# **Common-Emitter Amplifier**

The circuit diagram of a common-emitter (CE) amplifier is shown in Fig. 1 (a). The capacitor  $C_B$  is used to couple the input signal to the input port of the amplifier, and  $C_C$  is used to couple the amplifier output to the load resistor  $R_L$ . We are interested in the bias currents and voltages, mid-band gain, and input and output resistances of the amplifier.



Figure 1: Common-emitter amplifier: (a) circuit diagram, (b) circuit for DC bias calculation.

#### **Bias computation**

The term "bias" refers to the DC conditions (currents and voltages) inside the amplifier circuit. The capacitors  $C_B$ ,  $C_E$ , and  $C_C$  are replaced with open circuits under DC conditions, and the circuit reduces to that shown in Fig. 1 (b). If the transistor  $\beta$  is assumed to be large  $(\beta \to \infty)$ , the base current can be neglected, and the  $R_1$ - $R_2$  network is then simply a voltage divider, giving

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} \,. \tag{1}$$

For the circuit to operate as an amplifier, it is designed such that the BJT operates in its active region, with the B-E junction under forward bias and the B-C junction under reverse bias. The B-E voltage drop  $(V_{BE} = V_B - V_E)$  is about 0.7 V for a silicon BJT, and that gives us  $V_E$  as

$$V_E = V_B - 0.7 = \frac{R_2}{R_1 + R_2} V_{CC} - 0.7.$$
<sup>(2)</sup>

The emitter current  $I_E$  is then obtained as  $I_E = V_E/R_E$ , and  $I_C = \frac{\beta}{\beta+1} I_E \approx I_E$  since we have assumed  $\beta$  to be large. The DC collector-emitter voltage is

$$V_{CE} = V_C - V_E = V_{CC} - I_C R_C - I_E R_E \approx V_{CC} - I_C (R_C + R_E).$$
(3)

The above procedure gives a good estimate of the DC bias quantities. If the base current  $I_B$  is to be taken into account in the bias computation, the Thevenin equivalent circuit shown in Fig. 2 can be used. KVL in the B-E loop gives



Figure 2: Bias computation for the common-emitter amplifier with finite base current.

$$V_{\rm Th} = I_B R_{\rm Th} + V_{BE} + (\beta + 1) I_B R_E.$$
(4)

The collector current  $I_C$  is then given by

$$I_C = \beta I_B = \beta \frac{V_{\rm Th} - V_{BE}}{R_{\rm Th} + (\beta + 1)R_E}, \qquad (5)$$

where  $R_{\text{Th}} = (R_1 \parallel R_2)$ , and  $V_{\text{Th}} = \frac{R_2}{R_1 + R_2} V_{CC}$ .

AC representation of an amplifier



Figure 3: AC representation of an amplifier.

An amplifier can be represented by the AC equivalent circuit enclosed by the box in Fig. 3. Note that the signal source (voltage  $V_s$  with a series resistance  $R_s$ ) and the load resistance  $R_L$ are *external* to the amplifier. The coupling capacitors ( $C_B$  and  $C_C$ ) are not shown in the AC circuit since their impedances are negligibly small in the "mid-band" region (see Fig. 4). The amplifier equivalent circuit is characterised by the input resistance  $R_i$  (ideally infinite), output resistance  $R_o$  (ideally zero), and gain  $A_{V0}$ . When  $R_L \to \infty$  (open circuit), the output voltage is  $v_o = A_{V0} \times v_i$  (since the current through  $R_o$  is zero in that case). With a finite  $R_L$ , the gain is lower because of the voltage drop across  $R_o$ .

Our goal in this experiment is to measure  $A_{V0}$ ,  $R_i$ , and  $R_o$  of the CE amplifier and compare the experimental values with the theoretically expected values given in the following.

## Mid-band gain $(A_{V0})$



Figure 4: Frequency response of a common-emitter amplifier (representative plot).

The term "mid-band" refers to the frequency region in which the amplifier gain is constant (see Fig. 4). In this region, the impedances due to the coupling capacitors ( $C_B$  and  $C_C$ ) and of the bypass capacitor  $C_E$  are negligibly small (i.e., they can be replaced with short circuits), and the impedances due to the BJT device capacitances are very large compared to the other components in the circuit (i.e., they can be replaced with open circuits). With these simplifications, the smallsignal (AC) equivalent circuit of the CE amplifier shown in Fig. 5 (a) reduces to the circuit of Fig. 5 (b).

The BJT small-signal equivalent circuit (consisting of the resistances  $r_{\pi}$  and  $r_o$ , and the dependent current source) used in Fig. 5 is valid only if the time-varying B-E voltage  $v_{be}$  is much smaller than  $V_T = kT/q$ , the thermal voltage which is about 25 mV at room temperature. The parameters  $r_{\pi}$  and  $g_m$  depend on the bias current  $I_C$  as

$$g_m = \frac{I_C}{V_T}, \quad r_\pi = \frac{\beta}{g_m}.$$
 (6)

Since  $v_{be} = v_s$  (see Fig. 5 (b)), we get

$$v_o = (R_C \parallel R_L \parallel r_o) \times (-g_m v_{be}) \quad \to \quad A_{VL} \equiv \frac{v_o}{v_s} = -g_m (R_C \parallel R_L) = -\frac{\beta (R_C \parallel R_L)}{r_{\pi}}, \quad (7)$$

if the output resistance  $r_o$  of the BJT is large. The open-circuit gain  $A_{V0}$  of the amplifier is given by

$$A_{VO} \equiv \left. \frac{v_o}{v_s} \right|_{R_L \to \infty} = -g_m (R_C \parallel R_L)|_{R_L \to \infty} = -g_m R_C = -\frac{\beta R_C}{r_\pi} \,. \tag{8}$$



Figure 5: (a) Small-signal equivalent circuit of a CE amplifier, (b) simplified circuit after replacing the coupling and bypass capacitors with short circuits.

To measure  $A_{VL}$  and  $A_{V0}$ , we apply a sinusoidal input voltage<sup>1</sup> ( $v_s$  in Fig. 1 (a)) and measure  $v_o$  with  $R_L$  in place and with  $R_L \to \infty$  (i.e., open circuit), respectively.

## Input resistance $R_i$

The input resistance of an amplifier can be found by applying a voltage  $v_s$  and measuring by some means the current<sup>2</sup>  $i_{in}$  shown in Fig. 6 (a) to obtain  $R_i = v_s/i_{in}$ . From the AC equivalent circuit of Fig. 5 (b), we can see that the input resistance is

$$R_i = (R_1 \parallel R_2 \parallel r_\pi), \quad \text{where} \quad r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C}, \qquad (9)$$

with  $I_C$  being the bias (DC) value of the collector current.

A simple way for experimental measurement of  $R_i$  is shown in Fig. 6(b). We connect the input voltage source to the amplifier<sup>3</sup> through a variable resistance (pot)  $R_s$ . Keeping

 $<sup>^{1}</sup>v_{s}$  must be sufficiently small to ensure that the output voltage is purely sinusoidal.

<sup>&</sup>lt;sup>2</sup>Note that  $v_s$  and  $i_{\rm in}$  are AC quantities.

<sup>&</sup>lt;sup>3</sup>The coupling capacitor  $C_B$  is not shown explicitly in Fig. 6(b), but it must be connected so that the bias



Figure 6: (a) Theoretical interpretation of  $R_i$ , (b) practical technique to measure  $R_i$ .

 $v_s$  constant, we then vary  $R_s$  and measure  $v_o$ . If  $v'_o$  corresponds to  $R_s = 0 \Omega$ , then the input resistance  $R_i$  is equal to the value of  $R_s$  which gives  $v_o = v'_o/2$ .

### Output resistance $R_o$



Figure 7: (a) AC equivalent circuit of an amplifier with  $v_s = 0$ , (b) AC equivalent circuit of the CE amplifier with  $v_s = 0$ .

If we apply  $v_s = 0$  to the generic amplifier shown in Fig. 3, we obtain the circuit shown in Fig. 7 (a). Since  $v_i = 0$ , the dependent voltage source gets replaced by a short circuit, and looking at the circuit from the output port, we only see  $R_o$ . If we do that for the AC equivalent circuit of the CE amplifier (Fig. 7 (b)), we get

$$R_o = r_o \parallel R_C \approx R_C \,, \tag{10}$$

since  $r_o$  of a BJT is typically much larger than  $R_C$ .

To experimentally measure  $R_o$  of the CE amplifier, we can use a procedure similar to that discussed for  $R_i$ . We connect a variable load resistance  $R_L$  (through a suitably large coupling capacitor) as shown in Fig. 8. Keeping  $v_s$  constant, we first measure  $v_o \equiv v'_o$  with  $R_L \to \infty$ (open circuit).  $R_o$  is given by the value of  $R_L$  which gives  $v_o = v'_o/2$ .

values are not disturbed.



Figure 8: Circuit for experimental measurement of  $R_o$ .

### Distortion

An amplifier is expected to produce a faithful or undistorted version of the input voltage (except for the amplification factor) at the output. For the CE amplifier, an undistorted output voltage is obtained as long as the small-signal condition  $v_{be} \ll V_T$  is satisfied. This is because the BJT small-signal model is valid if the nonlinear terms (degree 2 and higher) are negligibly small compared to the linear term in

$$\exp\left(\frac{v_{be}}{V_T}\right) = 1 + \frac{v_{be}}{V_T} + \frac{1}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \cdots$$
(11)

Since the signal voltage  $v_s$  is the same as  $v_{be}$  in the CE amplifier (see Fig. 5), we must have  $v_s \ll V_T$  to avoid distortion in the output voltage. With  $V_T \approx 25 \text{ mV}$  at room temperature, the amplitude of  $v_s$  should therefore be restricted to about 5 mV.

#### Common-emitter amplifier with partial bypass



Figure 9: (a) CE amplifier with partially bypassed emitter resistance, (b) AC equivalent circuit.

A CE amplifier with partially bypassed emitter resistance is shown in Fig. 9(a). The bias point computation of the CE amplifier (Fig. 1) is valid for the partial bypass case if we replace

 $R_E$  with  $(R_{E1} + R_{E2})$ . For computing the AC quantities of interest (gain,  $R_i$ ,  $R_o$ ), we use the circuit shown in Fig. 9(b). Since  $i_e = (\beta + 1) i_b$ , the resistance  $R_{E1}$  appears as  $(\beta + 1)R_{E1}$  as seen from the base, and we can write

$$v_s = i_b \left[ r_\pi + (\beta + 1) R_{E1} \right]. \tag{12}$$

The output voltage is

$$v_o = -\beta \, i_b \times (R_C \parallel R_L), \tag{13}$$

and the gain with load  $A_{VL}$  is therefore

$$A_{VL} = \frac{v_o}{v_s} = -\frac{\beta \left(R_C \parallel R_L\right)}{r_{\pi} + (\beta + 1)R_{E1}}.$$
(14)

If  $(\beta+1)R_{E1} \gg r_{\pi}$ ,  $A_{VL} \rightarrow -\frac{(R_C \parallel R_L)}{R_{E1}}$ , and the open-circuit gain  $A_{V0} = A_{VL}|_{R_L \rightarrow \infty} = -\frac{R_C}{R_{E1}}$ . Note that the gain of the CE amplifier with partial bypass is less than that of the CE amplifier

(compare Eqs. 7 and 14) as we would expect from an amplifier with negative feedback<sup>4</sup>.

The input resistance, by inspection of Fig. 9(b) is

$$R_i = R_1 \parallel R_2 \parallel (r_{\pi} + (\beta + 1)R_{E1}), \tag{15}$$

and the output resistance is  $R_o \approx R_C$ , assuming  $r_o$  of the BJT to be large.

An important point to note is that the base-emitter small-signal voltage  $v_{be}$  in this case is much smaller than  $v_s$  (see Fig. 9 (b)):

$$\frac{v_{be}}{v_s} = \frac{r_\pi i_b}{r_\pi i_b + (\beta + 1)R_{E1}i_b} = \frac{r_\pi}{r_\pi + (\beta + 1)R_{E1}}.$$
(16)

As a result, the small-signal condition  $v_{be} \ll V_T$  means that  $v_s \frac{r_{\pi}}{r_{\pi} + (\beta + 1)R_{E1}} \ll V_T$  or  $v_s \ll V_T \left(1 + \frac{(\beta + 1)R_{E1}}{r_{\pi}}\right)$ , i.e., a larger  $v_s$  can be applied (as compared to the CE amplifier) without causing distortion in the output voltage.

### References

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<sup>&</sup>lt;sup>4</sup>The feedback involved in the CE amplifier with partial bypass is of the series-series type. On the output side, the output current  $i_c$  causes a voltage drop  $R_{E1}i_e \approx R_{E1}i_c$  across  $R_{E1}$ , and this voltage drop gets subtracted from the input voltage  $v_s$ .