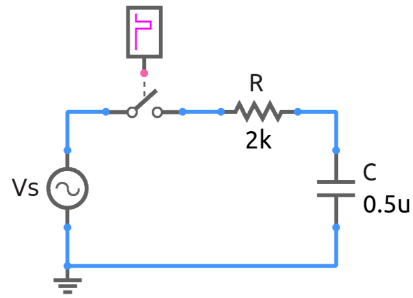
ee101_rc5.sqproj



In the RC circuit shown in the figure, the switch has been open for a long time and is closed at $t=0$. The initial capacitor voltage $V_C(0^-)$ is 0 V. The source voltage is given by $V_s = V_m \sin \omega t$, with $V_m = 1$ V and a frequency of 1 kHz. We are interested in the solution of the circuit equations for $t > 0$.

Exercise Set

1. Derive the following ODE from the circuit equations.

$$\frac{dV_C}{dt} + \frac{1}{\tau} V_C = \frac{V_s}{\tau}, \tag{1}$$

   where $\tau = RC$.

2. Let the solution be

$$V_C(t) = V_C^h(t) + V_C^p(t), \tag{2}$$

   where V_C^p is a particular solution of the ODE, and V_C^h is the general solution of the associated homogeneous differential equation (obtained by replacing the RHS with zero). Show that $V_C^h(t)$ is of the form $A \exp(-t/\tau)$.

3. Since we expect $V_C(t)$ to be sinousoidal in the steady state, let the particular solution be

$$V_C^p(t) = C_1 \sin \omega t + C_2 \cos \omega t. \tag{3}$$

   Find C_1 and C_2 by substituting $V_C^p(t)$ in Eq. 1, and equating the coefficients of $\sin \omega t$ and $\cos \omega t$ on both sides of the equation.

4. Now write the complete solution using Eq. 2. Obtain the constant A using the initial condition on V_C (i.e., $V_C(0^+) = 0$ V).

5. Using $V_C(t)$, obtain an expression for the current $I_C(t)$.

6. Compare your solutions with simulation results.