

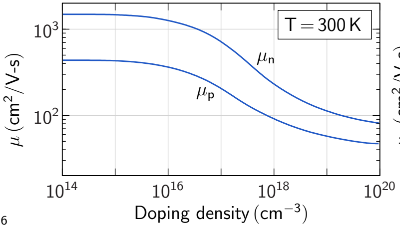

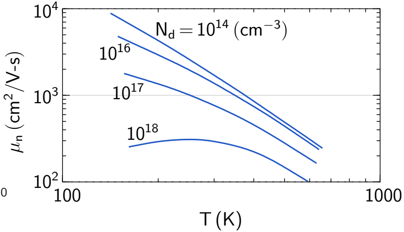

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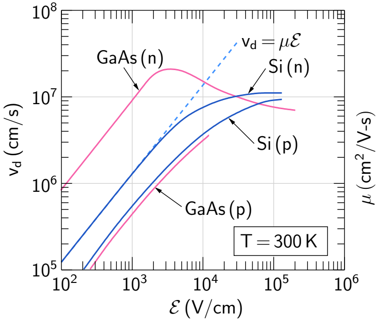

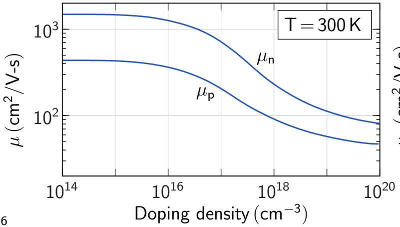

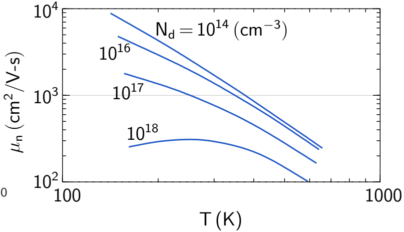

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  In GaAs, as $\mathcal{E}$ is increased, v_d increases, reaches a peak, and then decreases to saturate to a constant value.

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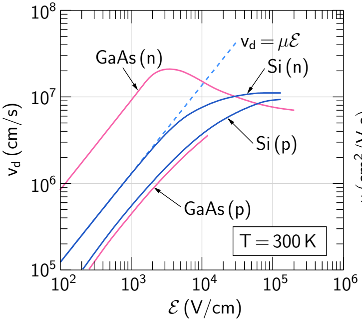

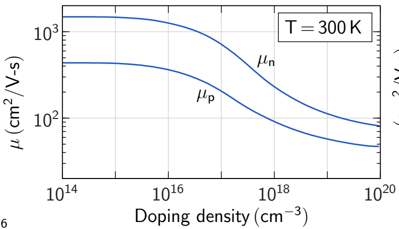

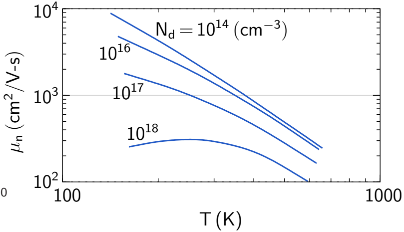

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- ∗ The mobility varies significantly with temperature and doping density.

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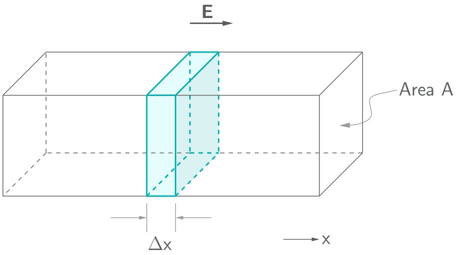

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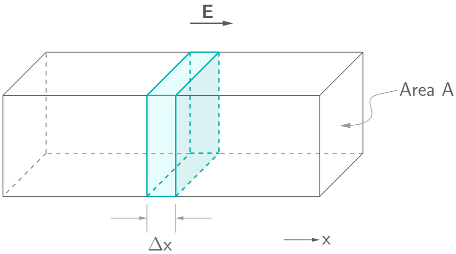

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* The electric field shown in the figure causes holes to drift in the $+x$ direction with a velocity v_d^p and electrons in the $-x$ direction with a velocity $-v_d^n$.

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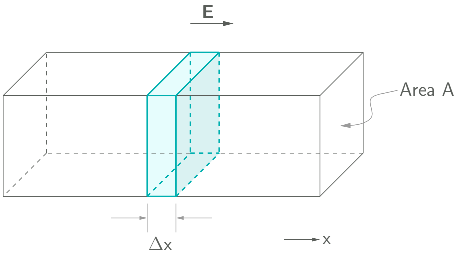

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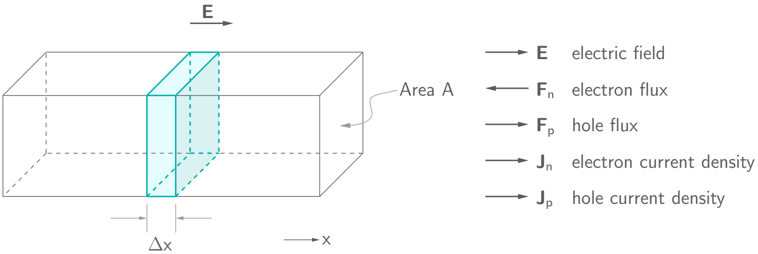

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- Consider the box in the figure.

$$\mathcal{F}_p = \frac{\text{number of holes in the box}}{\text{time to traverse } \Delta x} \times \frac{1}{A} = \frac{p\,A\,\Delta x}{\Delta x / v_d^p} \times \frac{1}{A} = p\,v_d^p .$$

Similarly, $\mathcal{F}_n = -n\,v_d^n$.

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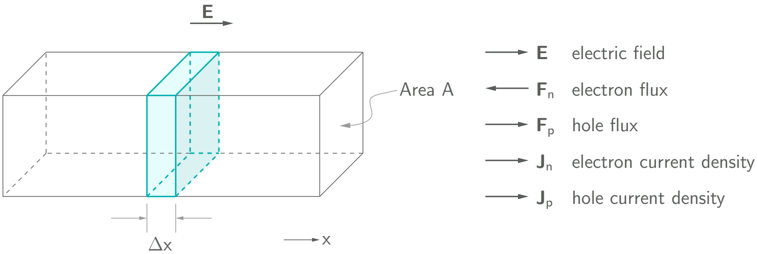

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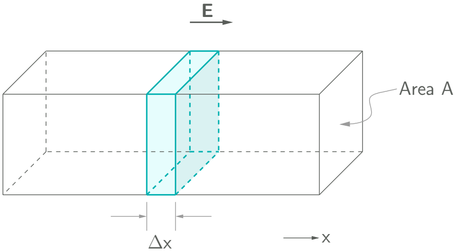

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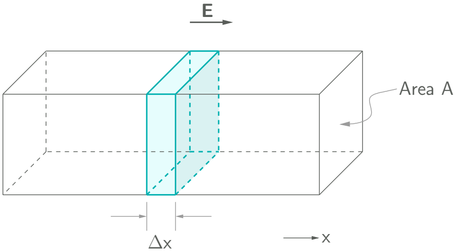

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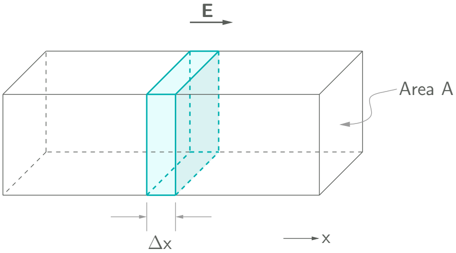

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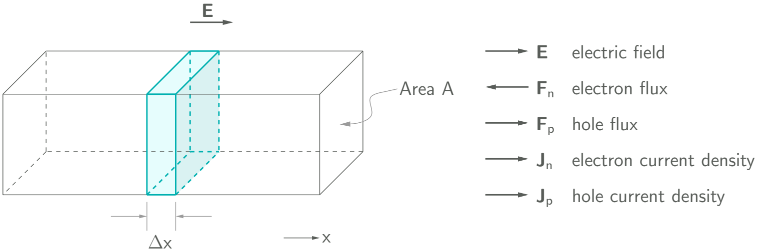

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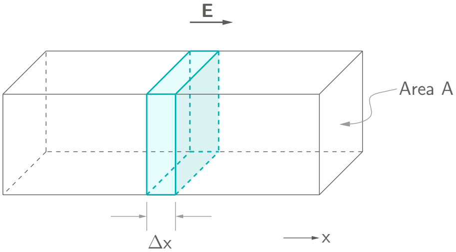

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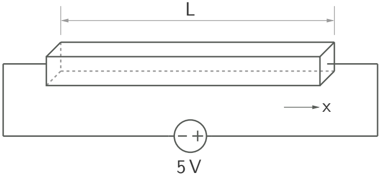
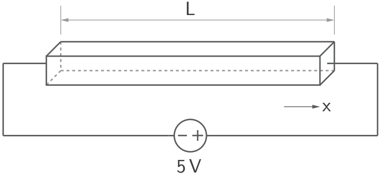
- Check units: Coul $\times \dfrac{\text{cm}^2}{\text{V-s}} \times \dfrac{1}{\text{cm}^3} \times \dfrac{\text{V}}{\text{cm}} = \dfrac{\text{Amp}}{\text{cm}^2}$ ✓

Example



For the rectangular silicon bar shown in the figure, $L = 50\,\mu\text{m}$, and the cross-sectional area is $20\,\mu\text{m}^2$. It is uniformly doped with $N_d = 5 \times 10^{17}\,\text{cm}^{-3}$. At $T = 300\,\text{K}$ and with an applied voltage of 5 V, find the following.

Example



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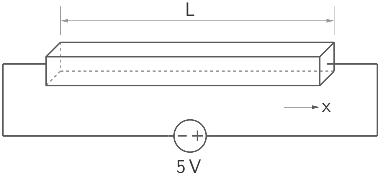
Example



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(a) electric field,

(b) current density,

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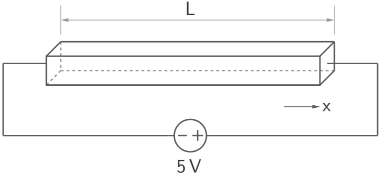

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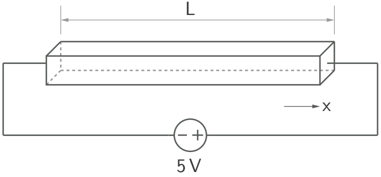

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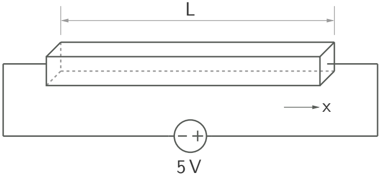

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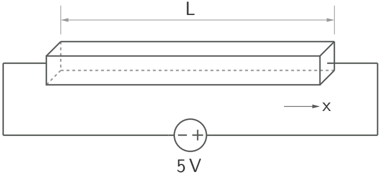

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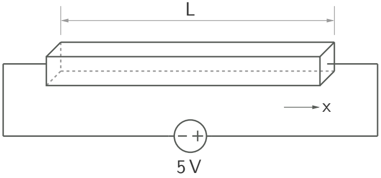

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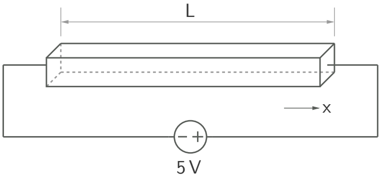
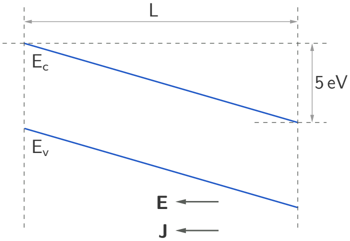
(Note: Such a bar does not exist in isolation, but we can fabricate a region inside a silicon wafer which would resemble this structure.)



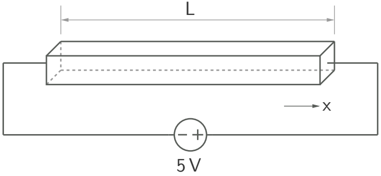


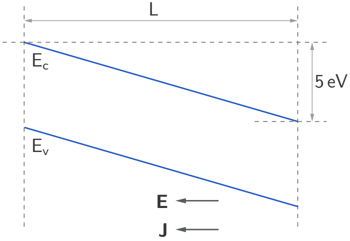
Assuming all donors to be ionised, $n = p + N_d^+ \approx N_d = 5 \times 10^{17}\,\text{cm}^{-3}$.

Assume that the metal-semiconductor contacts serve as a perfect source or sink for the carriers.

The applied voltage appears across the semiconductor, resulting in a uniform field and causing a drift current.

$$\mathcal{E} = -\frac{dV}{dx} = -\frac{V_0}{L} = -\frac{5\,\text{V}}{50 \times 10^{-4}\,\text{cm}} \text{ i.e., } -1\,\text{kV/cm} \equiv \mathcal{E}_0$$





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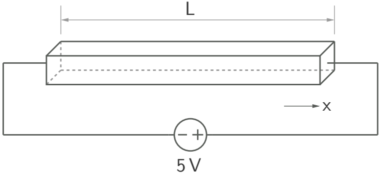

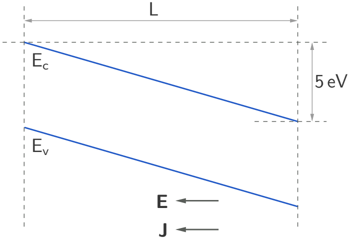

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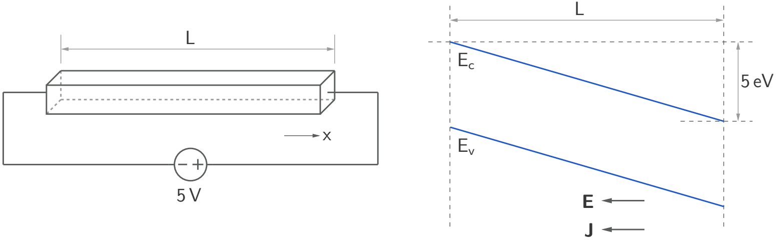

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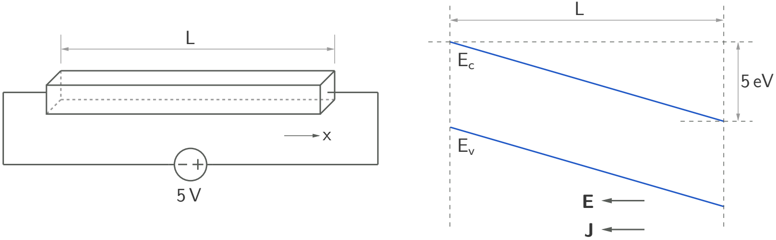

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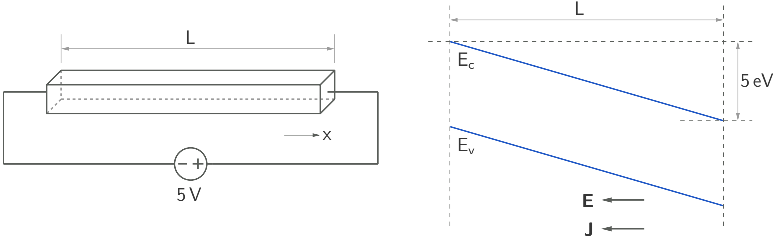
$$= 1.6 \times 10^{-19}\,\text{C} \times 5 \times 10^{17}\,\frac{1}{\text{cm}^3} \times 400\,\frac{\text{cm}^2}{\text{V-s}} \times \left(-10^3\,\frac{\text{V}}{\text{cm}}\right)$$



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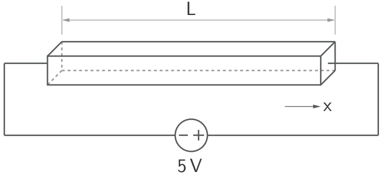

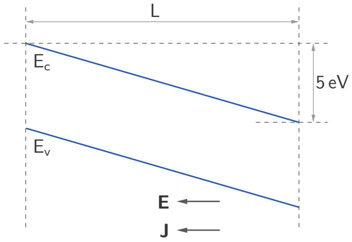

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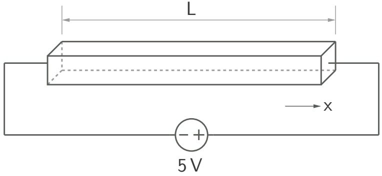

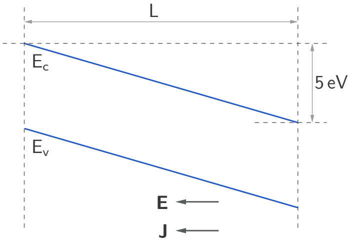

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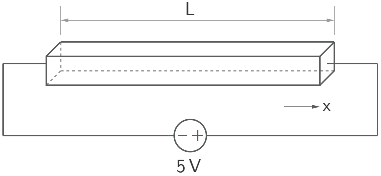

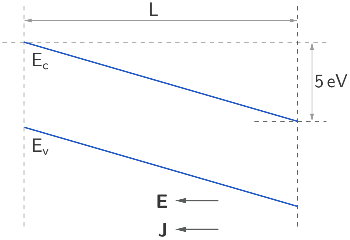

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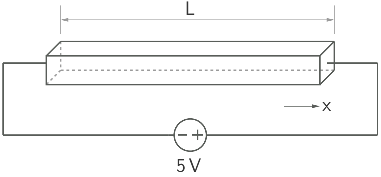

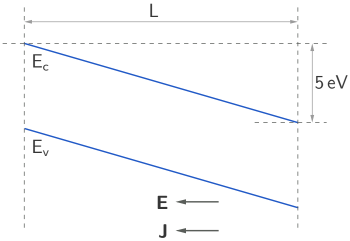

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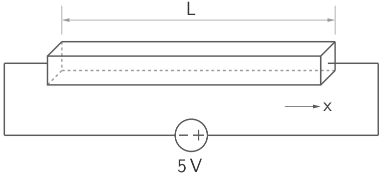

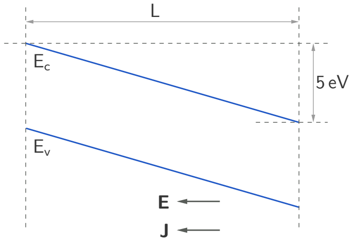

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$\rightarrow$ at $T = 310\,\text{K}$, $n_i \approx 2 \times 10^{10}\,\text{cm}^{-3}$, and σ will double.

$T\,(^\circ\text{C})$	$n_i\,(\text{cm}^{-3})$
25	8.1×10^9
35	1.7×10^{10}
45	3.5×10^{10}
55	6.9×10^{10}
65	1.3×10^{11}
75	2.3×10^{11}

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Let us compare the two cases we have discussed.

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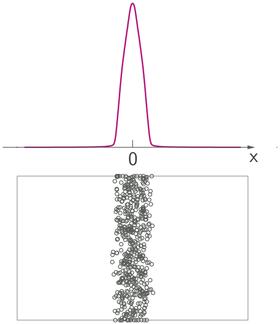
A huge change!

Diffusion



$t = t_0$

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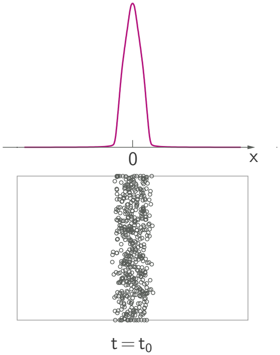

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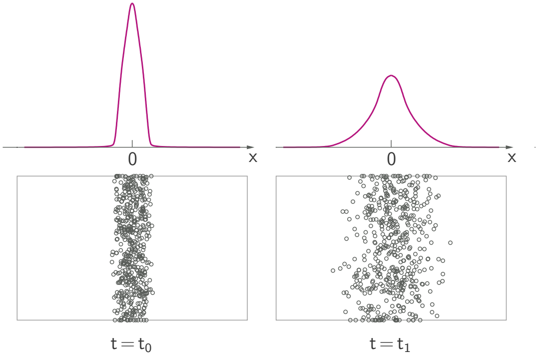

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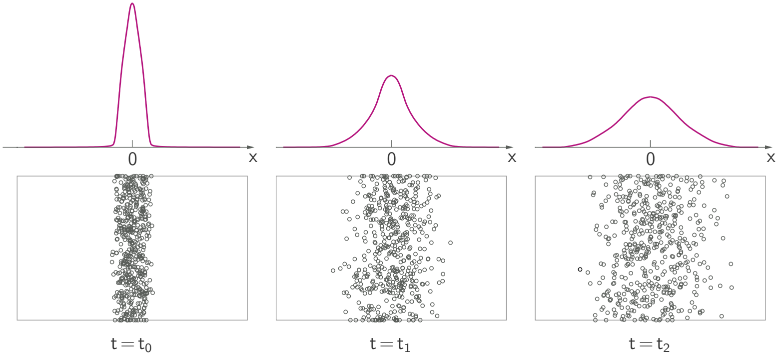

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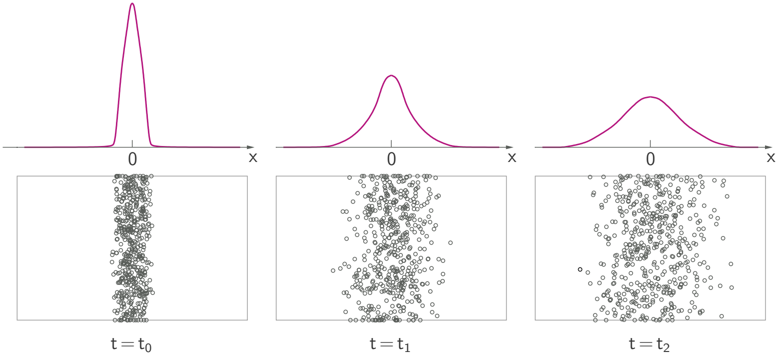

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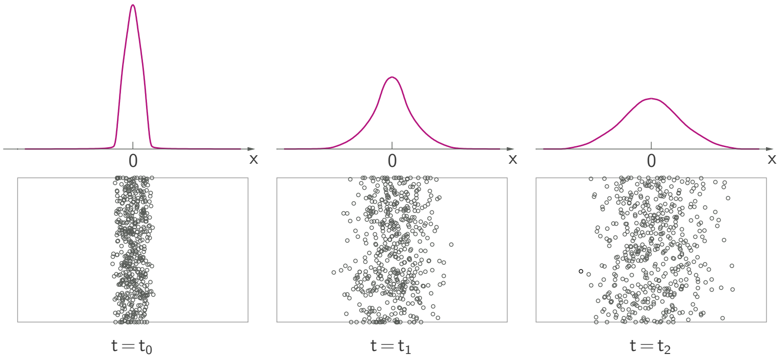

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- The particles are subjected to random scattering events.
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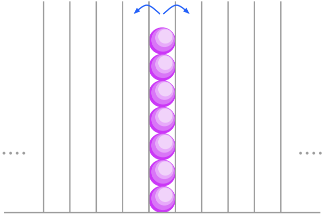
- The particles "diffuse" much like smoke emanating from a chimney.

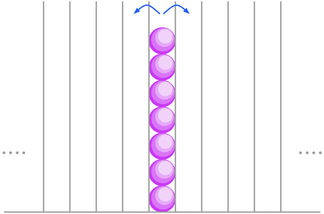
Diffusion



Consider the "balls and bins" system in the figure, governed by the following rules.

- A ball can move from its present bin to an adjacent bin (but not beyond that) in each time step Δt.

Diffusion



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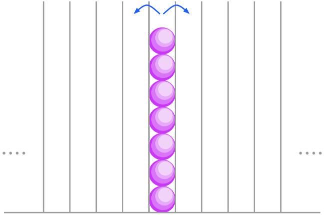

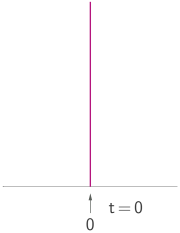

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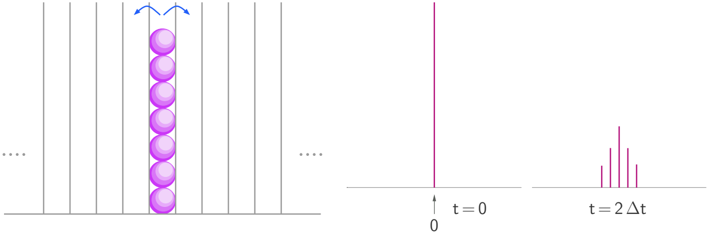

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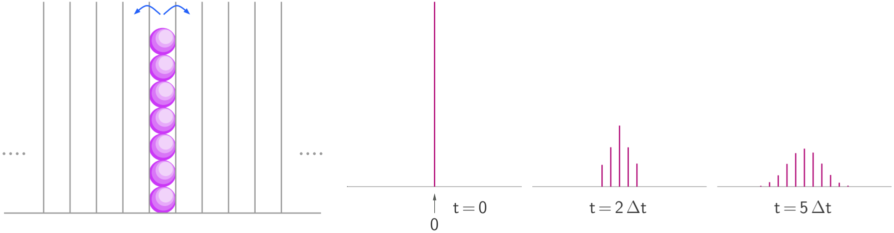

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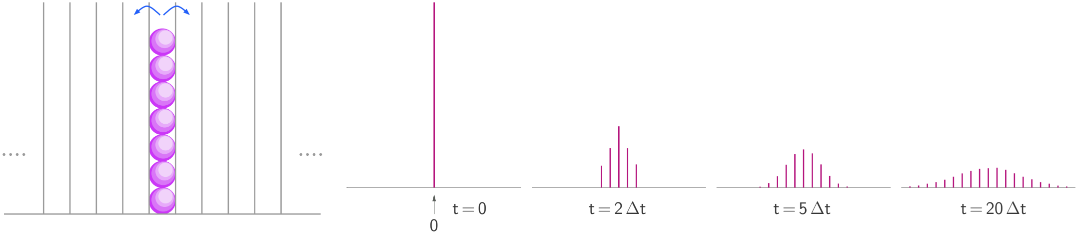

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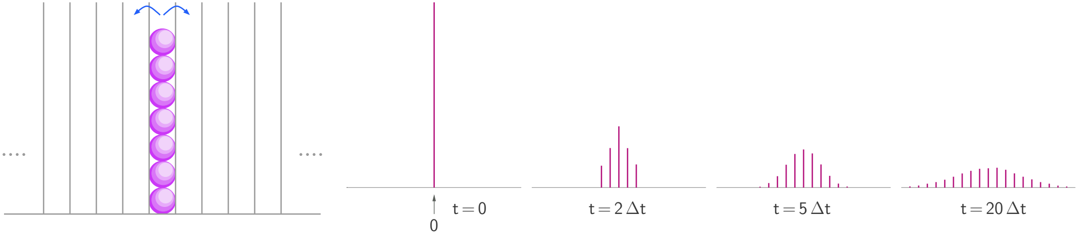

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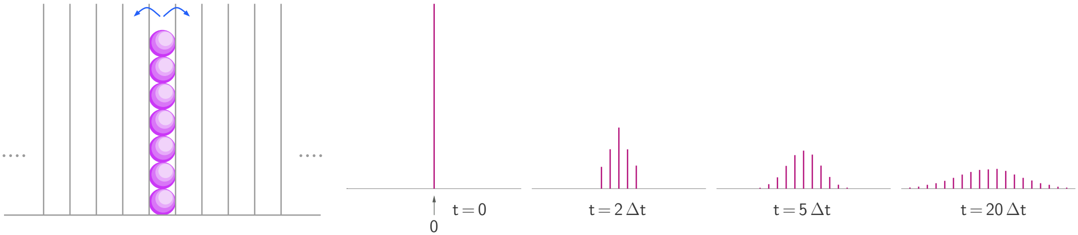

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- The driving force behind the process of diffusion is a concentration gradient.
- If all bins were equally populated, the number of balls going from bin k to an adjacent bin ($k-1$ or $k+1$) would be equal to the number of balls coming from that bin. As a result, the population of each bin would remain constant with time.

Diffusion

- The process of diffusion is described by Fick's law:

  $$\mathcal{F}_x = -D\,\frac{d\eta}{dx} \qquad (\mathsf{F} = -D\,\nabla\eta \text{ in three dimensions}),$$

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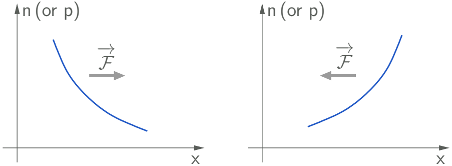

Total electron and hole current densities

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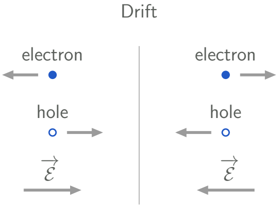

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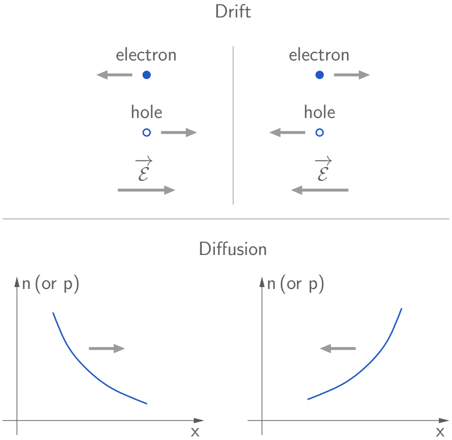

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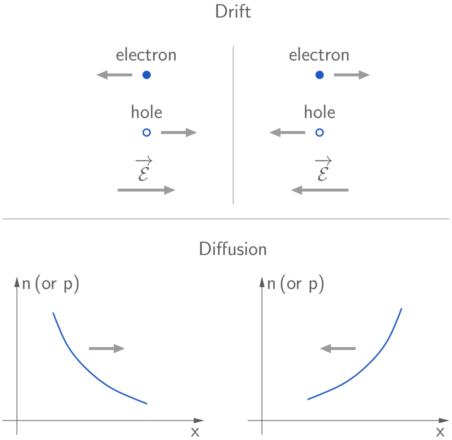

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- Current density (A/cm^2):

$$J_n = -q\,\mathcal{F}_n = qn\,\mu_n\mathcal{E} + qD_n\,\frac{dn}{dx},$$

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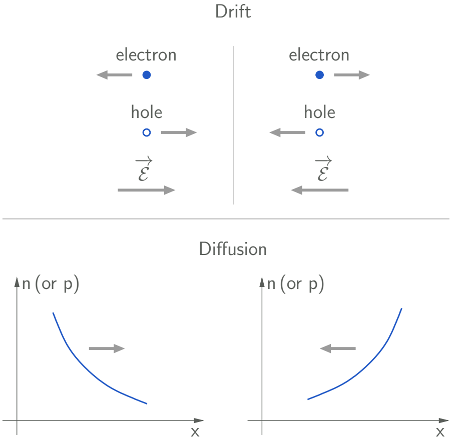
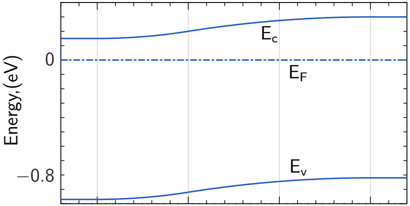


Example of equilibrium conditions

Consider a section of a silicon crystal in equilibrium, with the band diagram shown in the figure.



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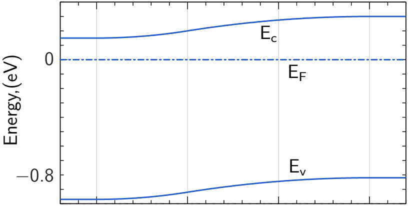

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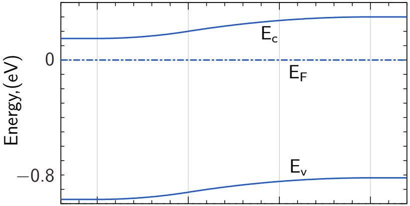

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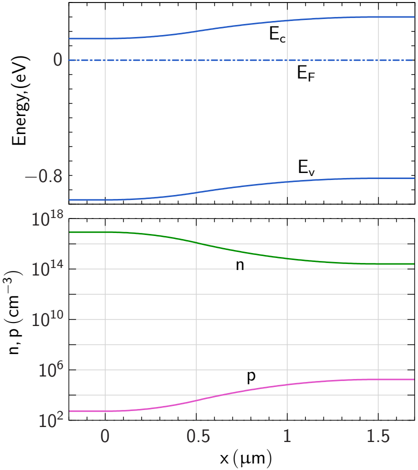

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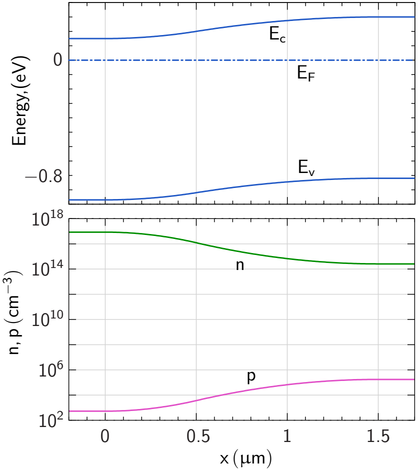

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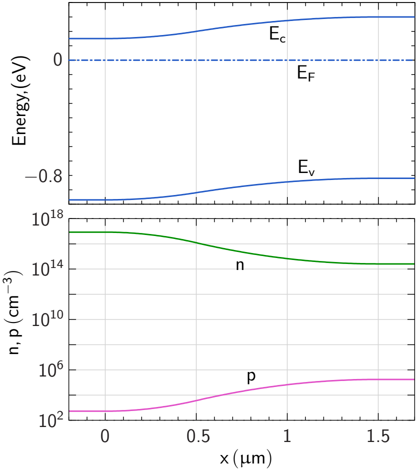
- Note that $np = n_i^2$ in equilibrium for non-degenerate conditions.



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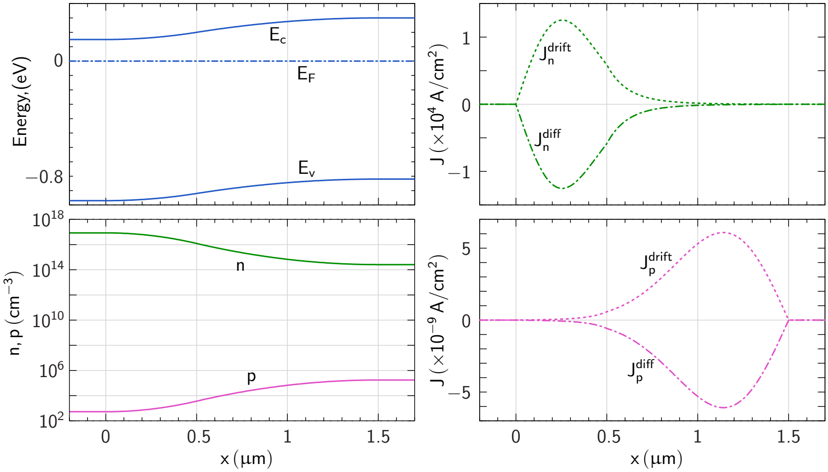

Current directions:

- J_n^{drift}: $\mathcal{E} = \dfrac{1}{q}\,\dfrac{dE_c}{dx} > 0 \rightarrow \mathcal{F}_n^{\text{drift}} < 0 \rightarrow J_n^{\text{drift}} > 0$.

  (Electrons flow "downhill" due to electric field.)

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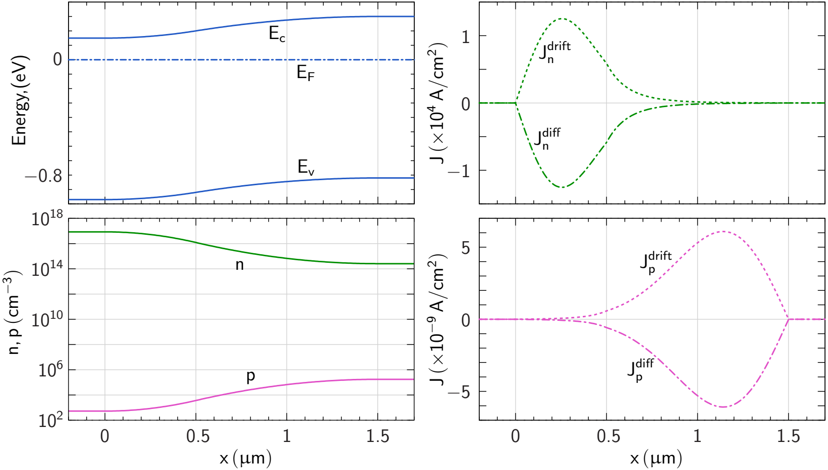

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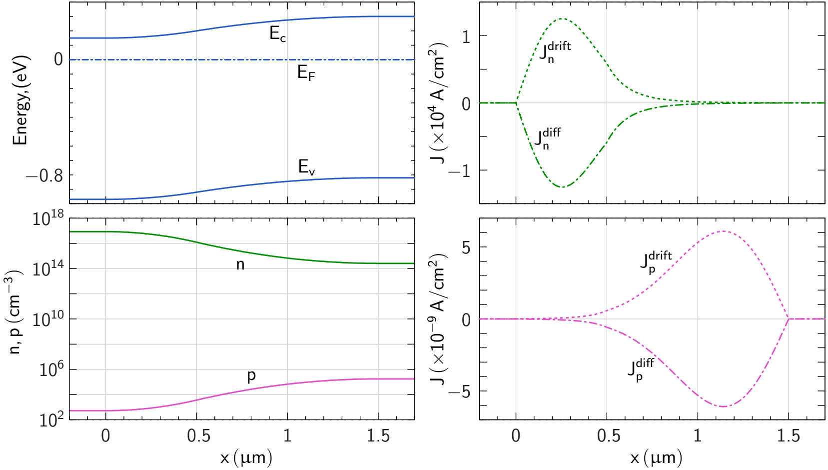

Drift current magnitudes:

- $$\frac{J_n^{\text{drift}}}{J_p^{\text{drift}}} = \frac{q\, n\, \mu_n\, \mathcal{E}}{q\, p\, \mu_p\, \mathcal{E}} = \frac{n\, \mu_n}{p\, \mu_p} \rightarrow |J_n^{\text{drift}}| \gg |J_p^{\text{drift}}|.$$

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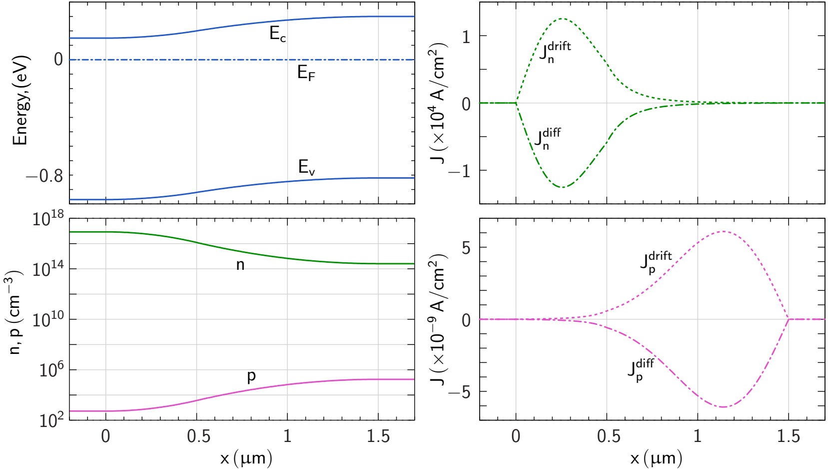

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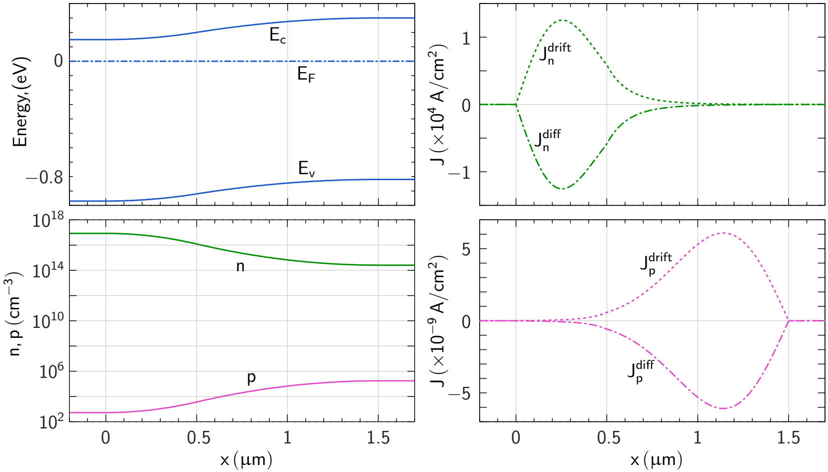

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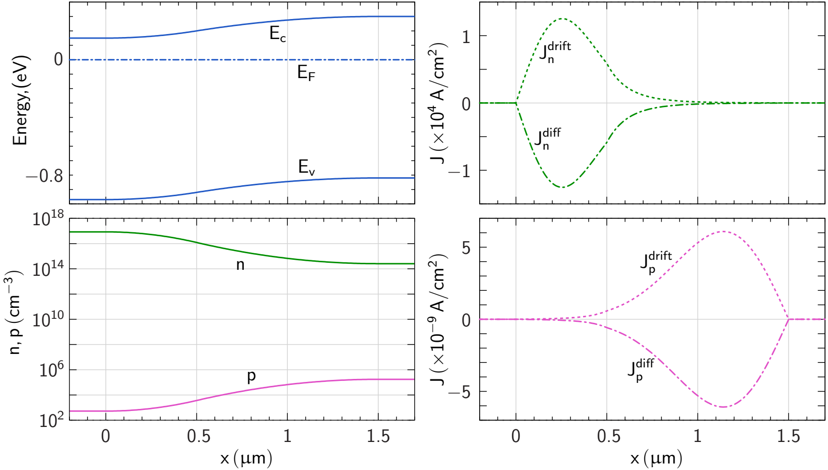
- J_n^{diff}: $\dfrac{dn}{dx} < 0 \rightarrow \mathcal{F}_n^{diff} > 0 \rightarrow J_n^{diff} < 0.$
- J_p^{diff}: $\dfrac{dp}{dx} > 0 \rightarrow \mathcal{F}_p^{diff} < 0 \rightarrow J_p^{diff} < 0.$

Example of equilibrium conditions



Detailed balance: At equilibrium, each elementary process should be equilibrated by its reverse process. (wiki)

- In equilibrium, $J^{\text{total}} = J_n + J_p = 0$.

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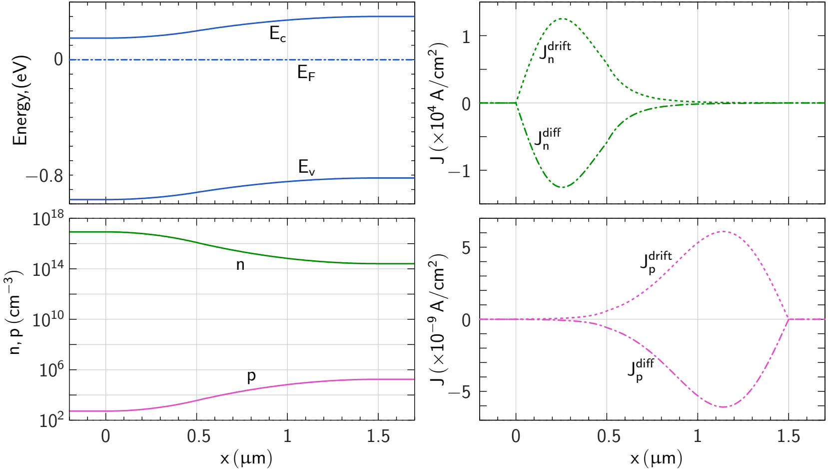

Detailed balance: At equilibrium, each elementary process should be equilibrated by its reverse process. (wiki)

- In equilibrium, $J^{\text{total}} = J_n + J_p = 0$.
- Also, $J_n = 0$ and $J_p = 0$ *individually* $\rightarrow J_n^{\text{drift}} = -J_n^{\text{diff}},\ \ J_p^{\text{drift}} = -J_p^{\text{diff}}$.

Equilibrium conditions: implication of $J_n^{\text{drift}} = -J_n^{\text{diff}}$

In a non-degenerate semiconductor in equilibrium,

$n = N_c e^{-(E_c - E_F)/kT}$,

where E_F is constant, and E_c may vary with x, as we saw in the last example.

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