ee101_reso_rlc_1.sqproj

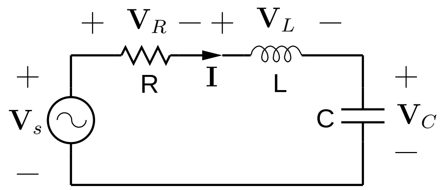

Figure 1: Series RLC circuit driven by a sinusoidal source.

The current $\mathbf{I}$ in the series RLC circuit shown in Fig. 1 is given by

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j(\omega L - 1/\omega C)} \,. \tag{1}$$

The following results hold for this circuit in the sinusoidal steady state:

(a) The total impedance seen by the source is minimum at $\omega_0 = 1/\sqrt{LC}$ rad/s, and is equal to R, giving $|\mathbf{I}|_{\text{max}} = \mathbf{V}_s/R$.

(b) The bandwidth of the circuit (i.e., $B = \omega_2 - \omega_1$, where ω_1 and ω_2 are frequencies at which $|\mathbf{I}| = |\mathbf{I}|_{\text{max}}/\sqrt{2}$) is given by $B = R/L$.

(c) The quality $Q = \dfrac{\omega_0}{B} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ is a measure of the sharpness of the $|\mathbf{I}|$ versus frequency curve. The quality of the circuit can be visually judged by plotting $|\mathbf{I}|$ versus $\log\omega$ (or $\log f$).

(d) For $\omega \ll \omega_0$, the total impedance is dominated by the capacitor, i.e., $\mathbf{Z} \approx 1/j\omega C$. Almost the entire source voltage appears across the capacitor, i.e., $\mathbf{V}_C \approx \mathbf{V}_s$, and the current $\mathbf{I}$ has a phase of $+\pi/2$.

(e) For $\omega \gg \omega_0$, the total impedance is dominated by the inductor, i.e., $\mathbf{Z} \approx j\omega L$. Almost the entire source voltage appears across the inductor, i.e., $\mathbf{V}_L \approx \mathbf{V}_s$, and the current $\mathbf{I}$ has a phase of $-\pi/2$.

Exercise Set

1. For $R = 1\,\Omega$, $L = 1\,\text{m}H$, and $C = 1\,\mu F$, calculate f_0 and B. Verify with simulation.

2. What happens to the bandwidth if R is doubled? Plot $|\mathbf{I}|$ versus $\log f$ for $R = 1\,\Omega$ and $R = 2\,\Omega$ on the same graph.

3. What is $|\mathbf{V}_C|$ for $\omega \ll \omega_0$, $\omega = \omega_0$, and $\omega \gg \omega_0$? Make an approximate sketch of $|\mathbf{V}_C|$ versus $\log \omega$, using the above information. Check with simulation.

4. What will happen to the $|\mathbf{V}_C|$ versus $\log \omega$ curve if R is doubled? Relate your answer to the Q of the circuit.

5. Sketch $|\mathbf{V}_L|$ and $|\mathbf{V}_C|$ versus $\log \omega$ together. Verify with simulation.

6. Sketch $\angle \mathbf{V}_L$ and $\angle \mathbf{V}_C$ versus $\log \omega$ together. Verify with simulation.

7. From the log-log simulation plot of $|\mathbf{V}_L|$, $|\mathbf{V}_C|$, $|\mathbf{V}_R|$ versus frequency, find the slope of each curve (in dB/decade) away from resonance on both sides (i.e., for $f \gg f_0$ and $f \ll f_0$). Explain your observations.