Boost Converter (`PE_boost_4.sqproj`)

Question: A boost converter feeding a resistive load is shown in Fig. 1. The switching frequency is 50 kHz and the duty ratio (D) is 0.45. The average and maximum current flowing through the inductor is 100 mA and 250 mA, respectively. Assume that all the components are ideal, and the capacitor and inductor are operating in steady state. Find

(i) the average output voltage.

(ii) the values of inductance and load resistance.

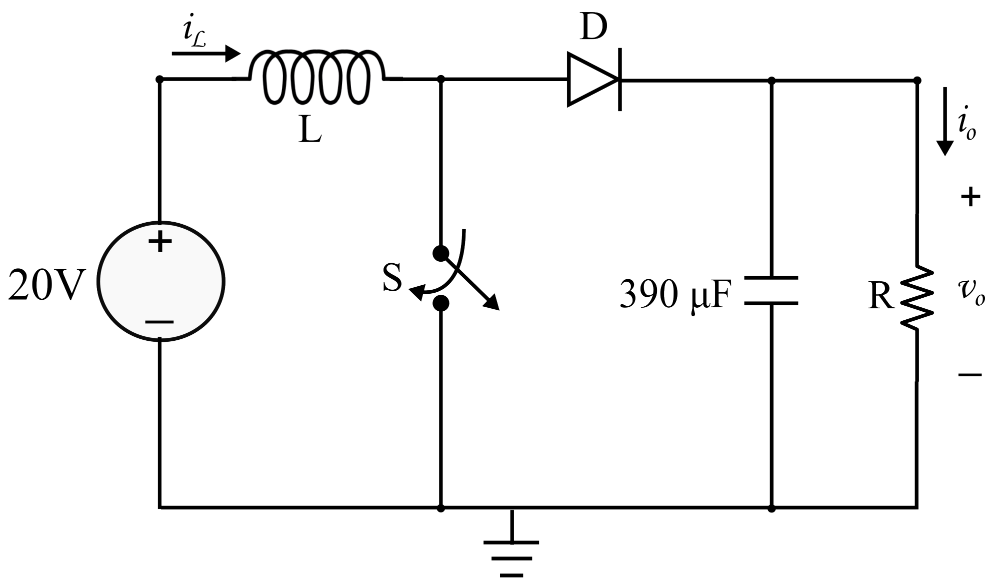

Figure 1: Boost converter

Solution :

Figs. 2 (a) and 2 (b) show ON and OFF conditions of switch S, respectively.

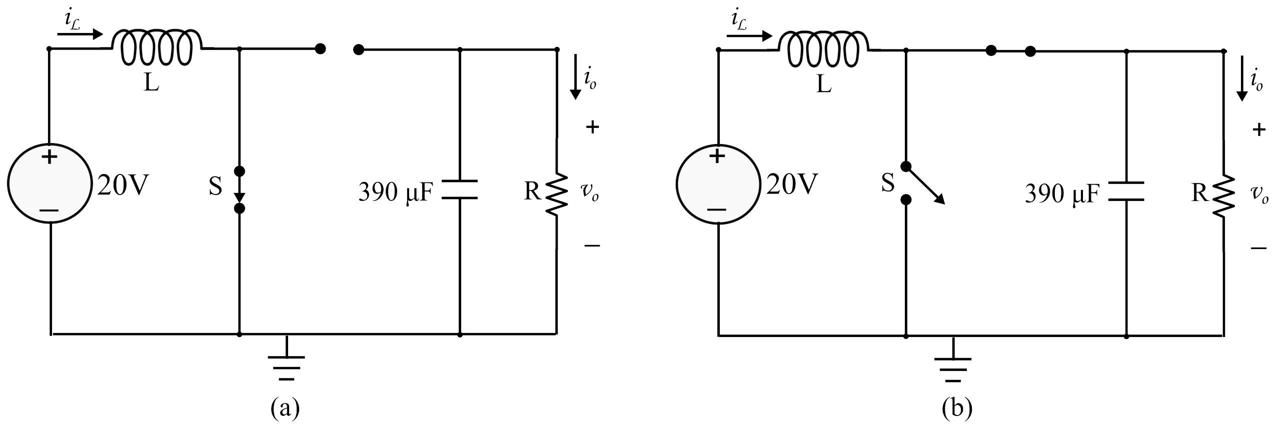

Figure 2: Boost converter circuit. (a) switch is ON, (b) switch is OFF and i_L is non-zero

When $I_L^{\text{avg}} < \dfrac{\Delta I_L}{2}$, the inductor current is discontinuous as shown in Fig. 3, where I_L^{avg} is the average inductor current and ΔI_L is the peak-to-peak ripple inductor current. The circuit is operating under steady state, i.e., the energy stored in the inductor during the ON interval should be released during the OFF interval. Since the inductor current is discontinuous, the inductor releases the complete stored energy before starting the next cycle .

Operation: When the switch is ON $(0 < t < DT)$, the diode is reverse biased and the inductor stores energy. When the switch is OFF and the inductor is releasing energy $(DT < t < \beta T)$, the diode is forward biased. When the inductor current is zero $(\beta T < t < T)$, the diode is again reverse biased.

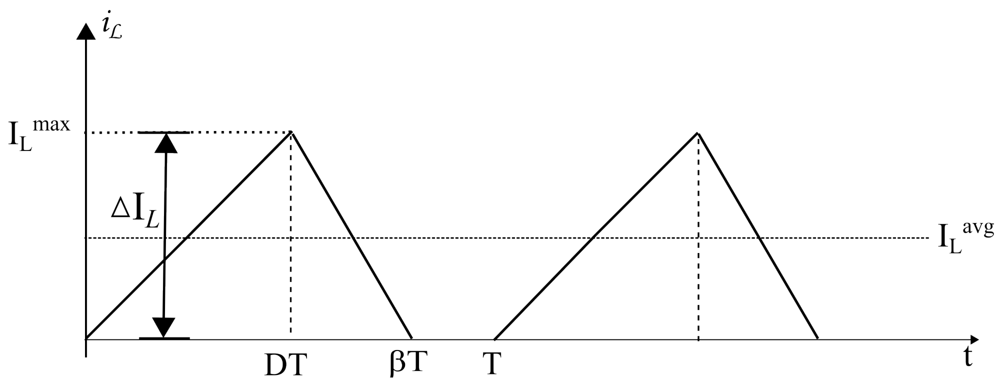

Figure 3: Inductor current waveform for discontinuous conduction

For $0 < t < DT$, $\quad i_L(t) = \frac{V_{in}}{L}\, t$

At $t = DT$, $\quad i_L(t) = I_L^{\max}$

$$\therefore I_L^{\max} = \frac{V_{in}\, DT}{L} \tag{1}$$

For $DT < t < \beta T$, $\quad i_L(t) = \frac{(V_{in} - V_o)}{L}\,(t - DT) + I_L^{\max}$

At $t = \beta T$, $\quad i_L(t) = 0$

$$\therefore I_L^{\max} = \frac{(V_o - V_{in})}{L}\,(\beta T - DT) \tag{2}$$

From Eqs. 1 and 2,

$$\frac{V_{in}\, DT}{L} = \frac{(V_o - V_{in})}{L}\,(\beta T - DT)$$

$$\boxed{\therefore V_o^{\text{DCM}} = V_{in}\left(\frac{\beta}{\beta - D}\right)} \tag{3}$$

where, V_o^{DCM} is the average voltage across load in discontinuous conduction mode. The value of β is less than 1 and greater than D.

$$\boxed{V_o^{\text{CCM}} = \frac{V_{in}}{1 - D}} \tag{4}$$

where, V_o^{CCM} is the average voltage across load in continuous conduction mode.

From Eqns. 3 and 4,

$$\boxed{V_o^{\text{DCM}} > V_o^{\text{CCM}}}$$

In boost converter, the output voltage in DCM mode is greater than output voltage in CCM mode.

(i) The average inductor current is given by

$$\begin{aligned} I_L^{\text{avg}} &= \frac{1}{T}\int_0^T i_L\, dt = \frac{1}{T}\int_0^{\beta T} i_L\, dt \\ &= \frac{1}{T}\left(\frac{1}{2}\,\beta T\, .I_L^{max}\right) \end{aligned}$$

$$\therefore\ 100 = 0.5\ \beta I_L^{\max}\ \Rightarrow \beta = 2 \times \frac{I_L^{\text{avg}}}{I_L^{\max}} \Rightarrow \beta = 2 \times \frac{100}{250} = 0.8$$

$$\therefore \mathrm{V}_0 = \frac{20 \times 0.8}{0.8 - 0.45} = 45.71\,\mathrm{V}$$

(ii) From eqn. 1, $I_L^{\max} = \Delta I_L = \dfrac{V_{in}\,DT}{L}$

$$\Rightarrow L = \frac{20 \times (0.45)}{(50 \times 10^3) \times (250 \times 10^{-3})} = 720\,\mu H$$

The overall circuit is lossless, i.e., $P_{out} = P_{in}$.

$$\therefore V_o I_o = V_{in} I_{in} \Longrightarrow I_o = \frac{V_{in} I_{in}}{V_o} = \frac{20 \times 100 \times 10^{-3}}{45.71} = 43.75\,\mathrm{mA}$$

$$V_o = R I_o \quad \Longrightarrow \quad R = \frac{V_o}{I_o} = 1.0448\,k\,\Omega$$

SequelApp Exercises:

1. Let the load resistance R be $2\,\mathrm{k}\,\Omega$. For an output voltage $V_o = 40\,\mathrm{V}$, what value of inductance is required? (All other circuit parameters remain same.)

Verify your answer using SequelApp.