ee101_rc8.sqproj

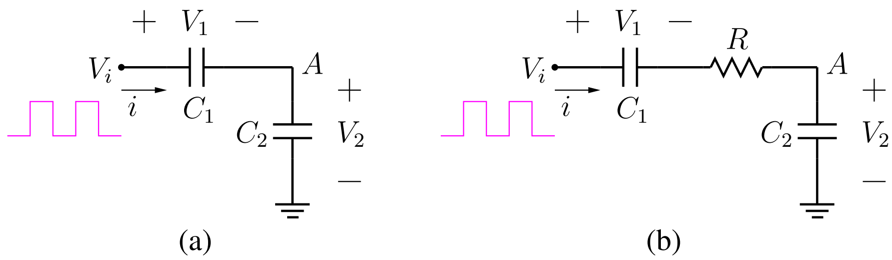

Figure 1: (a) Two capacitors in series driven by a square wave input, (b) Same circuit with a series resistor added.

Fig. 1 (a) shows two capacitors connected in series, with a square wave input voltage. Since there is no resistance in this ideal circuit, the capacitors will charge and discharge instantaneously, i.e., V_1 and V_2 will follow the changes in V_i without any delay. If V_i changes by ΔV_i, we must have $\Delta V_1 + \Delta V_2 = \Delta V_i$. How does ΔV_i get shared between V_1 and V_2? Since $i = C\,\dfrac{dV}{dt}$ for a capacitor, we can write

$$i = C_1 \frac{dV_1}{dt} \rightarrow \Delta V_1 = \frac{1}{C_1}\int_{t_1}^{t_2} i\,dt\,, \qquad i = C_2 \frac{dV_2}{dt} \rightarrow \Delta V_2 = \frac{1}{C_2}\int_{t_1}^{t_2} i\,dt\,. \tag{1}$$

The current is the same for the two capacitors, so is $\displaystyle\int_{t_1}^{t_2} i\,dt$, and therefore $\dfrac{\Delta V_1}{\Delta V_2} = \dfrac{C_2}{C_1}$.
If a resistor is added to the circuit (see Fig. 1 (b)), the above relationship between ΔV_1 and ΔV_2 continues to hold. In this case, the charging and discharging processes will not be instantaneous and will be governed by the time constant $R\,(C_1 + C_2)$.

Exercise Set

1. For a small value of R which makes the time constant negligibly small compared to the period T of the input square wave, run the simulation, and plot $V_i(t)$, $V_A(t)$ together for the following cases.

   (a) $C_1 = 1\,\text{pF}$, $C_2 = 1\,\text{pF}$.

   (b) $C_1 = 1\,\text{pF}$, $C_2 = 4\,\text{pF}$.

(c) $C_1 = 4\,\text{pF}$, $C_2 = 1\,\text{pF}$.

Explain your observations.

2. If R is increased, what changes do you expect in the above waveforms? For $R = 0.1\,\text{k}\Omega$, $1\,\text{k}\Omega$, $10\,\text{k}\Omega$, $100\,\text{k}\Omega$, run the simulation, and plot $V_i(t)$, $V_A(t)$ together. Check your answers.

References

1. W. H. Hayt and J. E. Kemmerly, *Engineering Circuit Analysis*, Prentice-Hall India, 1998. McGraw-Hill, 1971.