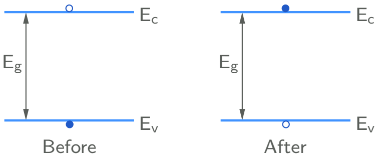
# Semiconductor Devices

## Carrier Transport: Part 2

M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering
Indian Institute of Technology Bombay

# Generation and recombination




Generation

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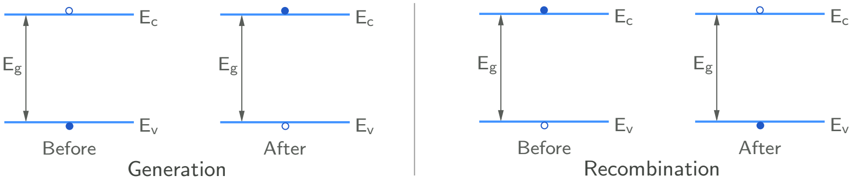

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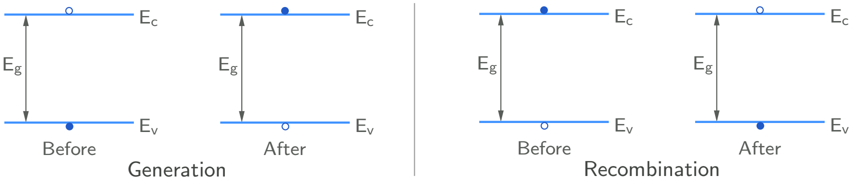


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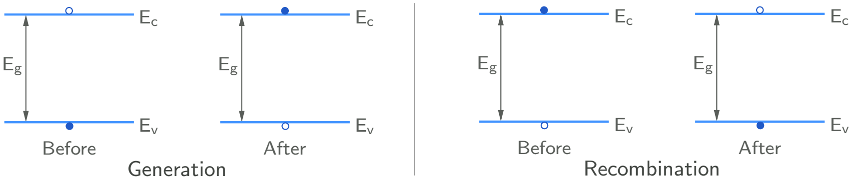
## Generation and recombination

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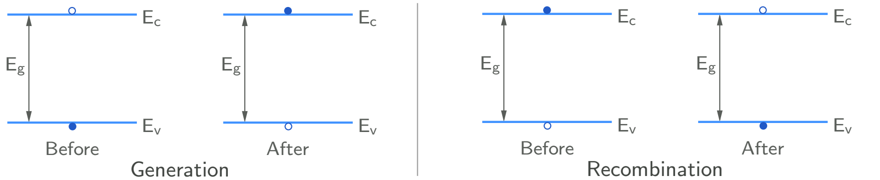


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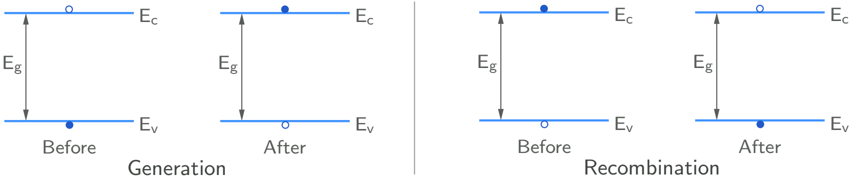


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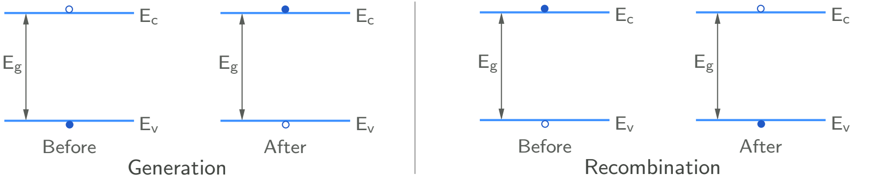


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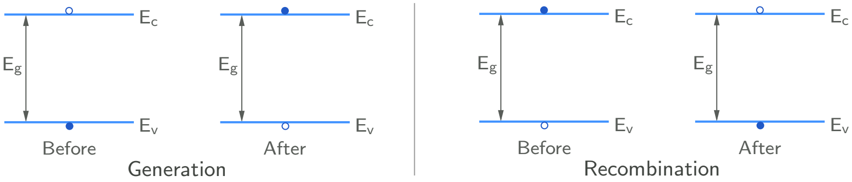


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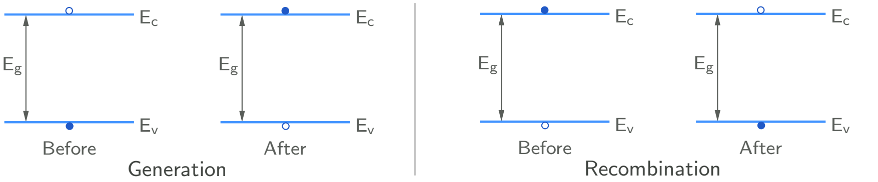


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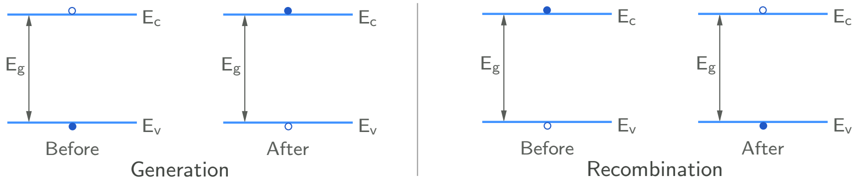


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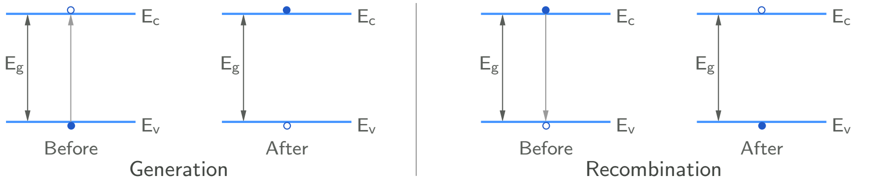
* Some of these processes, with very small rates, may be completely ineffective, while others may be dominant.

# Direct G-R




∗ Direct recombination

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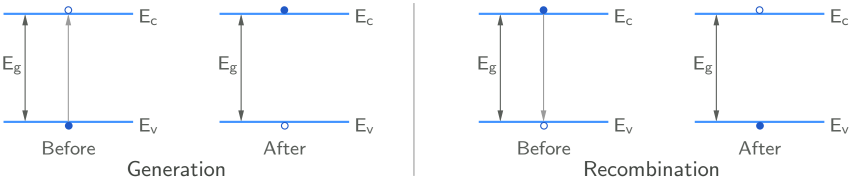


* Direct recombination
  - An electron from the conduction band combines directly with a hole in the valence band, thus destroying an EHP.

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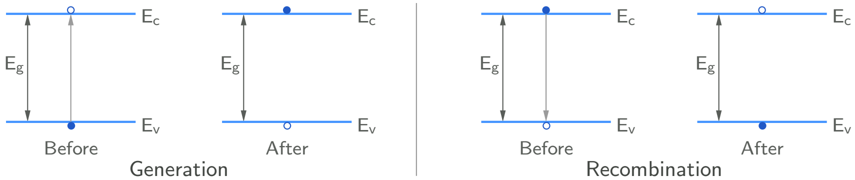


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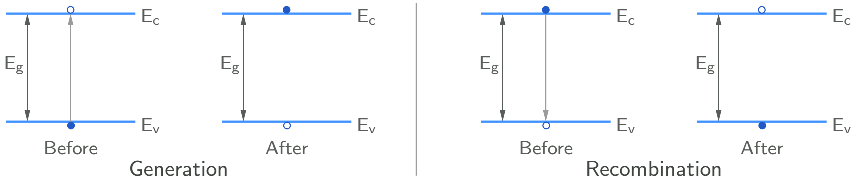


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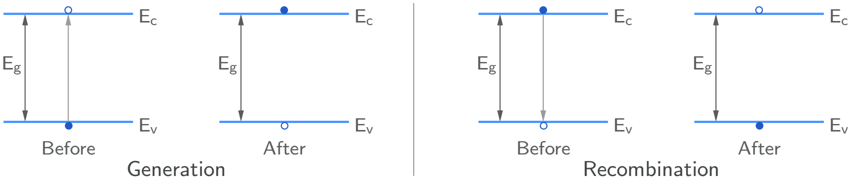


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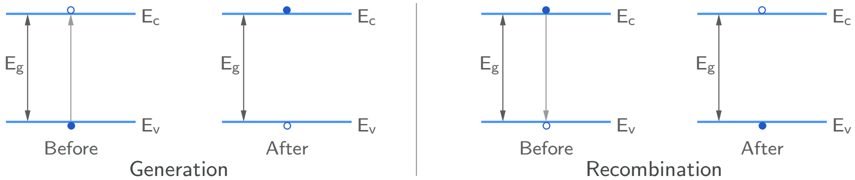


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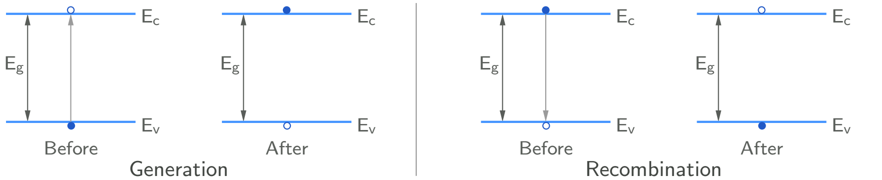


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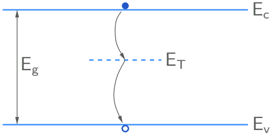
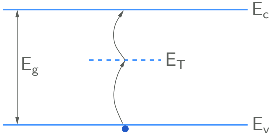
* Since particles from both condution and valence bands are simultaneously involved, the above processes are called "band-to-band" G-R.

# Indirect G-R




Recombination




Generation

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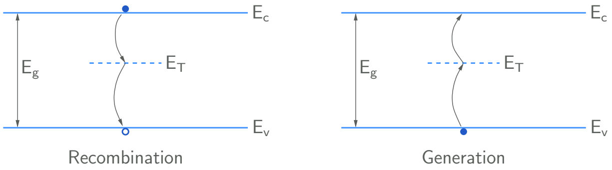


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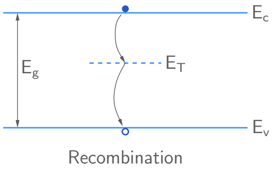


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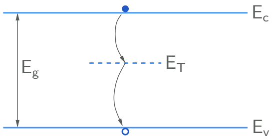


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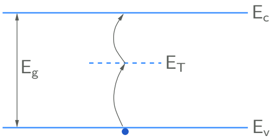


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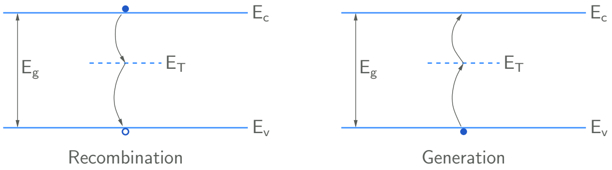


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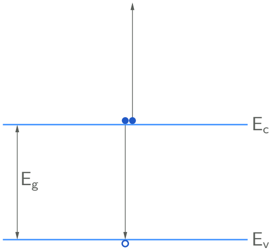
- Indirect G-R processes are particularly efficient when $E_T$ lies close to the middle of the band gap.

# G-R with three particles



Auger Recombination

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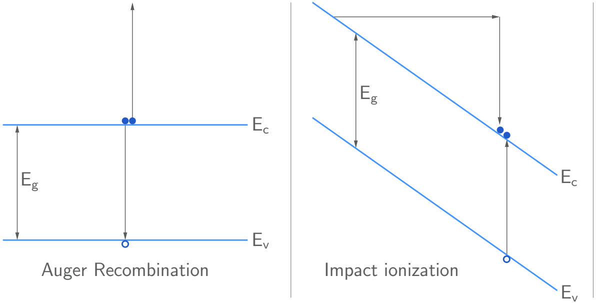


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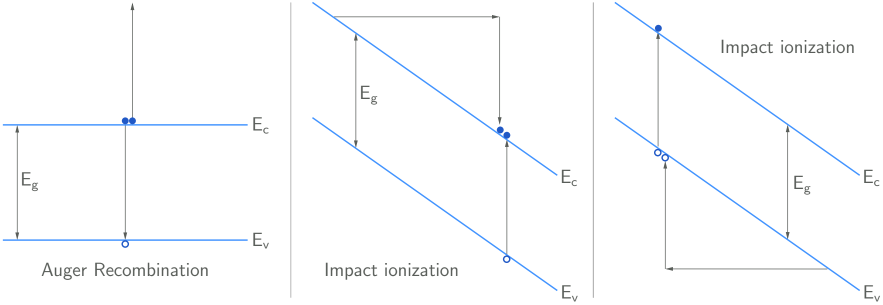


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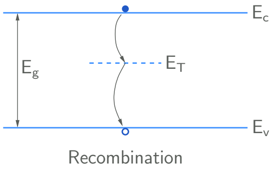


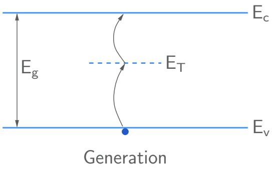


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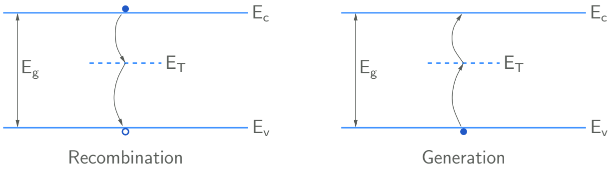


* In silicon, the dominant non-radiative G-R process is that involving a G-R centre.
* The *net* rate of recombination per unit volume is given by the Shockley–Read–Hall (SRH) formula, $R - G = \dfrac{n\,p - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$.

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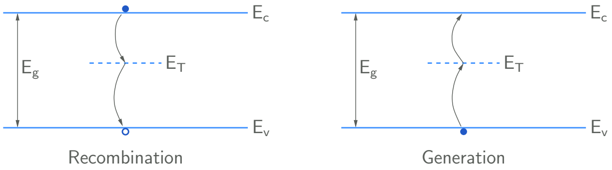


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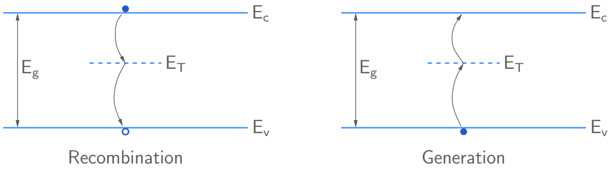


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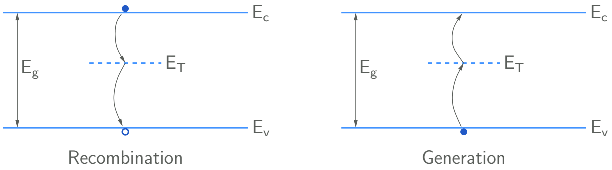


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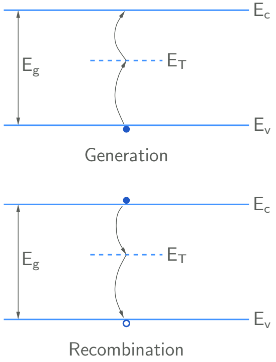
  $n_1 = n_i\, e^{(E_T - E_i)/kT}, \quad p_1 = n_i\, e^{(E_i - E_T)/kT}$.
* Since the most effective G-R centres have $E_T \approx E_i$, $n_1$ and $p_1$ are generally much smaller than the majority carrier density in a doped semiconductor.

# SRH formula: special case

$$R - G = \frac{n\,p - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}.$$

Let us consider a semiconductor in which electrons are the majority carriers, with the equilibrium values of $n$ and $p$ given by $n_0 = 10^{16}\,\text{cm}^{-3}$, $p_0 = 10^{4}\,\text{cm}^{-3}$.



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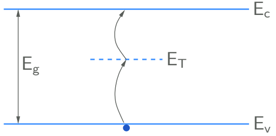
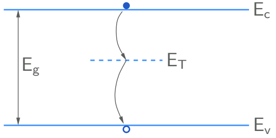


Generation



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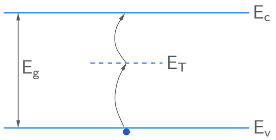


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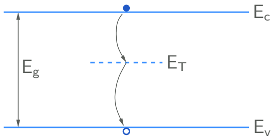


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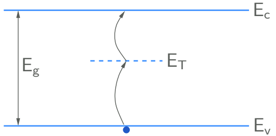

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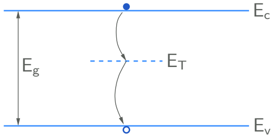

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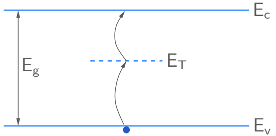
$$= \frac{p_0 \Delta n + n_0 \Delta p + \Delta n \Delta p}{\tau_p(n + n_1) + \tau_n(p + p_1)} \qquad \because n_0 p_0 = n_i^2$$

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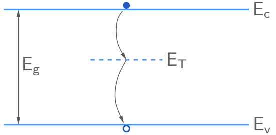


Recombination

# SRH formula: special case

$$R - G = \frac{n\,p - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}.$$

Let us consider a semiconductor in which electrons are the majority carriers, with the equilibrium values of $n$ and $p$ given by $n_0 = 10^{16}\,\text{cm}^{-3}$, $p_0 = 10^4\,\text{cm}^{-3}$.

Suppose that additional carriers are created in the semiconductor (e.g., by shining light on the sample), giving rise to "excess concentrations" $\Delta p = \Delta n = 10^{12}\,\text{cm}^{-3}$.

The total concentrations are $n = n_0 + \Delta n$ and $p = p_0 + \Delta p$,

Applying the SRH formula, we get

$$R - G = \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

$$= \frac{p_0 \Delta n + n_0 \Delta p + \Delta n \Delta p}{\tau_p(n + n_1) + \tau_n(p + p_1)} \qquad \because\ n_0 p_0 = n_i^2$$

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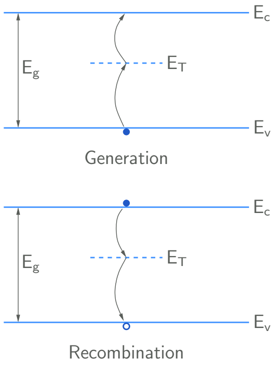

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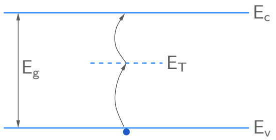

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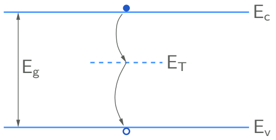

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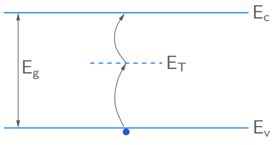


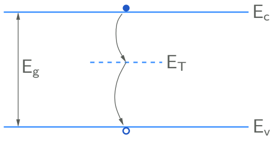

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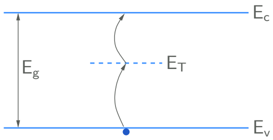

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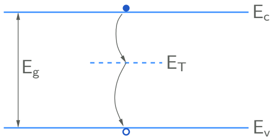

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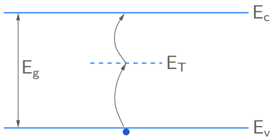
In conclusion,

* If the excess carrier densities $\Delta n$ and $\Delta p$ are small compared to the majority carrier density, the net recombination rate is governed by the <u>minority</u> carrier lifetime.



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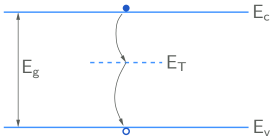

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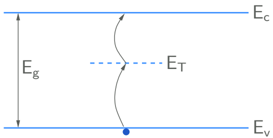
* We will find this result useful in our discussion of diodes and bipolar junction transistors.



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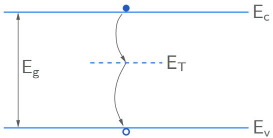

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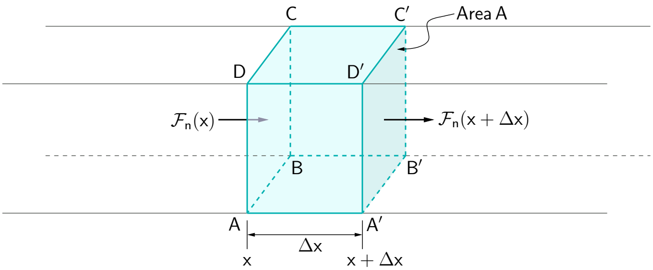
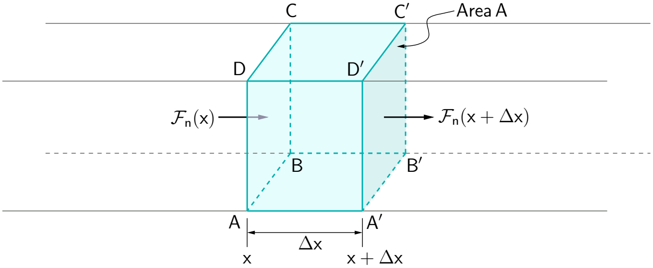



Two processes can change the number of electrons and holes in the box:

# Continuity equations

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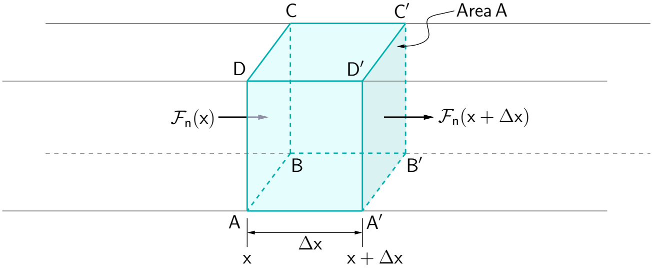


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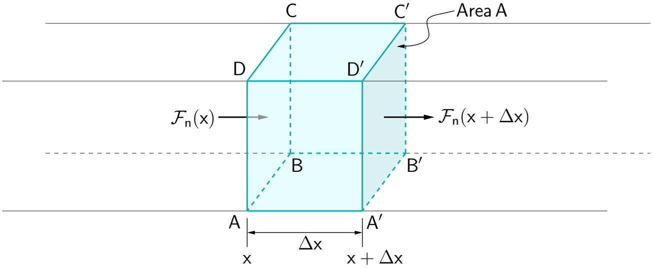


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The continuity equations serve to relate these phenomena.

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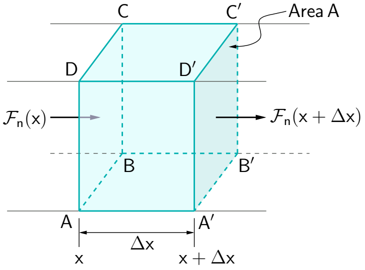

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The number of electrons in the box can change because of the following factors.

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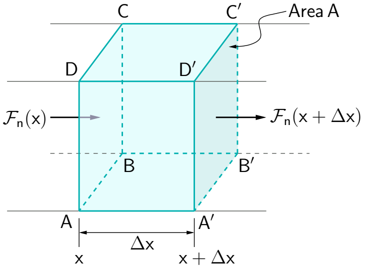

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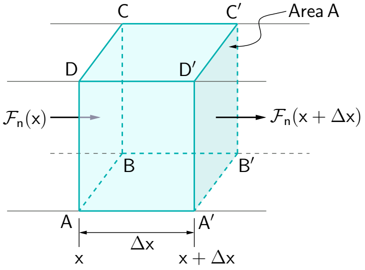

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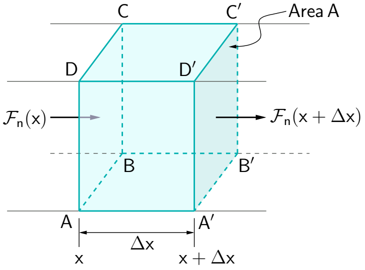

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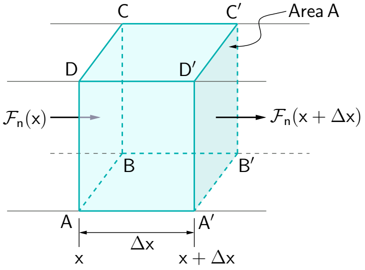
We can now relate the above factors to $\dfrac{\partial n}{\partial t}$:

$$(A\,\Delta x)\ \frac{\partial n}{\partial t} = \mathcal{F}_n(x)\,A - \mathcal{F}_n(x+\Delta x)\,A - (R-G)\,A\,\Delta x$$



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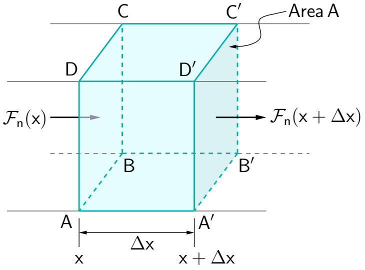


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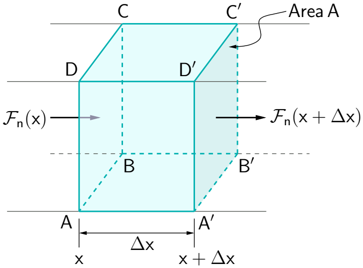

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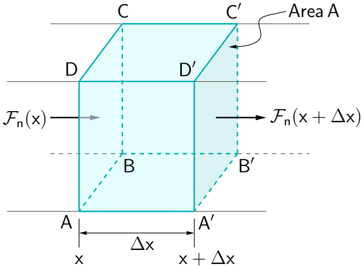

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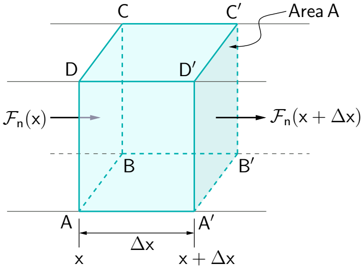

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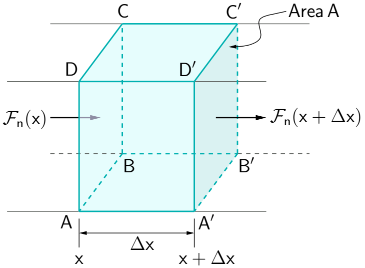

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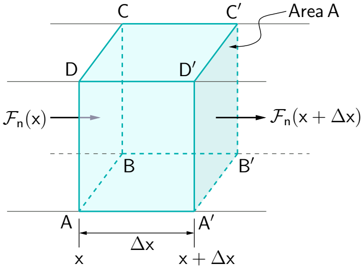

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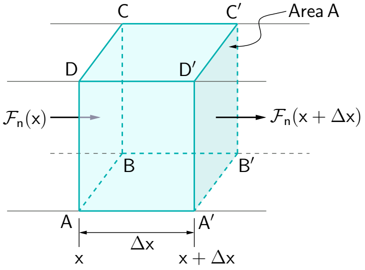


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# How does a semiconductor device simulator work?

A semiconductor device simulator solves the following coupled differential equations in a self-consistent manner:

Poisson's equation: $$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon} \quad \rightarrow \quad \frac{\partial^2 \psi}{\partial x^2} = -\frac{\rho}{\epsilon},$$

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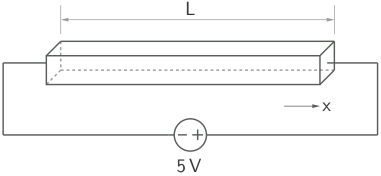
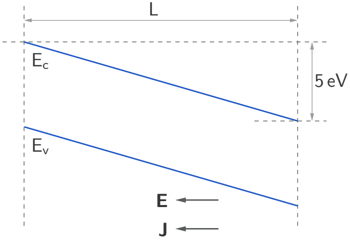
The general problem is very complex and needs to be solved numerically.

However, we can gain significant insight by considering examples which represent situations in real semiconductor devices.




$L = 50\ \mu m$

$N_d = 5 \times 10^{17}\ cm^{-3}$

$T = 300\ K\ (n_i = 10^{10}\ cm^{-3})$

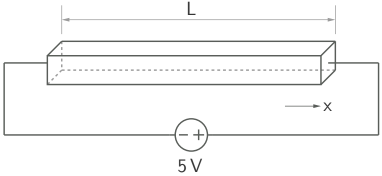
Solution:

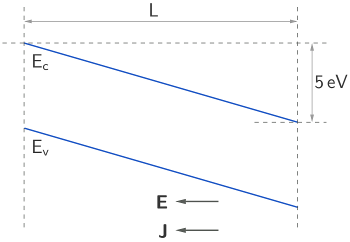
(a) $\mathcal{E} = -\dfrac{d\psi}{dx} = -1\ \dfrac{\text{kV}}{\text{cm}}$, i.e., $\psi(x) = -\mathcal{E}x + \text{constant}$, say $\psi(x) = -\mathcal{E}x$.

(b) $n \approx N_d = 5 \times 10^{17}\ \text{cm}^{-3}$ for $0 < x < L$.

(c) $\mathcal{E} = 1\ \text{kV/cm}$, $J \approx J_n^{\text{drift}} = 3.2 \times 10^4\ \text{A/cm}^2$ ($J_p^{\text{drift}} \approx 0$, assuming $p$ to be negligibly small).

# Silicon bar example (revisited): consistency check




$L = 50\,\mu\text{m}$

$N_d = 5 \times 10^{17}\,\text{cm}^{-3}$

$T = 300\,\text{K}$ $(n_i = 10^{10}\,\text{cm}^{-3})$

Solution:

(a) $\mathcal{E} = -\dfrac{d\psi}{dx} = -1\,\dfrac{\text{kV}}{\text{cm}}$, i.e., $\psi(x) = -\mathcal{E}x + \text{constant}$, say $\psi(x) = -\mathcal{E}x$.

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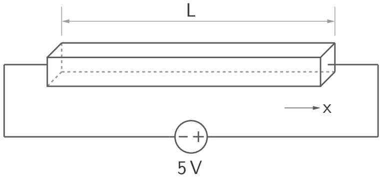


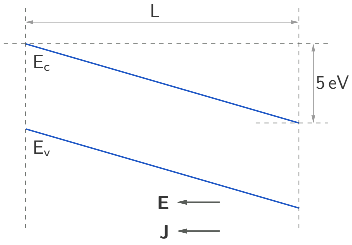


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  i.e., $\rho = q(N_d^+ - n + p)$ must be zero, which is satisfied by our solution.

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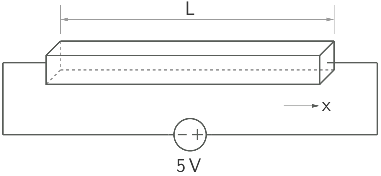


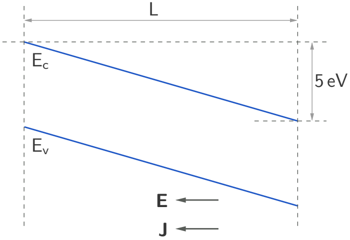


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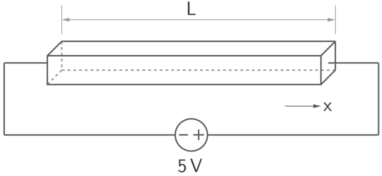


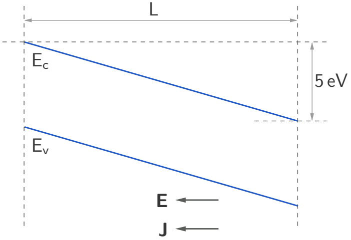


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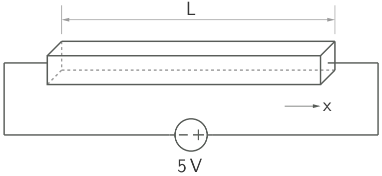


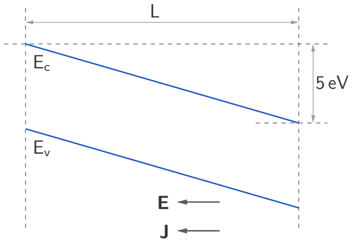


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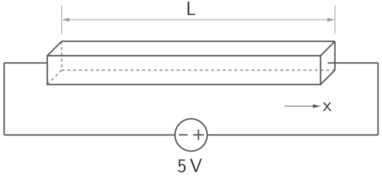


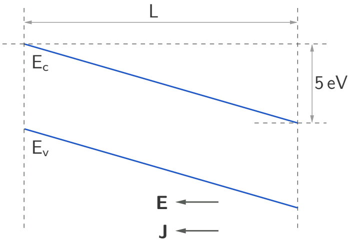


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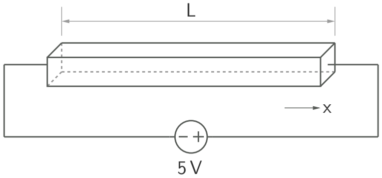


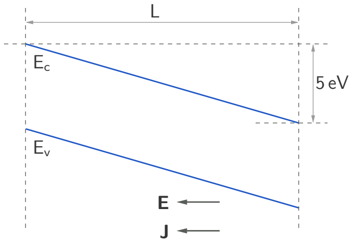


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  We see that $p$ is indeed negligibly small.

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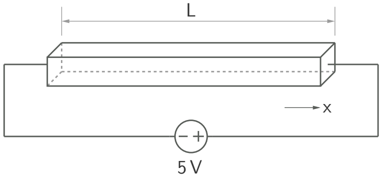


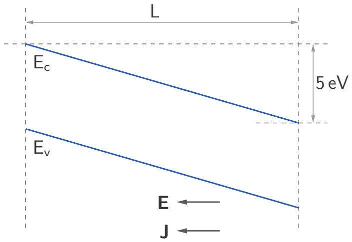


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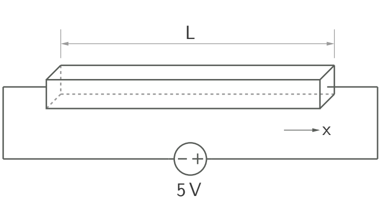


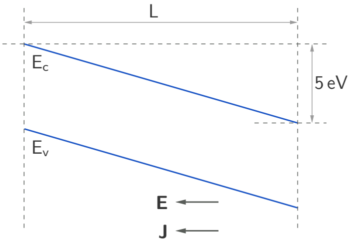


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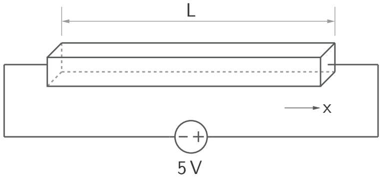


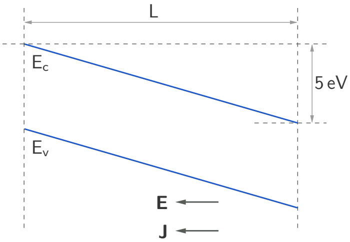


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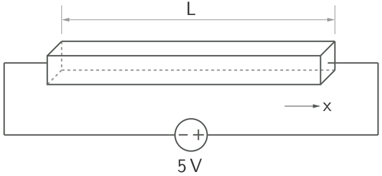


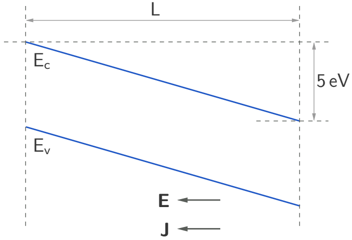


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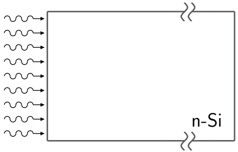
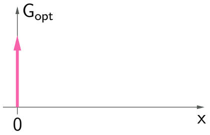
$$\frac{J_p}{J_n} \approx \frac{J_p^{\text{drift}}}{J_n^{\text{drift}}} = \frac{\mu_p}{\mu_n}\frac{p_0}{n_0} = \frac{\mu_p}{\mu_n}\frac{n_i^2}{n_0^2} = \frac{\mu_p}{\mu_n}\left(\frac{10^{10}}{5\times10^{17}}\right)^2 = \frac{\mu_p}{\mu_n}\times 4\times10^{-16}, \text{ really small!}$$

Consider an *n*-type silicon sample with $N_d = 10^{17}\,\text{cm}^{-3}$. Light is (continuously) incident on its surface, resulting in an optical generation rate shown in the figure.

(We are assuming here that the light is entirely absorbed in a very thin region near the semiconductor surface ($x = 0$) and does not penetrate deeper.)



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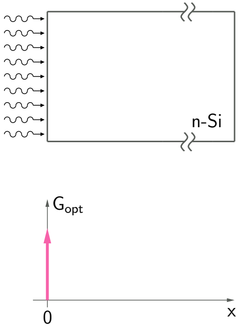

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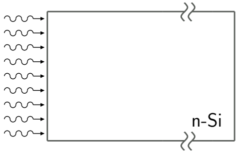
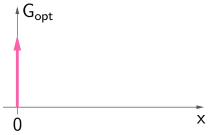
Solve the continuity equation for holes and obtain $\Delta p(x)$. ($T = 300\,\text{K}$)



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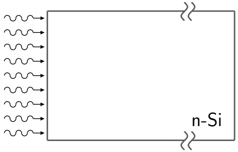


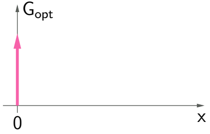


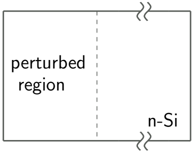

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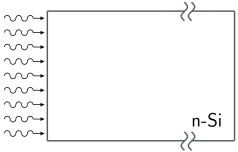


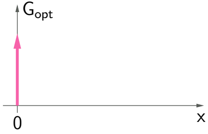


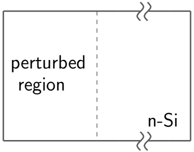

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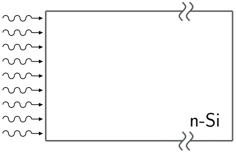


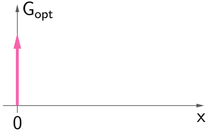


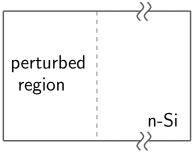

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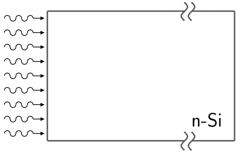
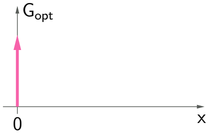
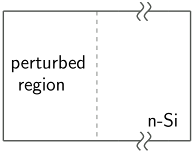
* Since only one end of the semiconductor is perturbed, we expect a region with a deviation from equilibrium conditions. We do not know at this point the extent of this region.
* We expect $p(x \rightarrow \infty) = p_0$, i.e., $\Delta p(x \rightarrow \infty) \equiv p(x \rightarrow \infty) - p_0 = 0$.
* At the surface $(x = 0)$, EHPs are continuously generated; therefore, we expect some excess hole concentration there, i.e., $p(0) = p_0 + \Delta p_1$.
* We assume steady-state situation in which all quantities have settled to their steady-state forms, not varying with time.

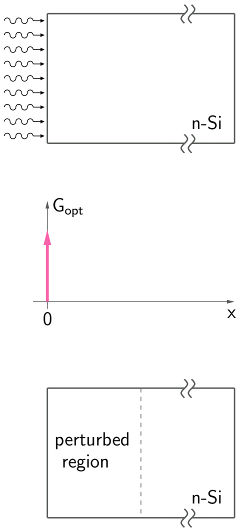


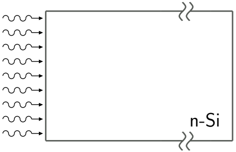
# Example

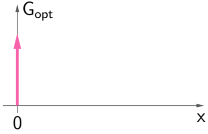
* Continuity equation for holes: $\dfrac{\partial p}{\partial t} = -\dfrac{\partial \mathcal{F}_p}{\partial x} - (R - G) = 0.$

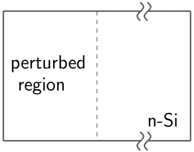


# Example

* Continuity equation for holes: $\dfrac{\partial p}{\partial t} = -\dfrac{\partial \mathcal{F}_p}{\partial x} - (R - G) = 0.$
* In steady state, the number of holes generated at $x = 0$ per second is equal to the total number of holes lost per second due to recombination.



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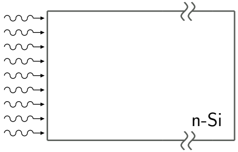
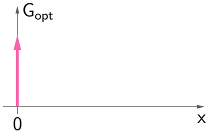
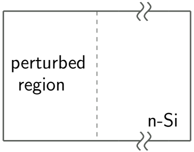
* In steady state, the number of holes generated at $x = 0$ per second is equal to the total number of holes lost per second due to recombination.
* Because of diffusion and recombination, the excess hole concentration decreases from $\Delta p_1$ at $x = 0$ to 0 at $x = \infty$.



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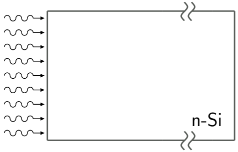
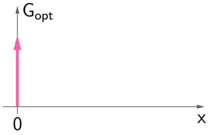
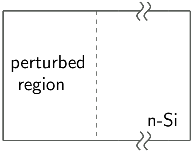
* Because of diffusion and recombination, the excess hole concentration decreases from $\Delta p_1$ at $x = 0$ to 0 at $x = \infty$.
* Since $\Delta p_1 (= 10^{10}\,\text{cm}^{-3}) \ll n_0 (= 10^{17}\,\text{cm}^{-3})$, we can approximate $(R - G)$ with $\Delta p/\tau_p$.



## Example

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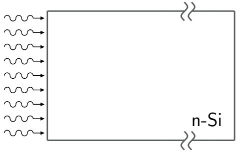
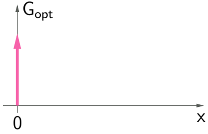
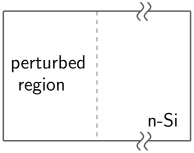
- Since $\Delta p_1 (= 10^{10}\,\text{cm}^{-3}) \ll n_0 (= 10^{17}\,\text{cm}^{-3})$, we can approximate $(R - G)$ with $\Delta p / \tau_p$.
- Let us assume that $J_p^{\text{drift}} \ll J_p^{\text{diff}}$ (to be verified later)

  $\rightarrow \mathcal{F}_p \approx \mathcal{F}_p^{\text{diff}} = -D_p \dfrac{\partial p}{\partial x} = -D_p \dfrac{\partial (p_0 + \Delta p)}{\partial x} = -D_p \dfrac{\partial \Delta p}{\partial x}.$



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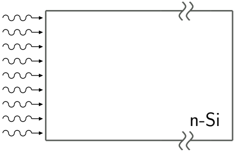


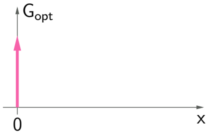


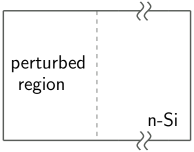

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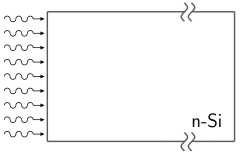


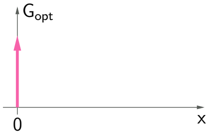


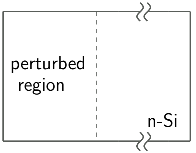

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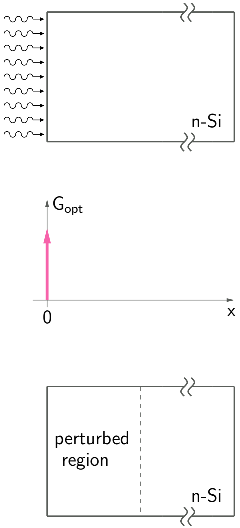
The quantity $\sqrt{D_p \tau_p}$ has units of $\sqrt{\dfrac{\mathrm{cm}^2}{\mathrm{s}} \times \mathrm{s}} = \mathrm{cm}$ and is called the "hole diffusion length" $L_p$ — also, in this case, the "minority carrier diffusion length" since holes are the minority carriers.



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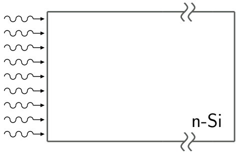


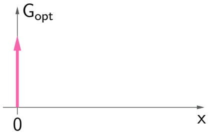


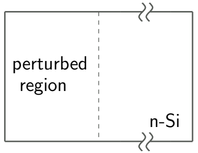

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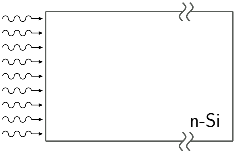


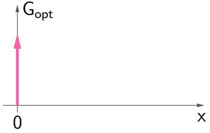


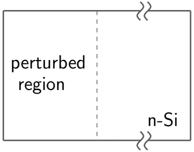

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Note:

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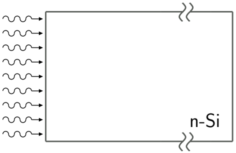


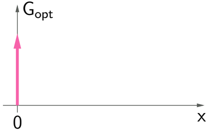


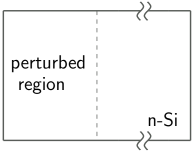

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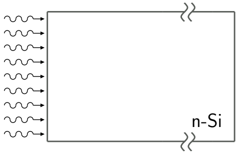


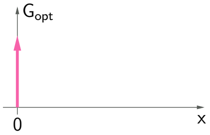


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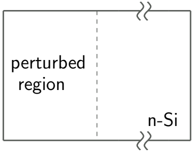
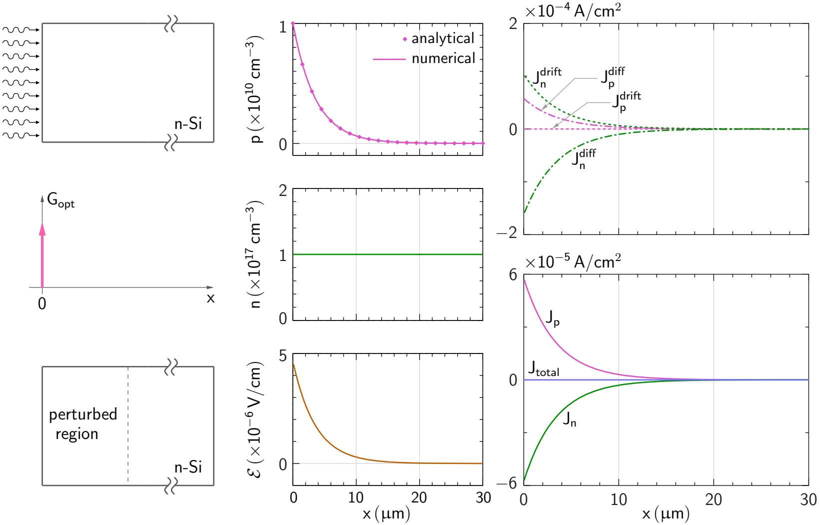



Note:

* Our solution is valid provided $J_p^{\text{drift}}$ is small compared to $J_p^{\text{diff}}$.
* We could not have solved the continuity equations for electrons as easily since $J_n^{\text{drift}}$ cannot be ignored; even a small electric field causes a significant $J_n^{\text{drift}}$ because $n$ is large.




$N_d = 10^{17}\ \mathrm{cm}^{-3}$

$\mu_n = 1400\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

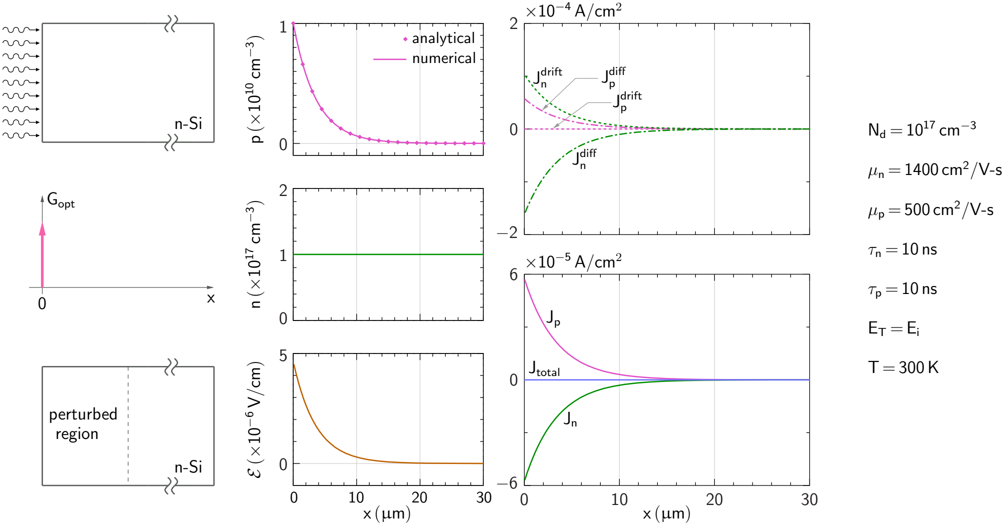
$\mu_p = 500\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

$\tau_n = 10\ \mathrm{ns}$

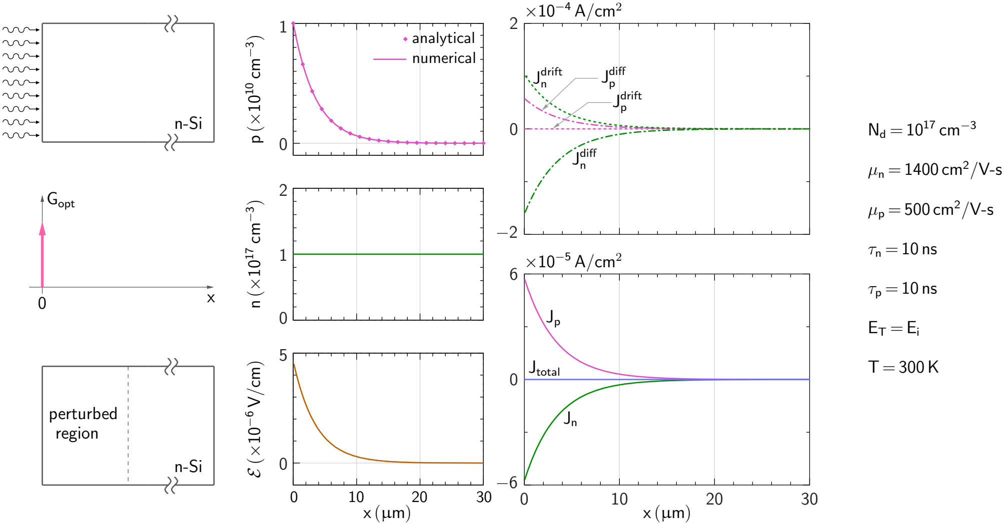
$\tau_p = 10\ \mathrm{ns}$

$E_T = E_i$

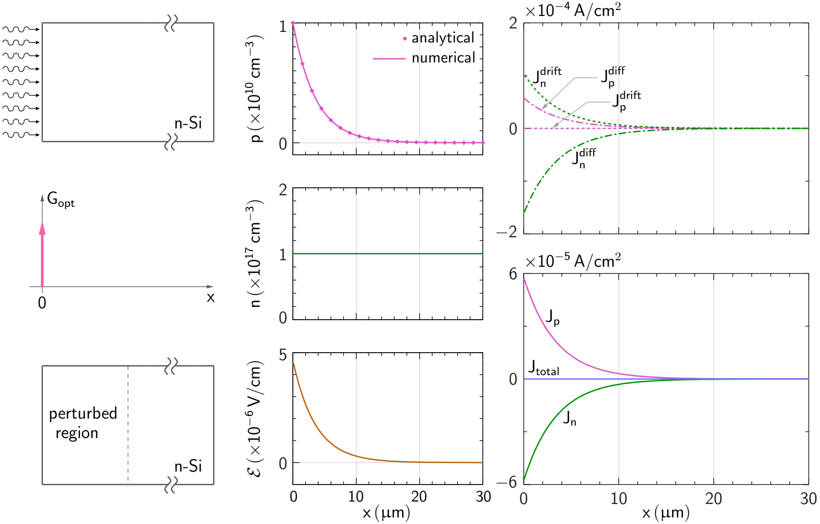
$T = 300\ \mathrm{K}$




* The analytical solution $\Delta p = \Delta p_1 e^{-x/L_p}$ matches very well with the numerical solution.




* The analytical solution $\Delta p = \Delta p_1 e^{-x/L_p}$ matches very well with the numerical solution.
* $J_p^{\text{drift}}$ is negligibly small, as we had assumed.

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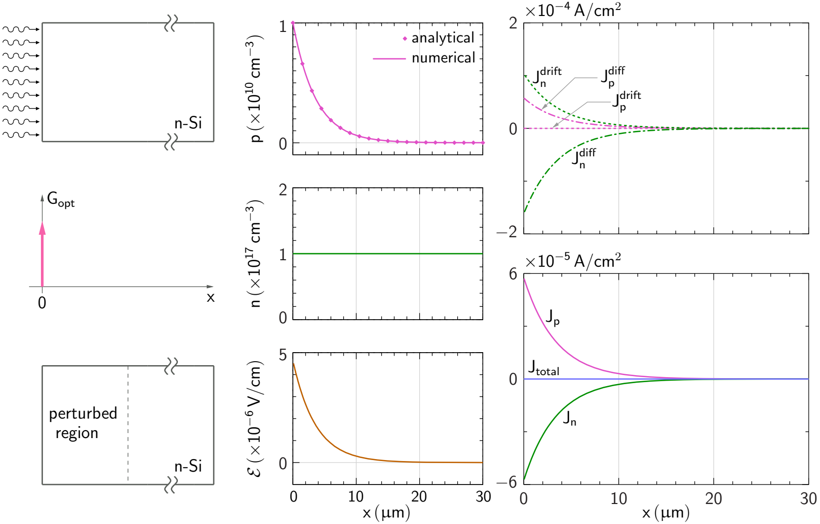


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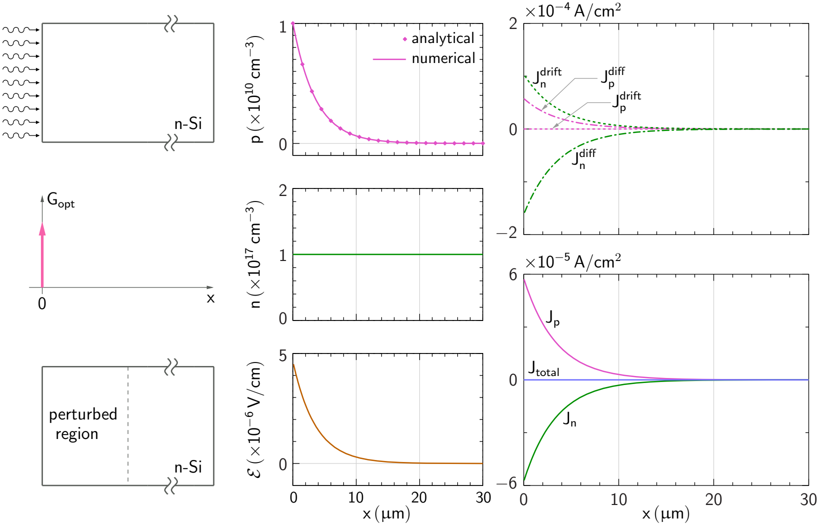
* $L_p = \sqrt{D_p \tau_p} = \sqrt{\mu_p\, V_T\, \tau_p} = \sqrt{500\ \dfrac{cm^2}{V\text{-}s} \times 0.0258\ V \times 10 \times 10^{-9}\ s} = 3.6 \times 10^{-4}\ cm = 3.6\ \mu m.$

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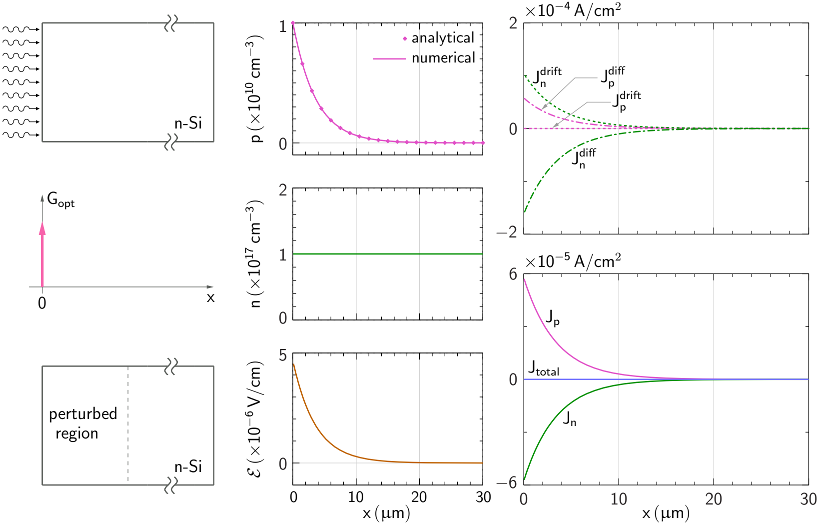
We expect the length of the perturbed region to be about $5\,L_p$ or $18\,\mu$m.

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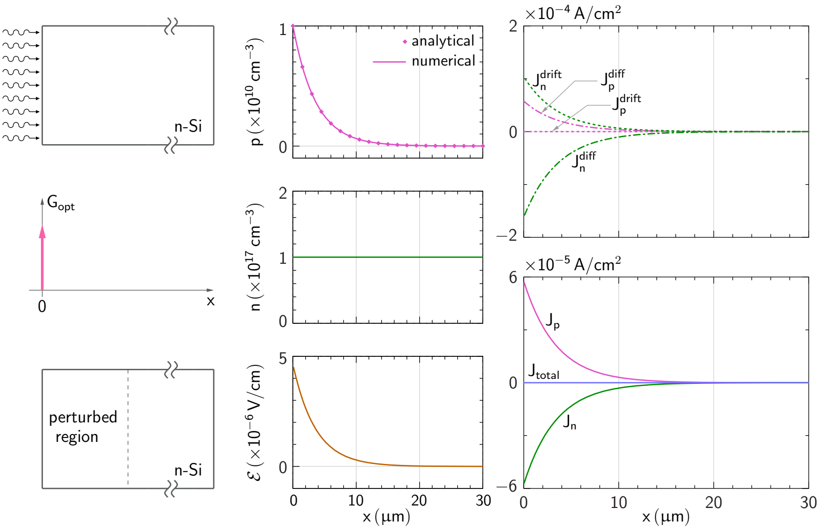


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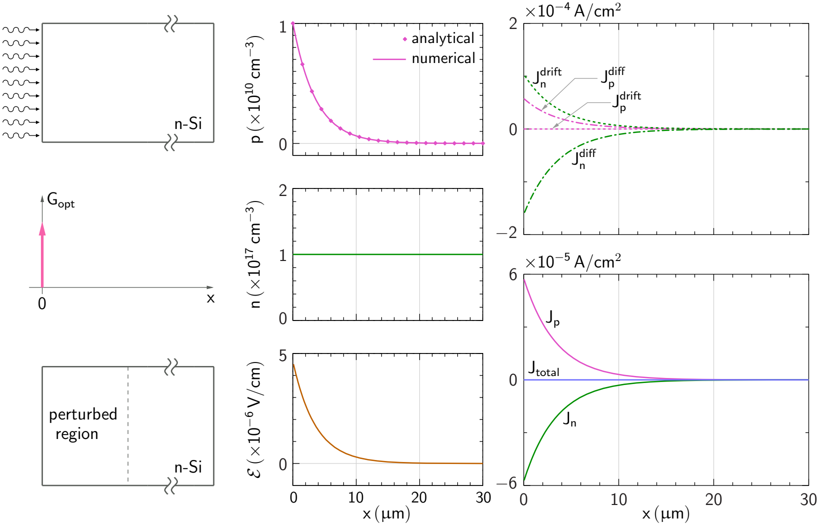
$T = 300\ \mathrm{K}$

* The condition $\Delta p \ll n$ (in general, the excess minority carrier concentration being much smaller than the majority carrier concentration) is called "low-level injection," i.e., injection of a small number of minority carriers in a sea of majority carriers.




$N_d = 10^{17}\ \mathrm{cm}^{-3}$

$\mu_n = 1400\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

$\mu_p = 500\ \mathrm{cm}^2/\mathrm{V\text{-}s}$

$\tau_n = 10\ \mathrm{ns}$

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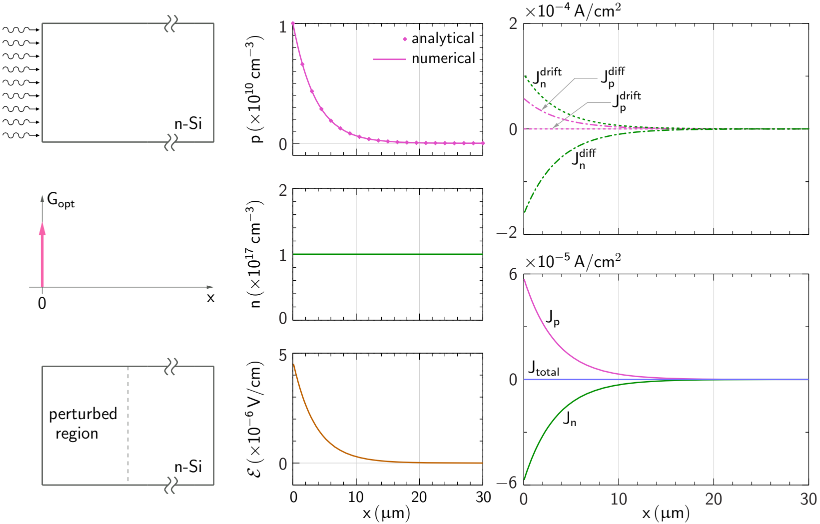
$E_T = E_i$

$T = 300\ \mathrm{K}$

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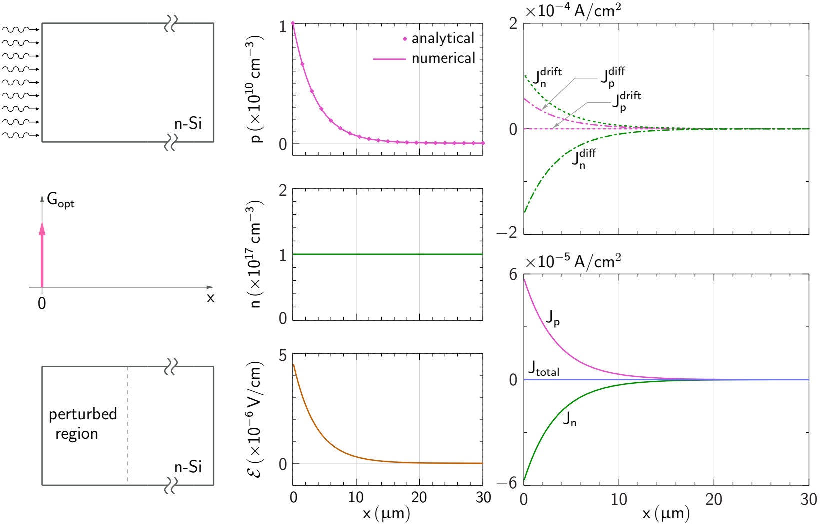
* The electron density is essentially constant ($= n_0$), but there is a small change in $n$ with respect to $x$ which gives rise to a non-zero $J_n^{\text{diff}}$. $\Delta n(x) \approx \Delta p(x)$ (not shown).

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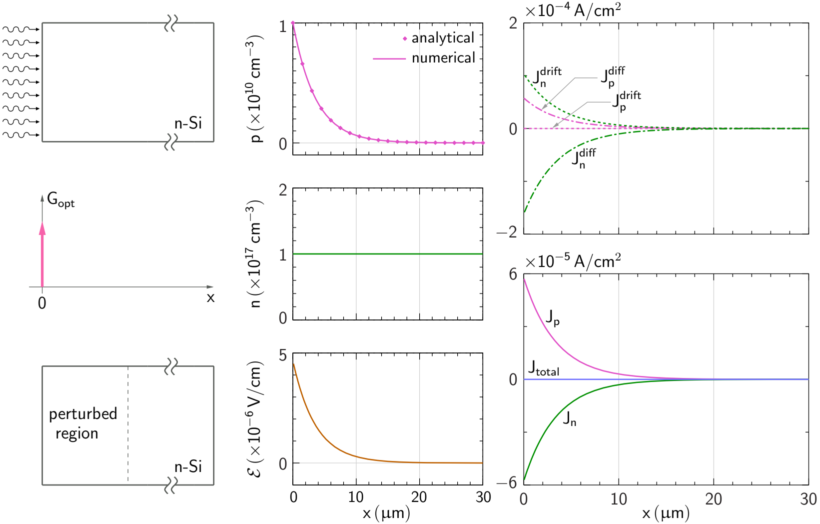
* The electric field is very small, but it is sufficient to cause a significant $J_n^{\text{drift}}$ because $n$ is large.

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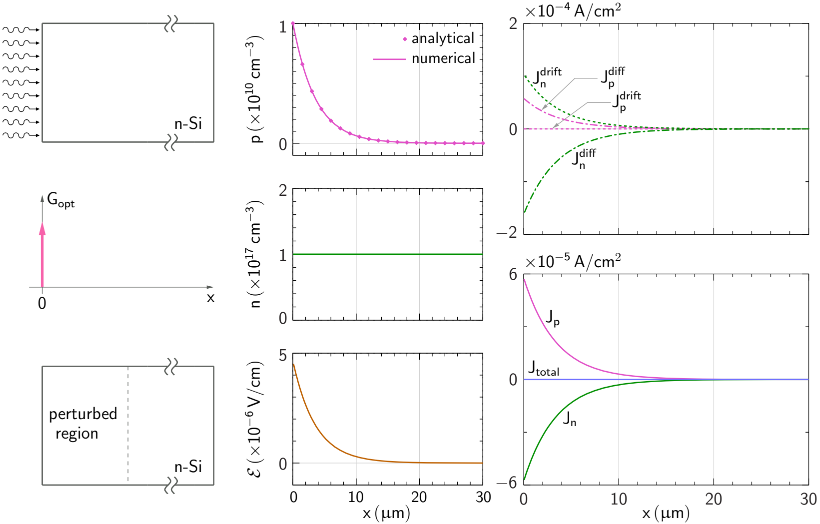


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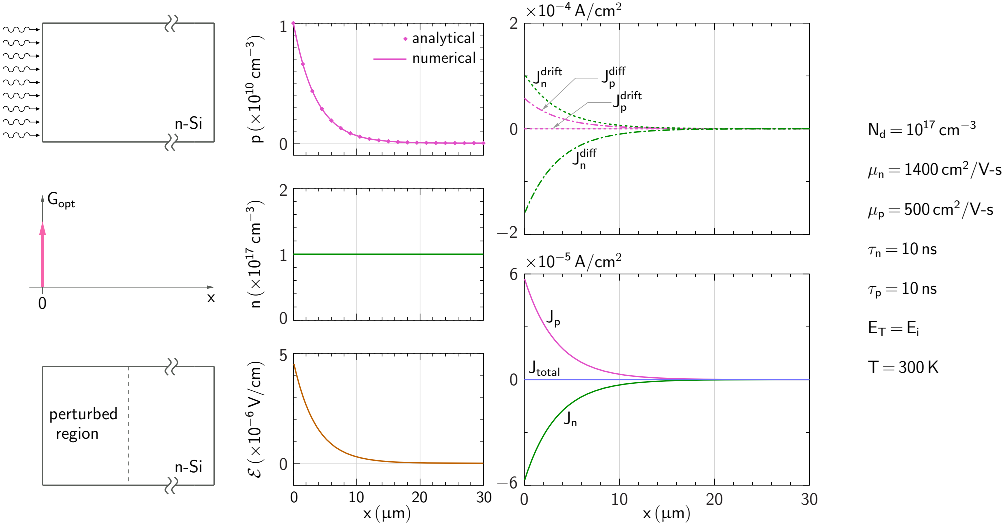


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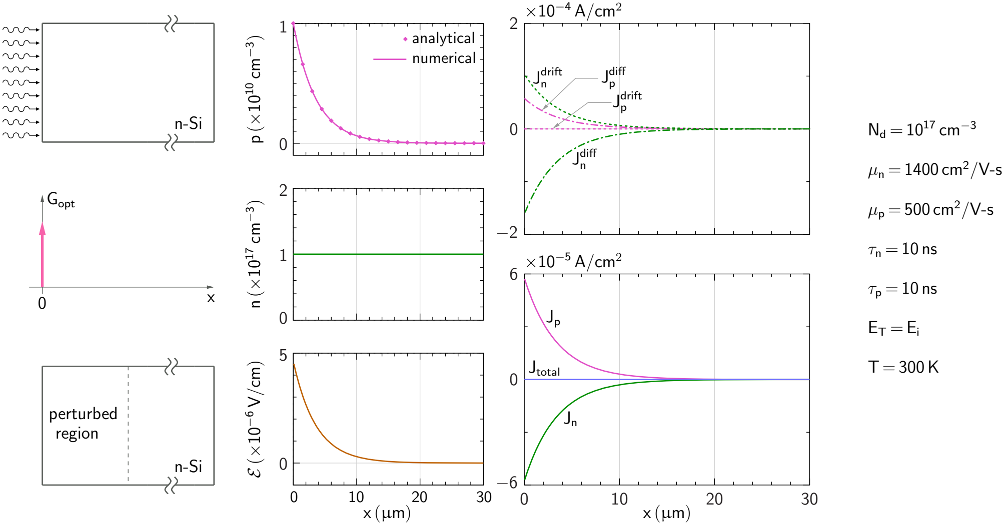
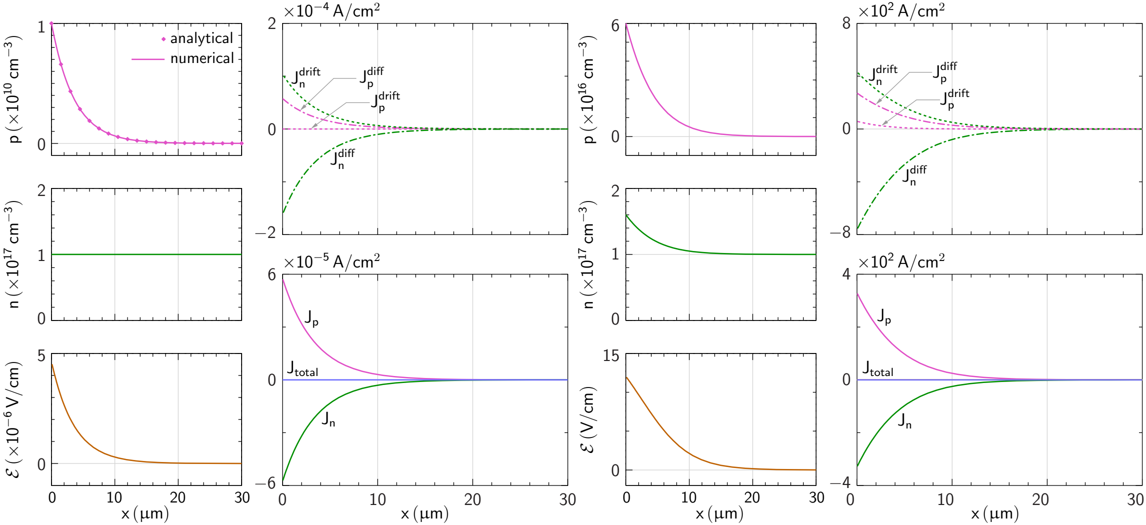
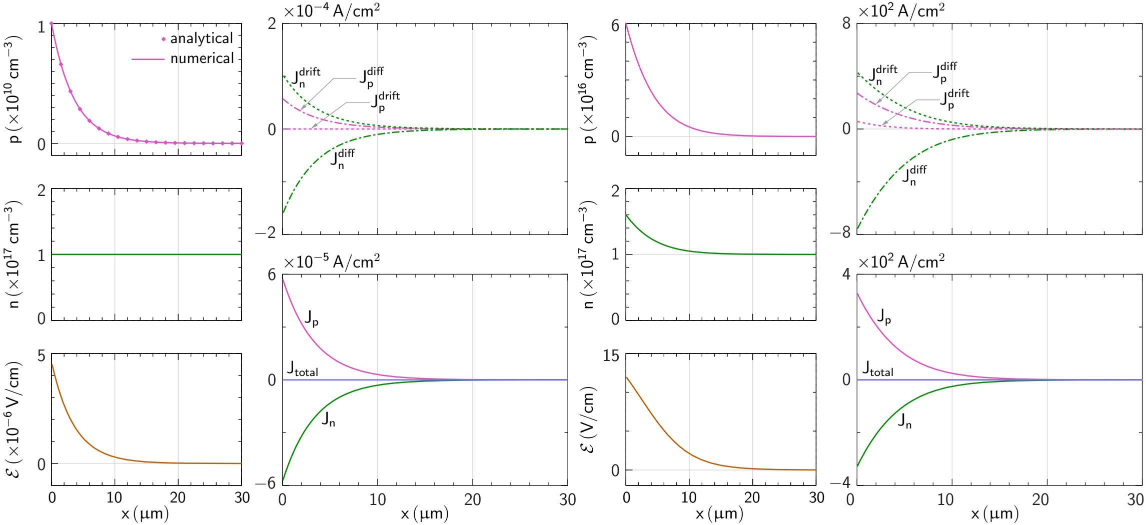


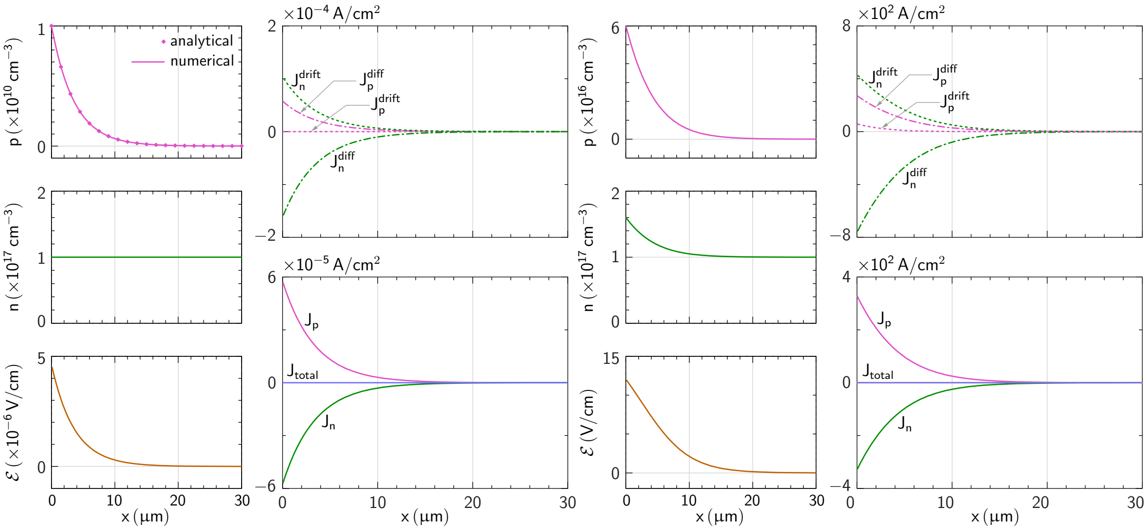

* The total current $J = J_n + J_p$ is zero throughout since we have an open-circuit condition.
* The voltage drop ($\int \mathcal{E} dx$ in magnitude) is very small.
* To summarise, $\Delta p \ll n_0$, $J_p \approx J_p^{\text{diff}}$, $\Delta n(x) \approx \Delta p(x) \rightarrow$ charge neutrality $\rightarrow$ small $\mathcal{E}$, and $J^{\text{total}} = 0$.



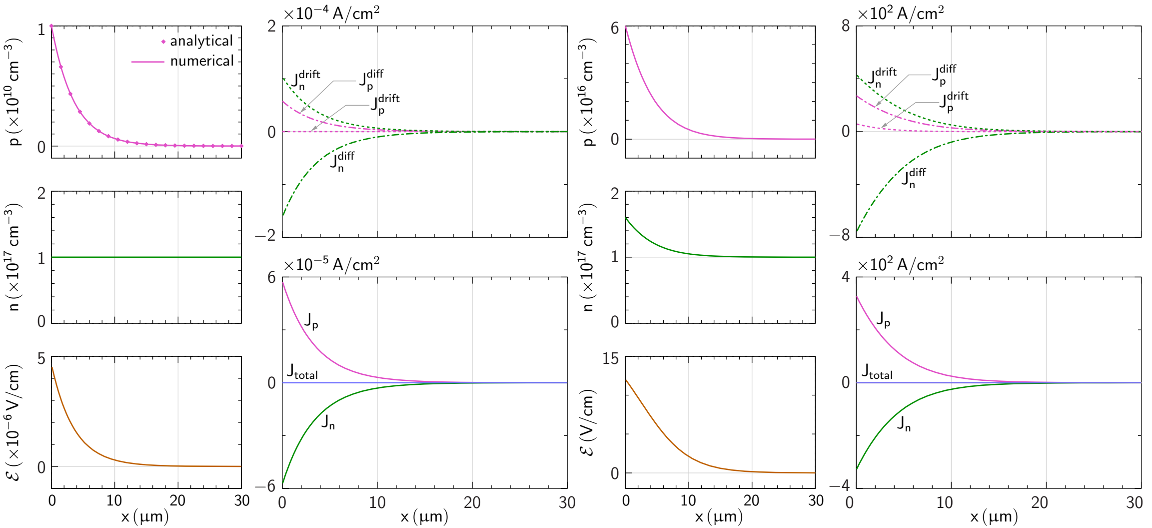
analytical
numerical
p (×10^10 cm^−3)
n (×10^17 cm^−3)
ℰ (×10^−6 V/cm)
×10^−4 A/cm^2
×10^−5 A/cm^2
p (×10^16 cm^−3)
n (×10^17 cm^−3)
ℰ (V/cm)
×10^2 A/cm^2
×10^2 A/cm^2
J_n^drift
J_p^diff
J_p^drift
J_n^diff
J_p
J_total
J_n
x (µm)




* Suppose $G_{opt}$ is increased such that $\Delta p_1 = 6 \times 10^{16}\ \text{cm}^{-3}$. $\Delta p(0)$ is now comparable to the majority carrier density, and we have a high-level injection situation.




* Suppose $G_{\text{opt}}$ is increased such that $\Delta p_1 = 6 \times 10^{16}\ \text{cm}^{-3}$. $\Delta p(0)$ is now comparable to the majority carrier density, and we have a high-level injection situation.
* $J_p^{\text{drift}}$ is comparable to $J_p^{\text{diff}}$ with high-level injection → We would not be able to solve the continuity equation for holes analytically.



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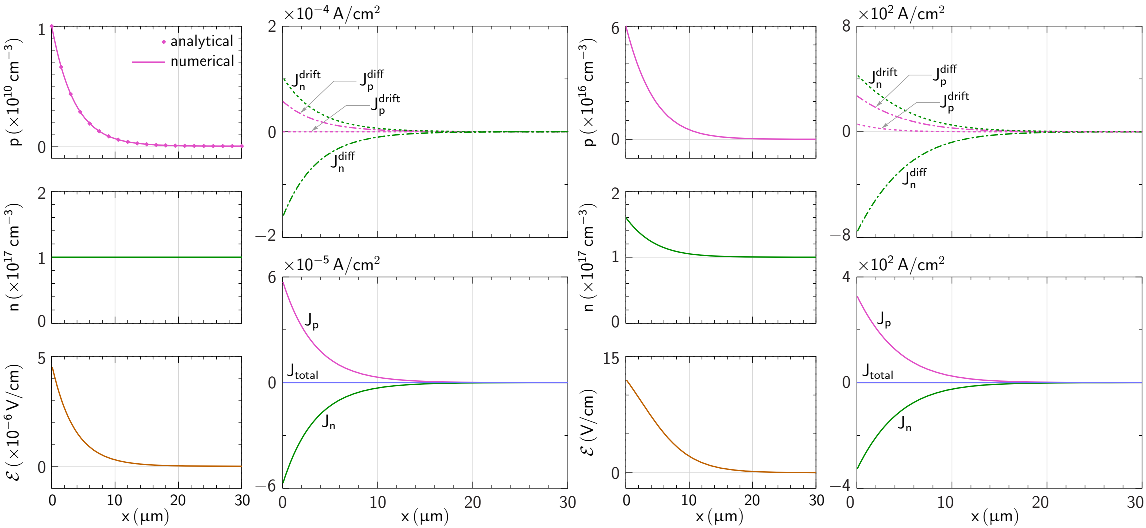


* Charge neutrality is still maintained almost perfectly because
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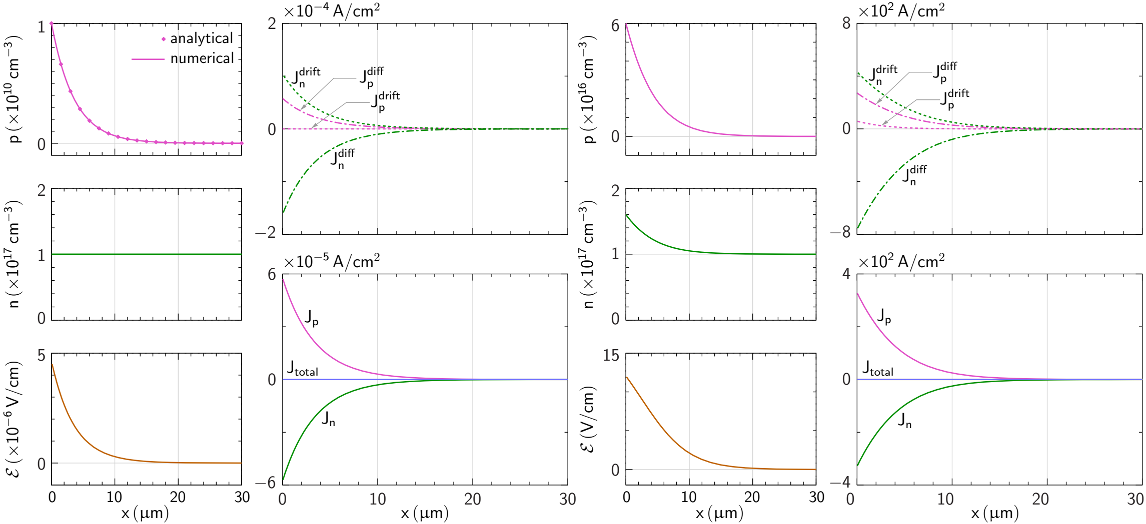


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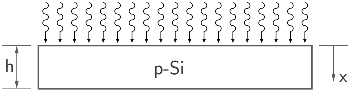
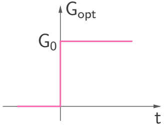
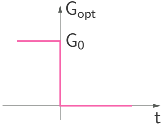
  $\Delta n \approx \Delta p \rightarrow \Delta n + n_0 \approx \Delta p + N_d^+ \rightarrow n \approx p + N_d^+$ (since $p_0$ is small.)
* $J^{\text{total}} = J_n + J_p$ remains equal to zero even with high-level injection.

Consider a $p$-type silicon sample with $N_a = 5 \times 10^{17}\,\text{cm}^{-3}$ at $T = 300\,\text{K}$. When it is illuminated uniformly with light of a certain wavelength, there is uniform generation throughout the sample at the rate of $G_0\,/\text{cm}^3$-s, where $G_0$ depends on the intensity of the incident light.




Case 1

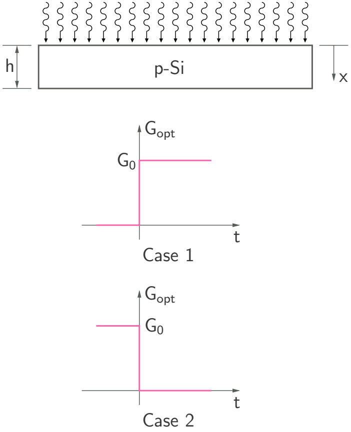



Case 2

# Example

Consider a $p$-type silicon sample with $N_a = 5 \times 10^{17}\,\text{cm}^{-3}$ at $T = 300\,\text{K}$. When it is illuminated uniformly with light of a certain wavelength, there is uniform generation throughout the sample at the rate of $G_0$ /cm$^3$-s, where $G_0$ depends on the intensity of the incident light.

For the excitations shown as Case 1 and Case 2, how does the excess electron concentration $\Delta n = n - n_0$ vary with time?



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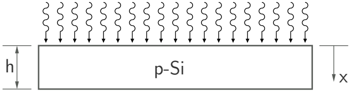
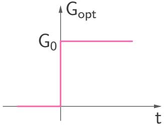
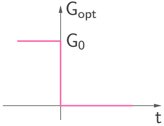
Note: The condition of uniform generation holds if the thickness of the sample $h$ is much smaller than $1/\alpha$, where $\alpha$ is the absorption coefficient of silicon at the wavelength of the incident light.




Case 1

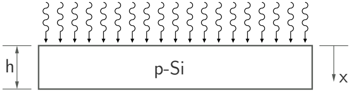


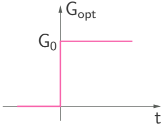

Case 2

# Example: Case 1

Continuity equation for the minority carriers (electrons) for $t > 0$ is,

$$\frac{\partial n}{\partial t} = -\frac{\partial \mathcal{F}_n}{\partial x} - (R - G)_{\text{SRH}} + G_{\text{opt}}$$



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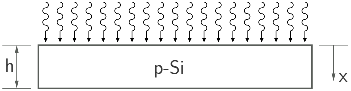


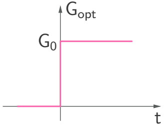

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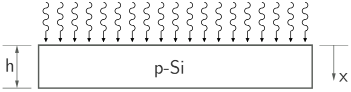


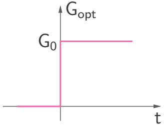

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\therefore \quad \frac{\partial (n_0 + \Delta n)}{\partial t} &= -\frac{\Delta n}{\tau_n} + G_0 .
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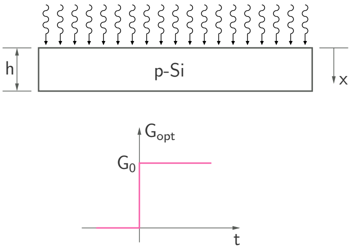

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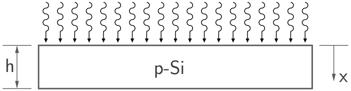


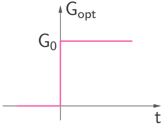

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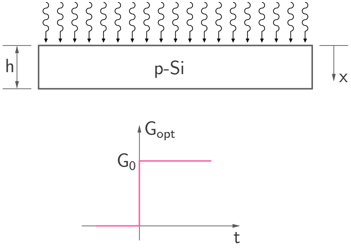


Homogeneous solution: $\Delta n^{(h)} = A\, e^{-t/\tau_n}$.

# Example: Case 1

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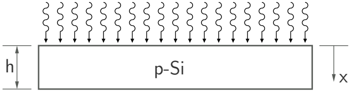


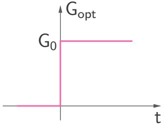

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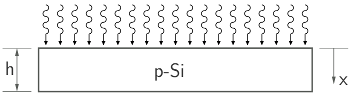


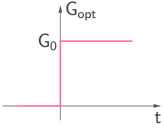


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Particular solution: $\Delta n^{(p)} = G_0 \tau_n$.

$\rightarrow \; \Delta n(t) = A\, e^{-t/\tau_n} + G_0 \tau_n, \quad t > 0.$

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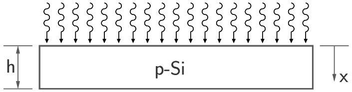


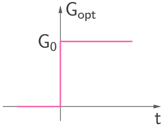

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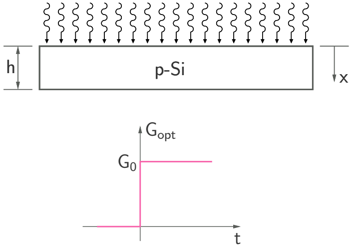

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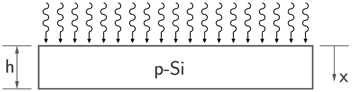


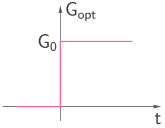

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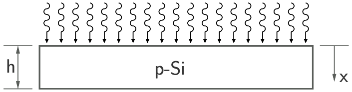


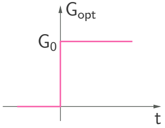


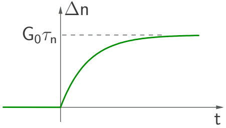

$\Delta n(t) = A\, e^{-t/\tau_n} + G_0 \tau_n, \quad t > 0.$

At $t = 0^-$, $G_{\text{opt}} = 0 \rightarrow \Delta n = 0$.

$\Delta n(0^+) = \Delta n(0^-)$ since $\Delta n$ cannot change suddenly (that would require an infinite electron flux or an infinite G-R rate).

Using $\Delta n(0^+) = 0$, we get $A = -G_0 \tau_n$, and

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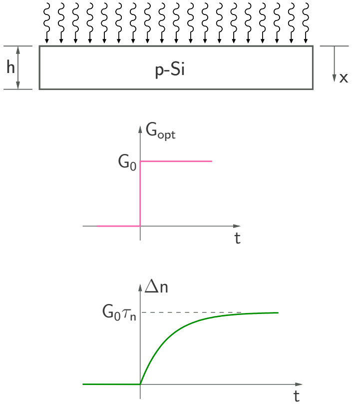
In steady state, as $t \rightarrow \infty$, the rate of optical generation must be equal to the rate of net thermal recombination.



# Example: Case 1

$\Delta n(t) = A\, e^{-t/\tau_n} + G_0 \tau_n, \quad t > 0.$

At $t = 0^-$, $G_{\text{opt}} = 0 \rightarrow \Delta n = 0$.

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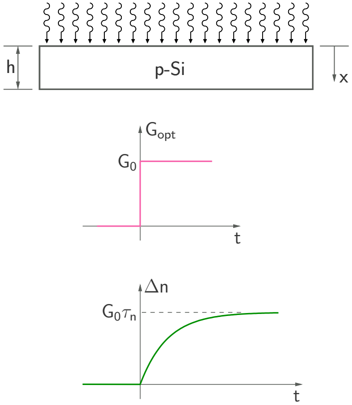
In steady state, as $t \rightarrow \infty$, the rate of optical generation must be equal to the rate of net thermal recombination.

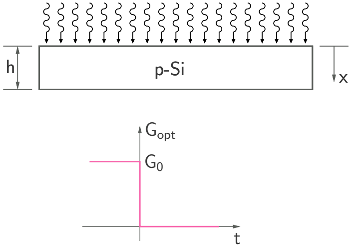
$\rightarrow G_0 = \dfrac{\Delta n}{\tau_n} \rightarrow \Delta n = G_0 \tau_n$, as predicted by the above equation.



# Example: Case 2

Continuity equation for the minority carriers (electrons) for $t > 0$ is,

$$\frac{\partial n}{\partial t} = -\frac{\partial \mathcal{F}_n}{\partial x} - (R - G)_{\mathrm{SRH}} \quad \text{(Note: no } G_{\mathrm{opt}} \text{ term here)}$$



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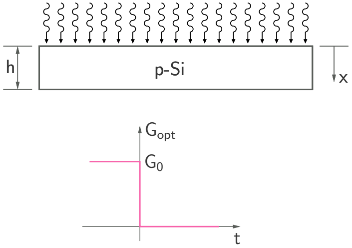

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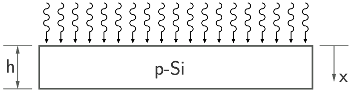


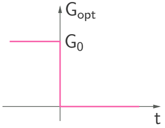

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Homogeneous solution: $\Delta n^{(h)} = A' \, e^{-t/\tau_n}$.

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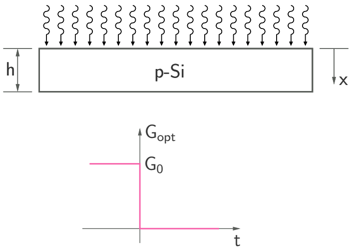

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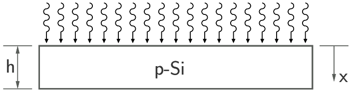


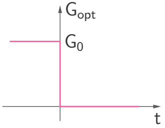

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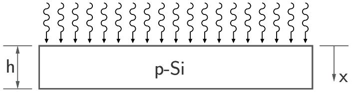


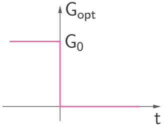

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