BJT Differential Amplifier

Common-mode and difference-mode voltages

A typical sensor circuit produces an output voltage between nodes A and B (see Fig. 1) such that

$$V_{o1} = V_c + \frac{V_d}{2}\,, \qquad V_{o2} = V_c - \frac{V_d}{2}\,, \tag{1}$$

where V_c is called the "common-mode" voltage and V_d the "difference-mode" or "differential" voltage. The common-mode voltage is a result of the biasing arrangement used within the sensor

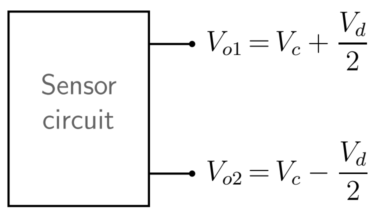


Figure 1: Example of common-mode and difference-mode voltages.

circuit, and it can be large (a few volts). The difference-mode voltage is the quantity of actual interest, and it is a measure of the quantity being sensed (such as temperature, pressure, or concentration of a species in a gas). The difference-mode voltage is typically much smaller (say, a few mV) as compared to the common-mode voltage.

Of these, the common-mode voltage is obviously a necessary devil, something we cannot simply wish away[1]. What we can do is to amplify only the differential quantity (V_d) while "rejecting" the common-mode quantity (V_c) by making use of a differential amplifier, as shown in Fig. 2. If V_{i1} and V_{i2} are expressed as $V_{i1} = V_{ic} + V_{id}/2$ and $V_{i2} = V_{ic} - V_{id}/2$, the output of

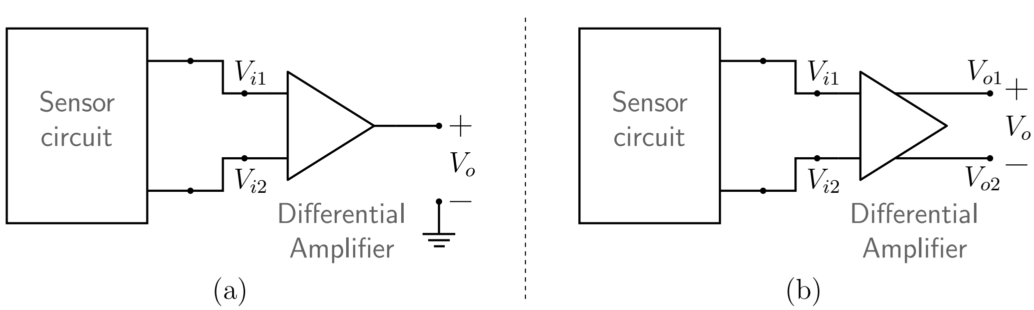


Figure 2: Sensor of Fig. 1 connected to a differential amplifier: (a) single-ended output, (b) differential output.

[1]Another situation in which we want to reject the common-mode signal is a telephone line where we want to amplify the true signal – the difference between the voltages on the two wires – but reject the spurious common-mode electromagnetic interference riding on each wire.

the differential amplifier is given by,

$$V_o = A_d V_{id} + A_c V_{ic}\,, \tag{2}$$

where A_d is the differential gain, and A_c is the common-mode gain. A good differential amplifier should reject V_{ic} entirely, i.e., it should have $A_c = 0$. In reality, A_c for a differential amplifier is small but finite, and a figure of merit called the "Common-Mode Rejection Ratio" (CMRR) is used to indicate the effectiveness of the amplifier in rejecting common-mode inputs. The CMRR is defined as

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right|\,. \tag{3}$$

In this experiment, our objective is to wire up a simple differential amplifier circuit and to see how its CMRR can be improved.

The BJT differential pair

The circuit shown in Fig. 3, known as the BJT differential pair, can be used to amplify only the differential input signal $V_{id} = (V_{i1} - V_{i2})$ while rejecting the common-mode signal $V_{iC} = \frac{1}{2}\,(V_{i1} + V_{i2})$. The two resistors are assumed to be matched and so are the BJTs[2] Q_1 and Q_2.

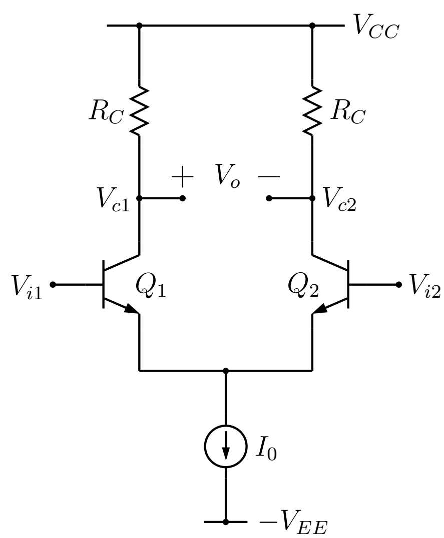


Figure 3: BJT differential pair.

Let us first look at the large-signal behaviour of the circuit. The two emitter currents are given by

$$I_{e1} = \frac{I_s}{\alpha}\exp\left(\frac{V_{i1} - V_e}{V_T}\right), \qquad I_{e2} = \frac{I_s}{\alpha}\exp\left(\frac{V_{i2} - V_e}{V_T}\right) \tag{4}$$

[2]Note that Q_1 and Q_2 should not only be identical in construction (fabrication), they must also operate at the same temperature.

(with $\alpha = \dfrac{\beta}{\beta+1} \approx 1$), and they must satisfy the constraint $I_{e1} + I_{e2} = I_0$. From Eq. 4, we can write

$$I_{e1} = \frac{I_0}{1 + e^{-V_{id}/V_T}}\,, \qquad I_{e2} = \frac{I_0}{1 + e^{+V_{id}/V_T}}\,. \tag{5}$$

The collector currents are $I_{c1} = \alpha I_{e1} \approx I_{e1}$ and $I_{c2} = \alpha I_{e2} \approx I_{e2}$.

Fig. 4 shows I_{c1} and I_{c2} as a function of $V_{id} = V_{i1} - V_{i2}$. Note the "steering" effect of the input voltage on the collector currents. When $V_{id} > 0\,\text{V}$, I_{c1} is larger than I_{c2}; when $V_{id} > 0\,\text{V}$, I_{c2} is larger. When V_{id} is about $4\,V_T$, almost the entire current I_0 is conducted by Q_1; when it is about $-4\,V_T$, it is conducted by Q_2.

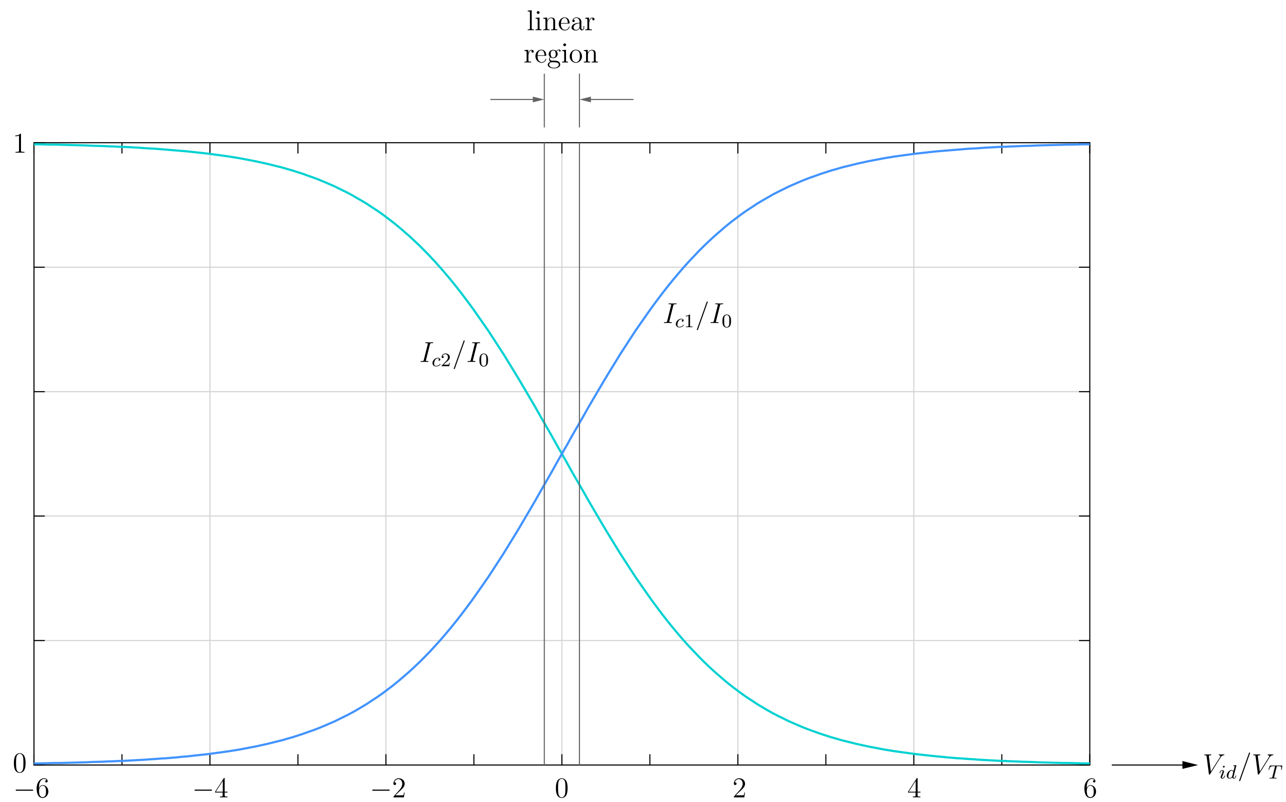


Figure 4: Normalised collector currents I_{c1} and I_{c2} versus V_{id}.

Our interest is in using the circuit as an amplifier, and we are therefore looking for the region of V_{id} where I_{c1} and I_{c2} vary linearly with V_{id} (see Fig. 4). In this linear region, the higher-order terms in

$$\exp\left(\frac{V_{id}}{V_T}\right) = \left(\frac{V_{id}}{V_T}\right) + \frac{1}{2}\left(\frac{V_{id}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{V_{id}}{V_T}\right)^3 + \cdots \tag{6}$$

must be sufficiently small, i.e., V_{id} must be much smaller than V_T, something like $V_{id} < 0.2\,V_T$, making the width of the linear region in Fig. 4 about $2 \times 0.2 \times V_T$ or $10\,\text{mV}$ at room temperature.

The output voltage V_o (see Fig. 3) is given by

$$V_o = V_{c1} - V_{c2} = (V_{CC} - R_C I_{c1}) - (V_{CC} - R_C I_{c2}) = R_C\,(I_{c2} - I_{c1})\,. \tag{7}$$

In the linear region,

$$I_{c1} = \frac{\alpha I_0}{1 + e^{-V_{id}/V_T}} \approx \frac{\alpha I_0}{1 + 1 - V_{id}/V_T} = \frac{\alpha I_0}{2 - V_{id}/V_T}\,, \tag{8}$$

$$I_{c2} = \frac{\alpha I_0}{1 + e^{+V_{id}/V_T}} \approx \frac{\alpha I_0}{1 + 1 + V_{id}/V_T} = \frac{\alpha I_0}{2 + V_{id}/V_T}\,. \tag{9}$$

The output voltage is

$$\begin{aligned} V_o &= R_C\,(I_{c2} - I_{c1}) \\ &= \frac{\alpha I_0\,[(2 - V_{id}/V_T) - (2 + V_{id}/V_T)]}{(2 + V_{id}/V_T)(2 - V_{id}/V_T)}\,. \end{aligned} \tag{10}$$

For $V_{id} \ll V_T$, we have

$$V_o = -\,\frac{\alpha I_0}{4} \times \frac{2\,V_{id}}{V_T}\,. \tag{11}$$

Since $\alpha I_0/2 = I_{c1} = I_{c2}$ when $V_{id} \approx 0$, and with $g_m = I_C/V_T = (I_0/2)/V_T$, we get

$$A_d \simeq \frac{V_o}{V_{id}} = -g_m R_C. \tag{12}$$

Note that V_o does not involve the common-mode voltage V_{ic} at all, exactly as we would like. In a real circuit, this is not quite true, as we shall see later.

Small-signal analysis

The expression in Eq. 12 can also be obtained using small-signal analysis of the BJT differential pair. The ideal current source in Fig. 3 is replaced with an open circuit in small-signal analysis. Using the T equivalent circuit for the BJTs, we then obtain the small-signal equivalent circuit shown in Fig. 5 (a). By symmetry, we have $i_{e1} = -i_{e2} = \dfrac{v_{id}}{2\,r_e}$. The output voltage is

$$v_o = v_{c1} - v_{c2} = -\alpha i_{e1} R_C - \alpha i_{e2} R_C = -2\alpha\,\frac{v_{id}}{2r_e}\,R_C = -g_m R_C\,v_{id} \rightarrow A_d = -g_m R_C\,, \tag{13}$$

using $g_m = \alpha/r_e$. The same relationship can be obtained using the equivalent circuit shown in Fig. 5 (b) where the hybrid-π equivalent circuit is used for the BJTs.

Implementation using discrete transistors

The BJT differential pair is an integral part of op amp integrated circuits. In this experiment, we will make up the circuit using discrete transistors. Since the transistors are supposed to be identical in all respects and also operating at the same temperature, it is best to use emitter-coupled transistors in a chip providing an array of BJTs, such as CA3096 or LM3086.

Fig. 6 shows a simple implementation of the circuit of Fig. 3 with a "crude" current source, viz., a simple resistor R_{EE}. With $V_{i1} = V_{i2} = 0\,\text{V}$, the emitter voltage is about $-0.7\,\text{V}$, and the

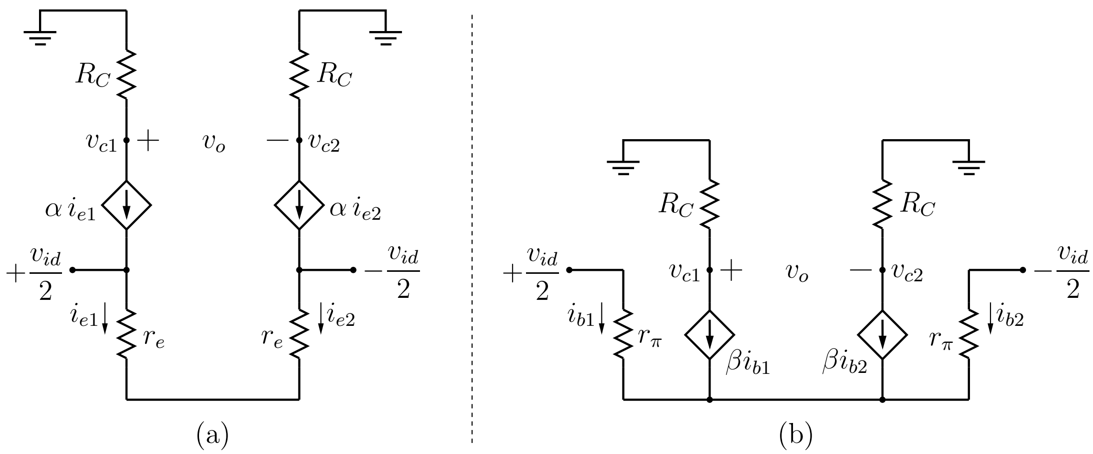


Figure 5: Small-signal equivalent circuit of the BJT differential pair: (a) using T equivalent circuit for the BJTs, (b) using the hybrid-π equivalent circuit for the BJTs (the output resistance of the BJTs r_o is assumed to be large).

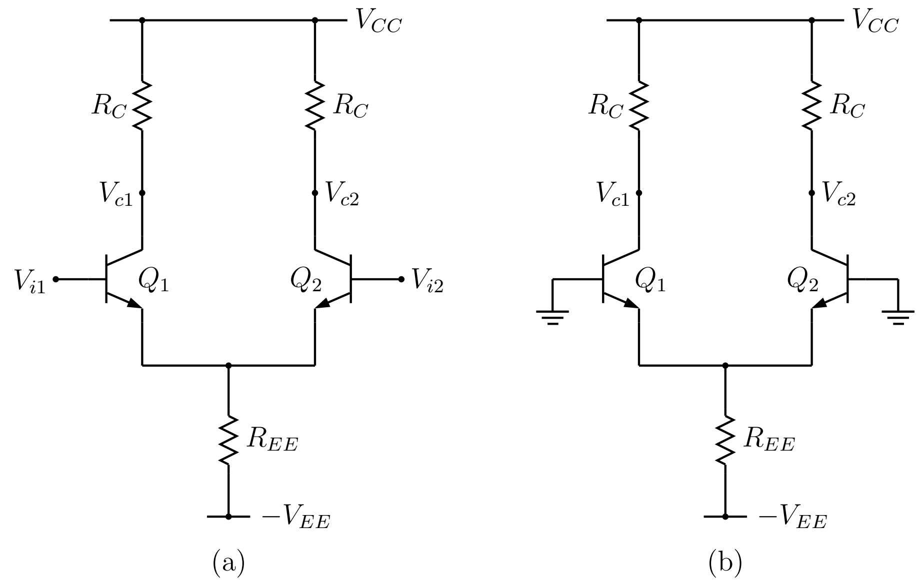


Figure 6: Practical implementation of the BJT differential pair: (a) with general inputs V_{i1} and V_{i2}, (b) with $V_{i1} = V_{i2} = 0\,\text{V}$.

bias current I_0 is therefore

$$I_0 = \frac{-0.7 - (-V_{EE})}{R_{EE}} = \frac{V_{EE} - 0.7}{R_{EE}}\,, \tag{14}$$

each transistor carrying $I_0/2$. Let us find the differential- and common-mode gains for this amplifier.

Consider $V_{i1} = v_{id}/2$ and $V_{i2} = -v_{id}/2$. The small-signal equivalent circuit for this situation is shown in Fig. 7. Writing KCL at the common emitter, we get

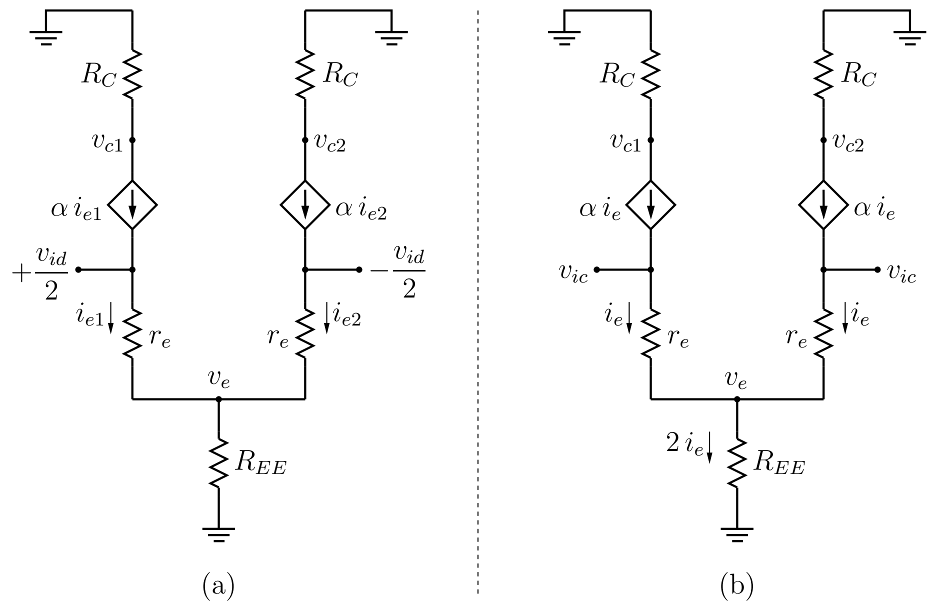


Figure 7: Small-signal equivalent circuit for the differential amplifier of Fig. 6: (a) computation of A_d, (b) computation of A_c.

$$\frac{1}{r_e}\left[\frac{v_{id}}{2} - v_e\right] + \frac{1}{r_e}\left[\frac{-v_{id}}{2} - v_e\right] - \frac{v_e}{R_{EE}} = 0\,, \tag{15}$$

giving $v_e = 0$, i.e., the common emitter as AC ground. The circuit now reduces to that shown in Fig. 5 (a), and we have

$$i_{e1} = -i_{e2} = \frac{v_{id}}{2\,r_e}\,, \quad v_{c1} = -\alpha\, i_{e1}\, R_C = -\alpha\, \frac{v_{id}}{2\,r_e}\, R_C = -\,\frac{1}{2}\, g_m R_C v_{id}\,. \tag{16}$$

Similarly, $v_{c2} = +\,\frac{1}{2}\, g_m R_C v_{id}$. If the output is taken in a single-ended fashion (say, $v_o \simeq v_{c1}$, we have $A_d = -\,\frac{1}{2} g_m R_C$. If it is taken in a differential fashion ($v_o \simeq v_{c1} - v_{c2}$), we have $A_d = g_m R_C$.

To find the common-mode gain, we apply a small signal $v_{i1} = v_{i2} = v_{ic}$, as shown in Fig. 7 (b). Because of symmetry, $i_{e1} = i_{e2} \simeq i_e$, $v_{ic} = i_e\,(r_e + 2\,R_{EE})$, and we get

$$v_{c1} = v_{c2} = -\,\alpha\, i_e\, R_C = -\,\alpha\, \frac{v_{ic}}{r_e + 2\,R_{EE}}\, R_C\,. \tag{17}$$

If the output is taken in a single-ended manner ($v_o = v_{c1}$ or v_{c2}), the common-mode gain is[3]

$$A_c = \frac{v_{c1}}{v_{ic}} = -\,\frac{\alpha\, R_C}{r_e + 2\,R_{EE}} \approx -\,\frac{R_C}{2\,R_{EE}}\,. \tag{18}$$

If the output is taken in a differential manner, $v_o = v_{c1} - vc2 = 0$ since v_{c1} and v_{c2} are the

[3]Note that A_c is independent of bias, and we can expect the expression to hold even for large values of the common-mode voltage.

same[4], giving $V_c = 0$. For this reason, the output of the first stage of an op amp is taken in a differential manner and fed to the second stage, thus resulting in a large value of CMRR.

Improved current source

The CMRR of the circuit in Fig. 6 is

$$\text{CMRR} = \left|\frac{A_d}{A_c}\right| = \frac{1}{2}\,g_m\,R_C \times \frac{1}{R_C/2R_{EE}} = g_m\,R_{EE}\,, \tag{19}$$

when the output is taken in a single-ended manner. By increasing R_{EE}, the CMRR can be improved; however, this will reduce the bias current I_0 (see Eq. 14) and therefore the differential gain $A_d = \frac{1}{2}\,g_m R_C$. Is there a way to provide the desired bias current and simultaneously achieve a high CMRR?

The key is to use a current source instead of the resistor R_{EE}. Fig. 8 shows a simple current mirror which can provide a current I_0 which is nearly independent of the voltage V_{C4}. The

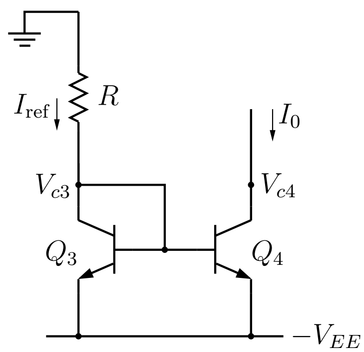


Figure 8: A simple current mirror.

operation of the current mirror is straightforward: The voltage V_{C3} is equal to V_{B3} which is about $-V_{EE} + 0.7\,\text{V}$, and the current I_{ref} is therefore

$$I_{\text{ref}} = \frac{0 - (-V_{EE} + 0.7)}{R} = \frac{V_{EE} - 0.7}{R}\,. \tag{20}$$

If transistors Q_3 and Q_4 are identical and are operating at the same temperature, the two collector currents given by

$$I_{C3} = I_s\,e^{V_{BE3}/V_T} \quad \text{and} \quad I_{C4} = I_s\,e^{V_{BE4}/V_T} \tag{21}$$

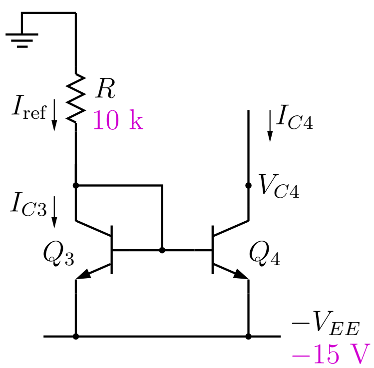
are identical. Furthermore, if β is sufficiently large, we can ignore the base currents and get

$$I_0 = I_{C4} = I_{C3} \approx I_{\text{ref}} \approx \frac{V_{EE} - 0.7}{R}\,. \tag{22}$$

[4]In practice, it is still possible for the output voltage to have a common-mode component (much smaller than the single-ended output case) due to mismatch in the resistance values and BJT parameters [1].

This is indeed what we are looking for – a constant current I_0 which is independent of V_{C4}. This is the basic idea behind a current source.

In practice, I_{C4} would show a small variation with V_{C4} due to the Early effect (see Fig. 9), with the slope $\dfrac{\partial I_{C4}}{\partial V_{C4}}$ equal to I_0/V_A, where V_A is the Early voltage (typically greater than 50 V) of the BJT[5]. Consequently, the small-signal equivalent circuit of this current source would simply be a resistance $r_o = V_A/I_0$, the output resistance of Q_4.



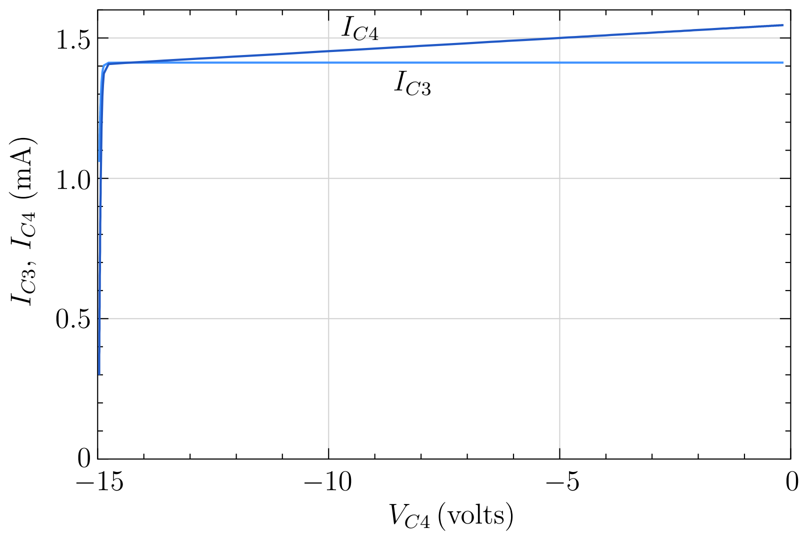


Figure 9: I_{C3} and I_{C4} versus V_{C4} for the simple current mirror of Fig. 8 (representative plot).

Fig. 10 (a) shows an improved differential amplifier circuit using the simple current mirror as the current source. The corresponding small-signal equivalent circuit is shown in Fig. 10 (b). Since the small-signal circuit is the similar to that of Fig. 7, we can use our earlier expressions for A_d and A_c by replacing R_{EE} with r_o. If single-ended output is considered, we get

$$|A_d| = \frac{1}{2}\, g_m\, R_C\,, \qquad |A_c| = \frac{R_C}{2\, r_o}\,, \tag{23}$$

where r_o is the output resistance of Q_4. Since r_o is typically much larger than R_{EE}, a significant reduction in A_c is obtained.

References

1. A.S. Sedra and K.C. Smith and A.N. Chandorkar, *Microelectronic Circuits Theory and Applications*. New Delhi: Oxford University Press, 2009.

2. P.R. Gray and R.G Meyer, *Analysis and Design of Analog Integrated Circuits*. Singapore: John Wiley and Sons, 1995.

[5]Note that near $V_{CE4} = 0$ V (i.e., $V_{C4} = -V_{EE}$), the currents drop since Q_4 is not in the active region any more. This region should of course be avoided.

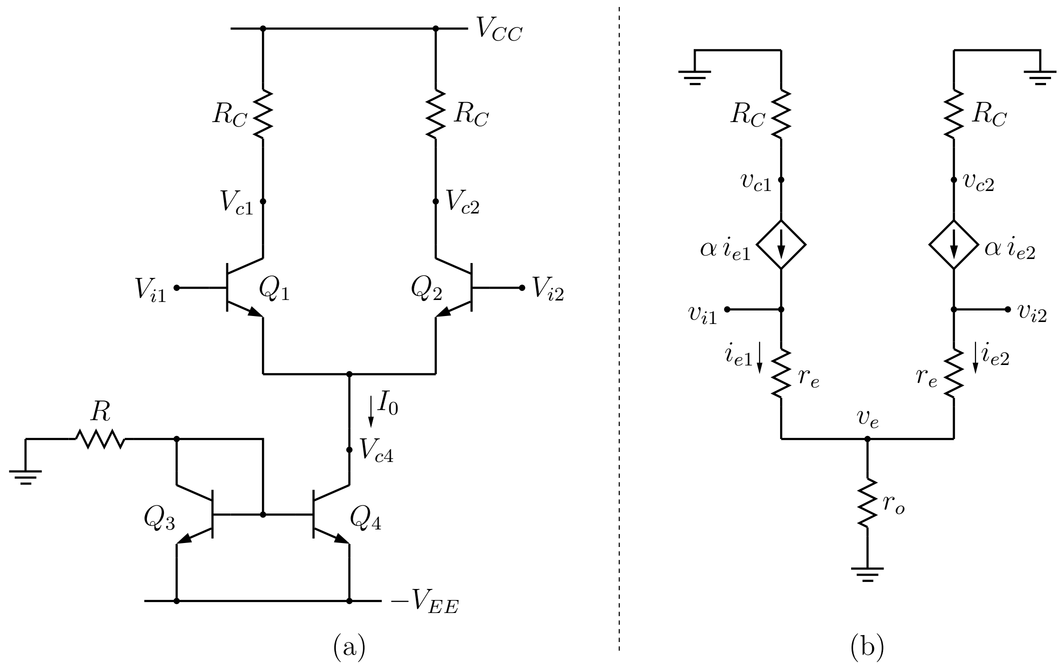


Figure 10: (a) Improved differential amplifier using a simple current mirror, (b) small-signal equivalent circuit.