DC−AC Converter (`PE_1ph_VSI_7.sqproj`)

Question: A half−bridge voltage source inverter (VSI) is supplying an RL load with $R = 40\,\Omega$ and $L = \dfrac{0.3}{\pi}\,\text{H}$ as shown in Fig. 1. The desired fundamental frequency of the load voltage is 50 Hz. The switch control signals of the converter are generated using sinusoidal pulse−width modulation with modulation index $m_a = 0.6$. The RL load draws an active power of 1.44 kW. The carrier frequency is 5 kHz. Find the value of the DC source voltage V_{dc}.

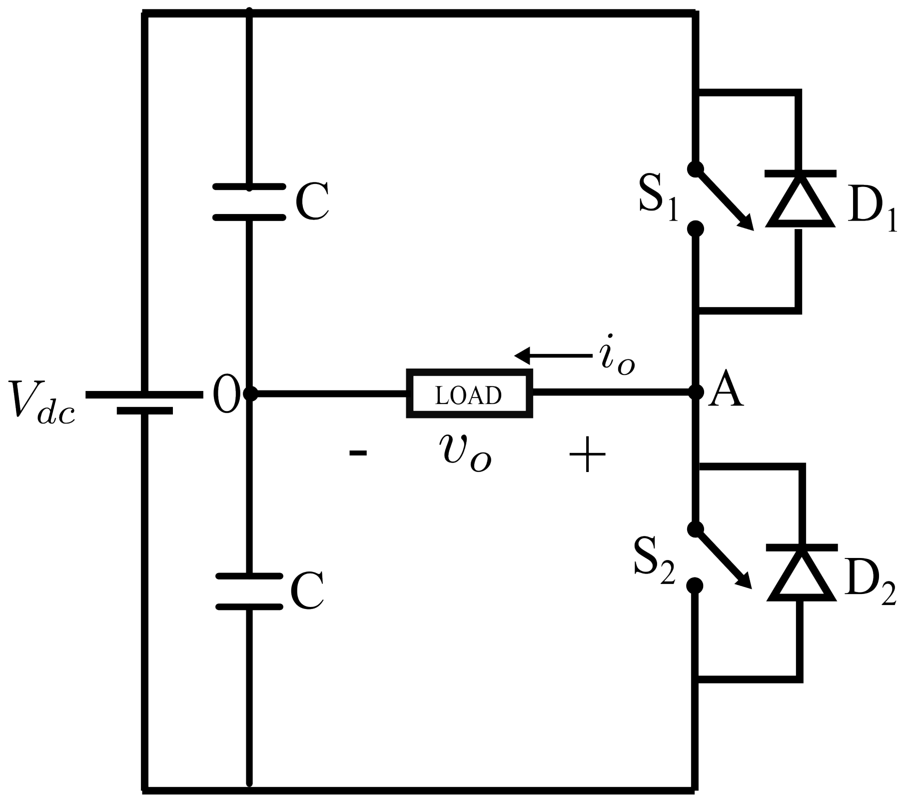

Figure 1: Half-bridge inverter

Solution :

Pulse−width modulation (PWM) techniques are employed for the control of output voltage. In PWM method, width of the output voltage gets modulated while maintaining amplitude as constant.The shaping of the output voltage waveform is generally achieved by having multiple pulses in each half period of the reference signal. Sinusoidal pulse−width modulation (SPWM) is a particular type of of multiple−pulse PWM.

Operation : SPWM technique uses a low−frequency (f_1) sinusoidal wave as the reference signal and a high−frequency (f_s) triangular wave as the carrier signal. From the intersections between the reference and the carrier signals, a number of pulses are generated. The output pulses are not identical i.e., they have variable pulse−width. The width of the pulses varies in accordance with the magnitude of sinusoidal waveform. These pulses functions as the trigger pulses for the respective switches. Modulated waveform with SPWM technique is shown in the Fig. 2.

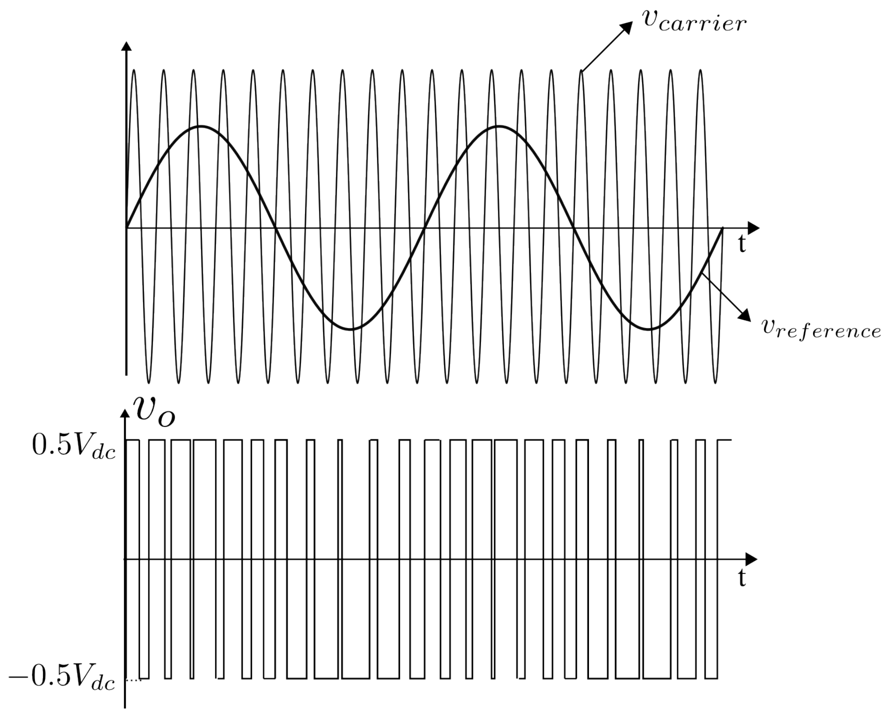

Figure 2: Output voltage waveform for half-bridge VSI using SPWM technique

In each half period, the pulse−width is maximum in the middle. From the center, pulse width decreases as shown in the Fig. 2.

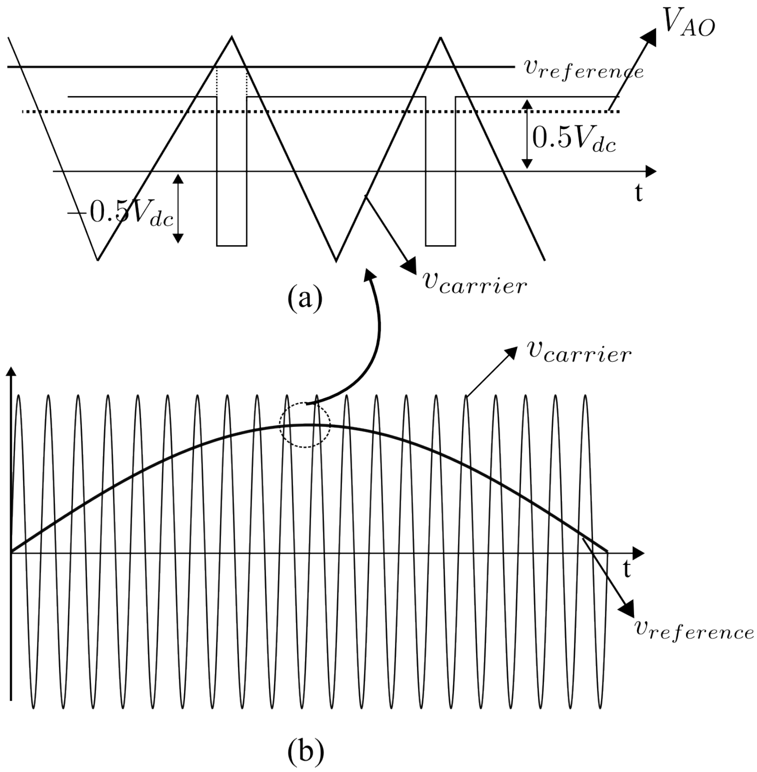

Figure 3: Output voltage waveform over one switching time period of carrier wave

The average output voltage (the output voltage averaged over one switching time period $(T_s = \frac{1}{f_s})$ V_{AO} is given by

$$V_{AO} = \frac{v_{control}}{\hat{v}_{tri}} \times \frac{V_{dc}}{2} \tag{1}$$

where $v_{control}$ is the reference sinusoidal waveform and $\hat{v}_{tri}$ is the peak value of carrier waveform.

Assuming $v_{reference}$ is constant over a switching time period, the eq. (1) indicates how the instantaneous average value of V_{AO} varies from one switching period to next. This instantaneous average is the same as the fundamental frequency component of V_{AO}.

$v_{control}$ vary sinusoidally at $f_1 = \frac{\omega_1}{2\pi}$, which is the fundamental frequency of inverter output.

$$v_{control} = \hat{v}_{control} \sin \omega_1 t \qquad , \qquad \text{where } \hat{v}_{control} \leq \hat{v}_{tri}$$

The fundamental frequency component of output voltage $(V_{AO})_1$ varies sinusoidally and is in phase with $v_{control}$. It is given by

$$(V_{AO})_1 = \frac{\hat{v}_{control}}{\hat{v}_{tri}} \sin\omega_1 t \frac{V_{dc}}{2} \tag{2}$$

The ratio of $\hat{v}_{control}$ to $\hat{v}_{tri}$ is defined as the amplitude modulation index (m_a), i.e.,

$$(V_{AO})_1 = m_a \sin\omega_1 t \frac{V_{dc}}{2}$$

Modulation index, $m_a = \dfrac{\hat{v}_{control}}{\hat{v}_{tri}}$=0.6.

Therefore the fundamental voltage waveform is given by

$$(V_{AO})_1 = \Big(\frac{0.6 \times V_{dc}}{2}\Big) \sin 100\pi t$$

and the RMS output voltage at fundamental frequency is given by

$$(V_{AO})_1^{rms} = \frac{m_a}{\sqrt{2}}.\frac{V_{dc}}{2} \tag{3}$$

The fundamental load current component is given by

$$\begin{aligned}(I_{AO})_1 &= \frac{\big(m_a \frac{V_{dc}}{2}\big)\sin\left(\omega_1 t - \Phi_1\right)}{Z_{L1}} \quad , \qquad \text{where} \qquad \Phi_1 = \tan^{-1}\Big(\frac{X_{L1}}{R}\Big)\\ &= \frac{\big(m_a \frac{V_{dc}}{2}\big)\sin\left(\omega_1 t - \Phi_1\right)}{\sqrt{R^2 + {X_{L1}}^2}}\end{aligned}$$

The RMS value of fundamental load current component is

$$(I_{AO})_1^{rms} = \frac{(V_{AO})_1^{rms}}{Z_{L1}} = \frac{(V_{AO})_1^{rms}}{\sqrt{R^2 + X_{L1}^2}}$$

Although there is significant amount of harmonic voltages, all these harmonics are at very high frequencies. The current contribution due to the harmonic voltages are very small because of the impedance offered by the inductance is very high at higher frequencies. Therefore, only the fundamental components are considered for active power calculation.

$$\begin{aligned}\therefore \text{Active Power} &= \Big(\frac{(V_{AO})_1^{rms}}{Z_{L1}}\Big)^2.R\\ 1440 &= \Big(\frac{(V_{AO})_1^{rms}}{50}\Big)^2.40 \qquad , \qquad R = 40\,\Omega, X_{L1} = 30\,\Omega\end{aligned} \tag{4}$$

From eqns. 3 and 4, $V_{dc} = \sqrt{\dfrac{1440 \times 50^2}{40}} \times \dfrac{2\sqrt{2}}{0.6} = 1000\sqrt{2}\,\mathrm{V} = 1414.21\,\mathrm{V}$

SequelApp Exercises:

(1) A half−bridge voltage source inverter is supplying an RL load. The DC source voltage V_{dc} is 800 V. The desired fundamental frequency of the load voltage is 50 Hz. The switch control signals of the converter are generated using sinusoidal pulse-width modulation with modulation index m_a=0.8. The RL load draws an active power of 0.5 kW. If the inductive reactance of the load at 50 Hz is 40 Ω, find the value of the load resistance. The carrier frequency is 5 kHz.

Verify your answer using SequelApp.