DC−AC Converter (`PE_1ph_VSI_6.sqproj`)

Question: The switches S_1 and S_2 in Fig. 1 (a) are switched in a complementary fashion with sinusoidal pulse width modulation. The modulating voltage $v_m(t) = 0.8 \sin 200\pi t$ and the triangular carrier (v_c) are as shown in Fig. 1 (b). The carrier frequency is 5 kHz. The load is an inductive load with $R = 12\,\Omega$ and $X_L = 16\,\Omega$ at 100 Hz. Determine

(i) the RMS output voltage at fundamental frequency.

(ii) the peak value of the 100 Hz component of the load current.

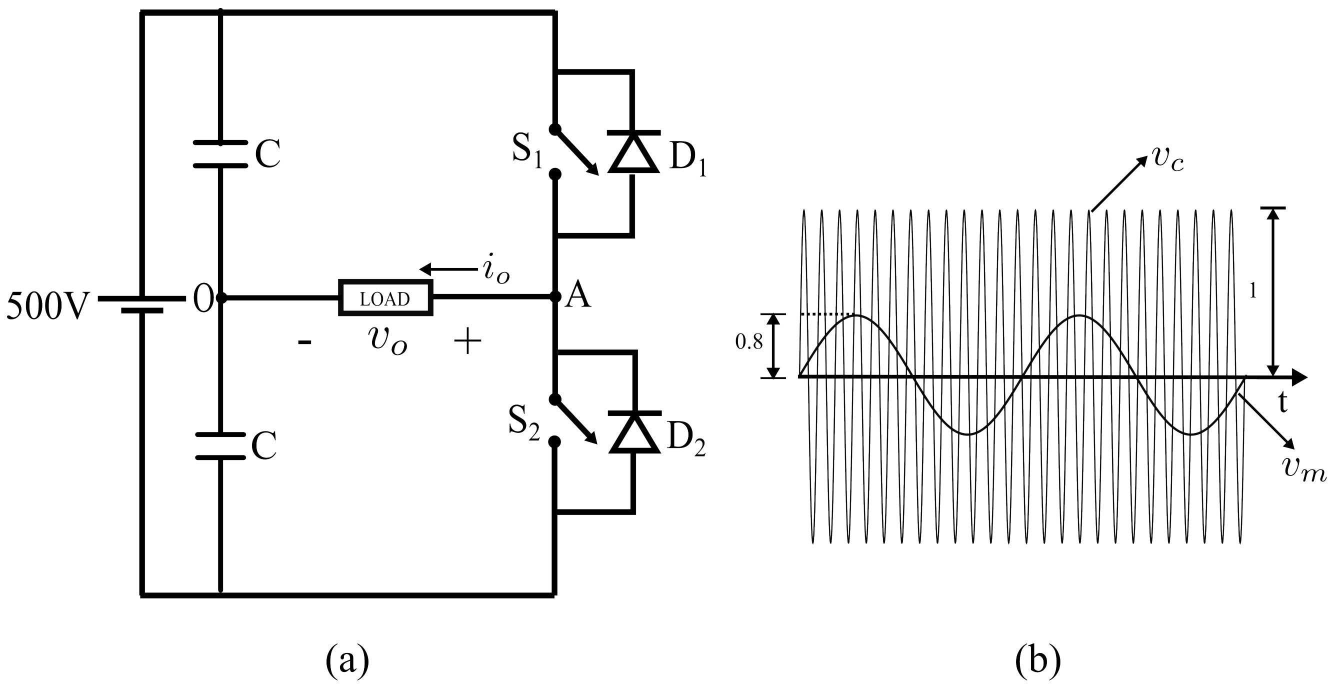

Figure 1: Half−bridge inverter

Solution :

Pulse−width modulation (PWM) techniques are employed for the control of output voltage. In PWM method, width of the output voltage gets modulated while maintaining amplitude as constant.The shaping of the output voltage waveform is generally achieved by having multiple pulses in each half period of the reference signal. Sinusoidal pulse−width modulation (SPWM) is a particular type of of multiple−pulse PWM.

Operation : SPWM technique uses a low−frequency (f_1) sinusoidal wave as the reference signal and a high−frequency (f_s) triangular wave as the carrier signal. From the intersections between the reference and the carrier signals, a number of pulses are generated. The output pulses are not identical i.e., they have variable pulse−width. The width of the pulses varies in accordance with the magnitude of sinusoidal waveform. These pulses functions as the trigger pulses for the respective switches. Modulated waveform with SPWM technique is shown in the Fig. 2.

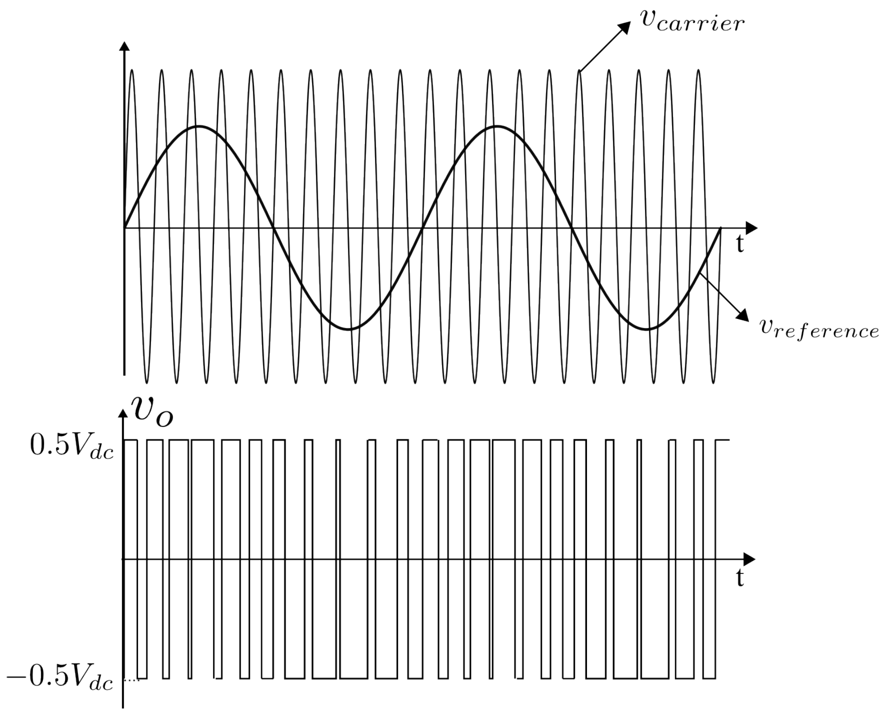

Figure 2: Output voltage waveform for half−bridge VSI using SPWM technique

In each half period, the pulse−width is maximum in the middle. From the center, pulse−width decreases as shown in the Fig. 2.

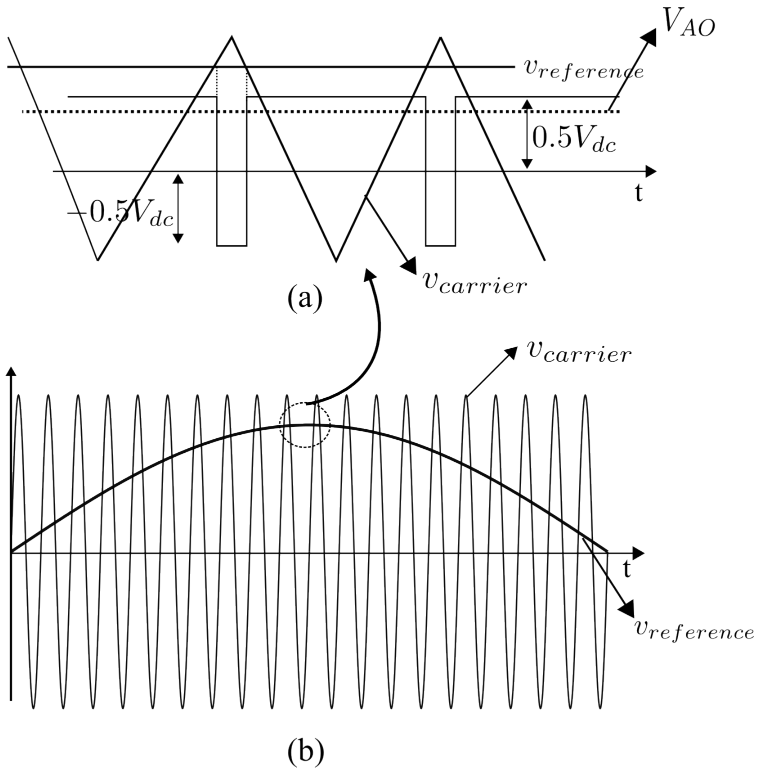

Figure 3: Output voltage waveform over one switching time period of carrier wave

The average output voltage (the output voltage averaged over one switching time period ($T_s = \frac{1}{f_s}$) V_{AO} is given by

$$V_{AO} = \frac{v_{control}}{\hat{v}_{tri}} \times \frac{V_{dc}}{2} \tag{1}$$

where $v_{control}$ is the reference sinusoidal waveform and $\hat{v}_{tri}$ is the peak value of carrier waveform.

Assuming $v_{reference}$ is constant (as shown in Fig. 3 (a)) over a switching time period, the eq. (1) indicates how the instantaneous average value of V_{AO} varies from one switching period to next. This instantaneous average is the same as the fundamental frequency component of V_{AO}.

$v_{control}$ vary sinusoidally at $f_1 = \frac{\omega_1}{2\pi}$, which is the fundamental frequency of inverter output.

$$v_{control} = \hat{v}_{control} \sin \omega_1 t \qquad , \qquad \text{where } \hat{v}_{control} \leq \hat{v}_{tri}$$

The fundamental frequency component of output voltage $(V_{AO})_1$ varies sinusoidally and is in phase with $v_{control}$. It is given by

$$(V_{AO})_1 = \frac{\hat{v}_{control}}{\hat{v}_{tri}} \sin \omega_1 t \frac{V_{dc}}{2} \tag{2}$$

The ratio of $\hat{v}_{control}$ to $\hat{v}_{tri}$ is defined as the amplitude modulation index (m_a), i.e.,

$$(V_{AO})_1 = m_a \sin \omega_1 t \frac{V_{dc}}{2}$$

(i) Modulation index, $m_a = \dfrac{\hat{v}_{control}}{\hat{v}_{tri}}$. From the Fig. 1. (b), $m_a = 0.8$.
Therefore the fundamental voltage waveform is given by

$$(V_{AO})_1 = \left(\frac{0.8 \times 500}{2}\right) \sin 200\pi t$$

and the RMS output voltage at fundamental frequency is given by

$$(V_{AO})_1^{\text{rms}} = \frac{m_a}{\sqrt{2}} . \frac{V_{dc}}{2} = \frac{0.8}{\sqrt{2}} \times 250 = 141.42\,\text{V}$$

(ii) The fundamental load current component is given by

$$\begin{aligned} (I_{AO})_1 &= \frac{\left(m_a \frac{V_{dc}}{2}\right) \sin(\omega_1 t - \Phi_1)}{Z_{L1}} \quad , \quad \text{where} \quad \Phi_1 = \tan^{-1}\left(\frac{X_{L1}}{R}\right) \\ &= \frac{\left(m_a \frac{V_{dc}}{2}\right) \sin(\omega_1 t - \Phi_1)}{\sqrt{R^2 + {X_{L1}}^2}} \end{aligned}$$

Therefore the peak value of the 100 Hz component of the load current is given by

$$\begin{aligned} (\hat{I}_{AO})_1 &= \frac{m_a . \frac{V_{dc}}{2}}{\sqrt{R^2 + {X_{L1}}^2}} \\ &= \frac{0.8 \times 250}{\sqrt{12^2 + 16^2}} = 10\,\text{A} \end{aligned}$$

SequelApp Exercises:

(1) A half−bridge voltage source inverter is supplying an RL load with $R = 8\,\Omega$ and $X_L = 6\,\Omega$ at 100 Hz. The desired fundamental frequency of the load voltage is 100 Hz. The switch control signals of the converter are generated using sinusoidal pulse−width modulation with modulation index $m_a = 0.7$. The carrier frequency is 5 kHz. If the peak value of the 100 Hz component of the load current is 20 A, find the dc voltage V_{dc}.

Verify your answer using SequelApp.