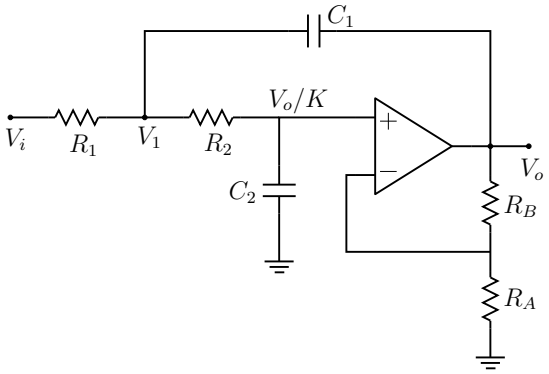
`ee101_op_filter_5.sqproj`



A second-order low-pass filter can be described by [1] $H(j\omega) = H_{0LP} H_{LP}(j\omega)$, where H_{0LP} is a constant, and

$$H_{LP}(j\omega) = \frac{1}{1 - (\omega/\omega_0)^2 + (j\omega/\omega_0)/Q} \,. \tag{1}$$

For a "KRC" or "Sallen-Key" low-pass second-order filter (shown in the figure) [1], the gain block (which determines H_{0LP}) is a non-inverting Op Amp amplifier circuit with gain

$$K = H_{0LP} = 1 + \frac{R_B}{R_A} \,. \tag{2}$$

To derive the complete transfer function, we use

$$V_o = K \, \frac{1}{R_2 C_2 s + 1} \, V_1 \,. \tag{3}$$

Writing KCL at node V_1 and eliminating V_1, we get

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{K}{R_1 C_1 R_2 C_2 s^2 + [(1-K) R_1 C_1 + R_1 C_2 + R_2 C_2] \, s + 1} \,. \tag{4}$$

Comparing with Eq. 1, we can write

$$\omega_0 = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}} \,, \tag{5}$$

$$Q = \frac{1}{(1-K)\sqrt{R_1 C_1 / R_2 C_2} + \sqrt{R_1 C_2 / R_2 C_1} + \sqrt{R_2 C_2 / R_1 C_1}} \,. \tag{6}$$

Note that K (the low-frequency gain) and Q (which determines the shape of $H(j\omega)$) depend on component ratios while ω_0 depends on the actual values of the components.

For $R_1 = R_2 = R$ and $C_1 = C_2 = C$, the above equations simplify to

$$H_{0LP} = K \,, \; \omega_0 = \frac{1}{RC} \,, \; Q = \frac{1}{3-K} \,. \tag{7}$$

Exercise Set

1. Plot the magnitude frequency response (log-log plot) and verify the circuit functionality. In particular, verify the value of K and the asymtotic behaviour of $H(j\omega)$, viz., a drop of -40 dB/decade at high frequencies.

2. Change component values, keeping their ratios the same, and observe how $H(j\omega)$ is affected.

3. Increase C_1 by a factor of 2, and observe its effect on $H(j\omega)$.

4. Change C_1 and C_2 such that their product remains constant, but their ratio is different. For example, (i) $C_1 = 10\,\text{nF}$, $C_2 = 10\,\text{nF}$, (ii) $C_1 = 5\,\text{nF}$, $C_2 = 20\,\text{nF}$, (iii) $C_1 = 8\,\text{nF}$, $C_2 = 12.5\,\text{nF}$. Observe its effect on $H(j\omega)$.

References

1. S. Franco, *Design with Operation Amplifiers and Analog Integrated Circuits,* McGraw-Hill, 1998.

2. J. Millman and A. Grabel, *Microelectronics,* McGraw-Hill, 1988.