

Figure 1: (a) inverting amplifier, (b) equivalent circuit, (c) equivalent circuit with $R_i \to \infty$, $R_o \to 0$.

Shown in Fig. 1(a) is an inverting amplifier for which

$$V_o(t) = -\frac{R_2}{R_1} V_i(t) \,. \tag{1}$$

If a sinusoidal input voltage $\mathbf{V}_i(j\omega) = \hat{V}_i \angle 0$ is applied, the output voltage is expected to be

$$\mathbf{V}_o(j\omega) = \frac{R_2}{R_1} \,\widehat{V}_i \,\angle \pi \,. \tag{2}$$

Eq. 2 is valid if the open-loop gain of the Op Amp (A_V) is large at all frequencies of interest. For a real Op Amp, however, A_V falls at high frequencies, thus limiting the gain $|\mathbf{V}_o|/|\mathbf{V}_i|$ of the inverting amplifier. The open-loop gain of a 741 Op Amp is given by,

$$A_V(s) = \frac{A_0}{1 + s/\omega_c},\tag{3}$$

with $A_0 \approx 10^5$ (i.e., 100 dB), $\omega_c \approx 2\pi \times 10$ rad/s.

By replacing the Op Amp in Fig. 1 (a) by its equivalent circuit (see Fig. 1 (b)) and then further simplifying it by assuming R_i to be large and R_o to be small (see Fig. 1 (c)), we get

$$-V_i(s) = V_s(s) \frac{R_2}{R_1 + R_2} + V_o(s) \frac{R_1}{R_1 + R_2}.$$
(4)

Using $V_o(s) = A_V(s) V_i(s)$,

$$\frac{V_o(s)}{V_s(s)} = -\frac{R_2}{R_1} \frac{1}{\left[1 + \left(\frac{R_1 + R_2}{R_1}\right)\frac{1}{A_0}\right] + \left(\frac{R_1 + R_2}{R_1 A_0}\right)\frac{s}{\omega_c}} \qquad (5)$$

$$\approx -\frac{R_2}{R_1} \frac{1}{1 + s/\omega_c'}, \qquad (6)$$

with $\omega_c' = \frac{\omega_c A_0}{1 + R_2/R_1} = \frac{\omega_t}{1 + R_2/R_1}.$

In other words, as ω increases, the gain of the inverting amplifier is no longer constant, but falls at the rate of 20 dB/dec.

If $R_2/R_1 \gg 1$, the product of the gain (i.e., R_2/R_1) and the bandwidth ω'_c is a constant, resulting in the well-known trade-off between gain and bandwidth of amplifiers.

Exercise Set

- 1. For $R_1 = 1 \text{ k}\Omega$, $R_2 = 50 \text{ k}\Omega$, and a 741 Op Amp model, obtain a plot of $\log |\mathbf{V}_o|$ versus $\log f$ by AC simulation, keeping $|\mathbf{V}_i| = 0.01 V$. What is the 3-dB frequency for this amplifier?
- 2. Repeat (1) for three other values of R_2 : (a) $25 \text{ k}\Omega$, (b) $10 \text{ k}\Omega$, (c) $5 \text{ k}\Omega$. Plot $\log |\mathbf{V}_o|$ versus $\log f$ for the various values of R_2 together and correlate your observations with the theoretical considerations given above.
- 3. Verify that the product of the low-frequency gain A_V^0 and the 3-dB frequency f_H of the inverting amplifier is approximately constant, irrespective of the R_2 value in (1) and (2).