Boost Converter `(PE_boost_6.sqproj)`

Question : In Fig. 1, a cascaded boost converter feeds a resistive load from a low voltage battery source. Switches S_1 and S_2 are simultaneously switched with a frequency of 10 kHz and a duty ratio (D) of 0.5. The load draws a constant power of 200 W and all the elements of circuit are assumed to be ideal. Find

(i) the average value of output voltage

(ii) the value of load resistance

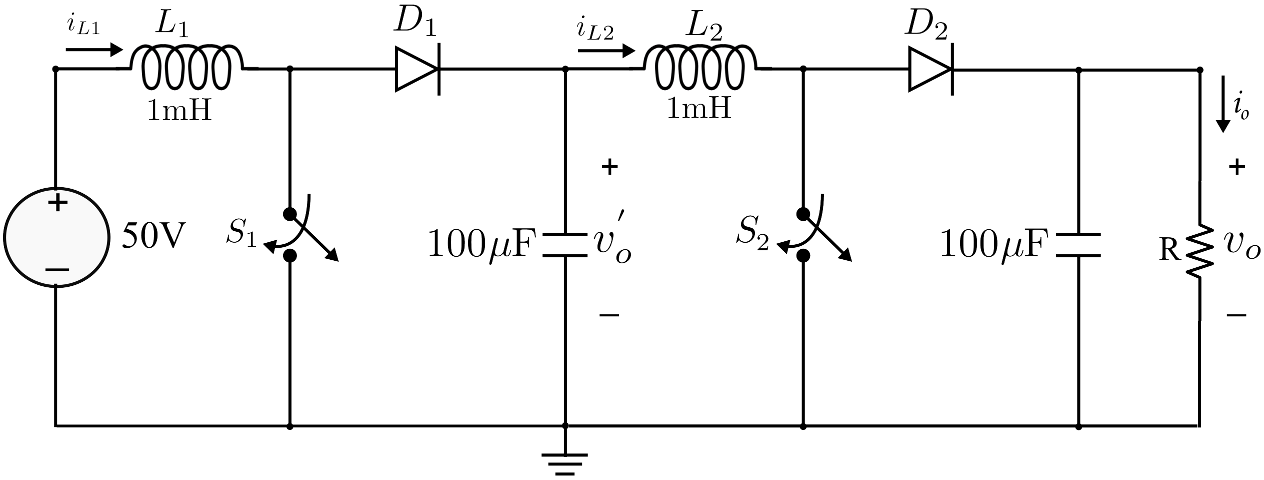

Figure 1: Cascaded boost converter

Solution :

The DC – DC converter shown in Fig. 1 has two boost converters connected in a cascaded fashion. The first converter's output is the input to the second converter and the two converters can be considered separately.

Assume that the inductor currents (i_{L1} and i_{L2}) are continuous.

Figs. 2 (a) and 2 (b) shows ON and OFF conditions of switches, respectively.

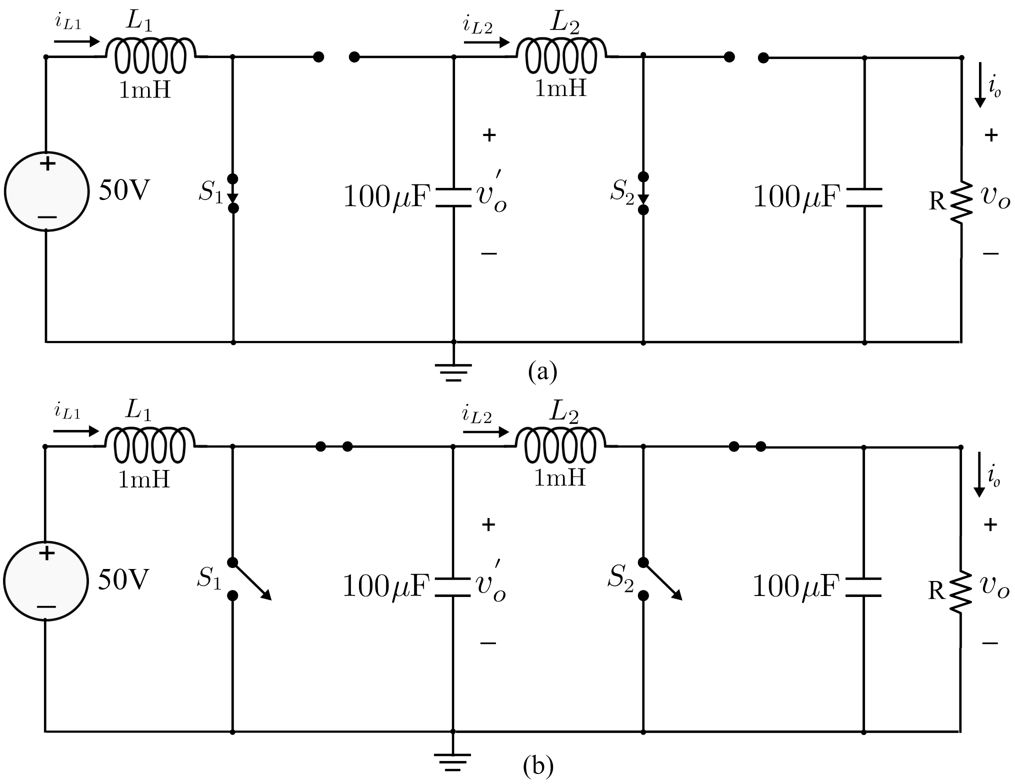

Figure 2: Cascaded boost converter circuit. (a) switches are ON, (b) switches are OFF

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The overall circuit is lossless, i.e., $P_{out} = P_{in}$.

$$\therefore P_{out} = V_{in} I_{in} \implies I_{in} = I_{L1}^{\text{avg}} = \frac{P_{out}}{V_{in}} = \frac{200}{50} = 4\,\text{A}$$

When the inductor current is continuous, $I_L^{\text{avg}} > \dfrac{\Delta I_L}{2}$, where I_L^{avg} is the average inductor current and ΔI_L is the peak – to – peak ripple inductor current as shown in Fig. 3.

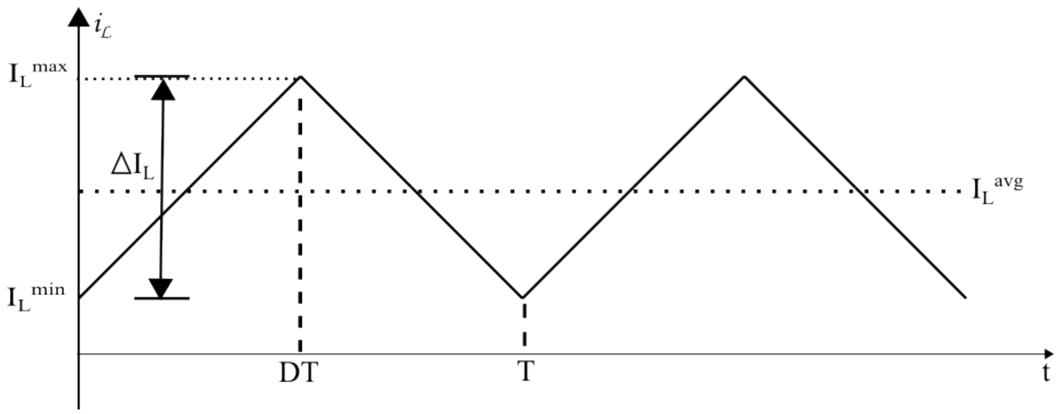

Figure 3: Inductor current waveform for continuous conduction

From Fig. 3, we can see that the peak – to – peak inductor current is

$$\Delta I_L = I_L^{\max} - I_L^{\min}$$

For $0 < t < DT$, $\quad i_L(t) = \dfrac{V_{in} \times t}{L} + I_L^{min}$

At $t = DT$, $\quad i_L(t) = I_L^{\max}$

$$\therefore\ I_L^{\max} = \frac{V_{in}\,DT}{L} + I_L^{\min}$$

$$\boxed{\Delta I_L = \frac{V_{in}\,D}{L\,f}}$$

$$\therefore \text{The peak} - \text{to} - \text{peak current through } L_1\,(\Delta I_{L1}) = \frac{V_{in}\,D}{L\,f}$$

$$= \frac{50 \times 0.5}{(1 \times 10^{-3}) \times (10 \times 10^3)} = 2.5\,\text{A}$$

From the above calculations $I_{L1}^{\text{avg}} > \dfrac{\Delta I_{L1}}{2}$, i.e., the current through inductor L_1 is continuous and the first converter is operating in continuous conduction mode (CCM).

Operation of CCM : When the switch is ON $(0 < t < DT)$, the diode is reverse biased and the inductor stores energy. Alternatively, when the switch is off $(DT < t < T)$, the diode is forward biased and the inductor releases energy.

The circuit is operating under steady state, i.e., the energy stored in the inductor during the ON interval should be released during the OFF interval.

The average voltage across capacitor C_1 can be derived as follows:

When the switch is ON, applying KVL gives

$$L\,\frac{di_{L1}}{dt} = V_{in}$$

Similarly, when the switch is OFF, applying KVL gives

$$L\,\frac{di_{L1}}{dt} = V_{in} - v_o'(t)$$

Applying the volt-sec balance equation,

$$\boxed{V_o' = \frac{V_{in}}{1-D}} \qquad \Longrightarrow \qquad \mathrm{V}_o' = \frac{50}{0.5} = 100\,\mathrm{V}$$

For the second converter, the average and peak − to − peak inductor current (I_{L2}^{avg} & ΔI_{L2}) can be calculated as follows:

$$V_{in}'\,(V_o') = 100\,\mathrm{V} \text{ and } P_{out} = 200\,\mathrm{W} \qquad \therefore\ I_{L2}^{avg} = \frac{P_{out}}{V_{in}'} = 2\,\mathrm{A}$$

$$\text{The peak} - \text{to} - \text{peak current through } L_2\,(\Delta I_{L2}) = \frac{V_{in}'\,D}{L\,f}$$

$$= \frac{100 \times 0.5}{(1 \times 10^{-3}) \times (10 \times 10^{3})} = 5\,\mathrm{A}$$

From the above calculations, $I_{L2}^{\mathrm{avg}} < \dfrac{\Delta I_{L2}}{2}$, i.e., the current through inductor L_2 is discontinuous (as shown in Fig. 4) and therefore the second converter is operating in discontinuous conduction mode (DCM).

Operation of DCM: When the switch is ON ($0 < t < DT$), the diode is reverse biased and the inductor stores energy. When the switch is OFF and the inductor is releasing energy ($DT < t < \beta T$), the diode is forward biased. When the inductor current is zero ($\beta T < t < T$), the diode is again reverse biased.

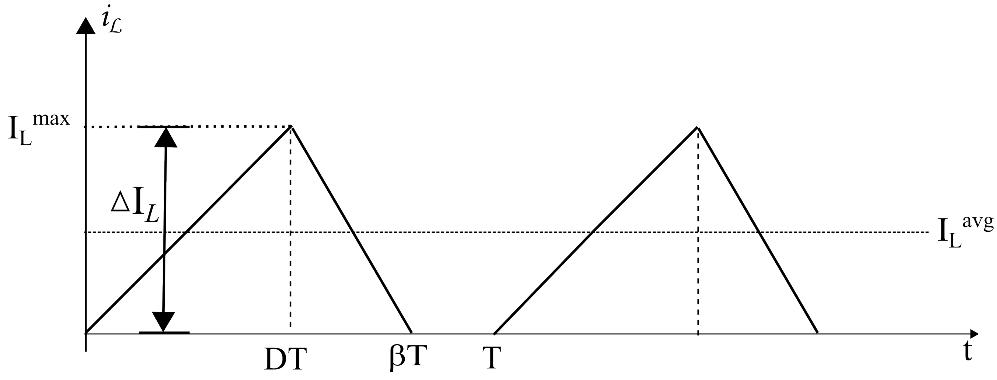

Figure 4: Inductor current waveform for discontinuous conduction

(i) The average output voltage for a boost converter in DCM can be derived as follows:

For $0 < t < DT$, $\quad i_L(t) = \frac{V_{in}'}{L}\,t$

At $t = DT$, $\quad i_L(t) = I_L^{\max}$

$$\therefore I_L^{\max} = \frac{V_{in}'\,DT}{L} \tag{1}$$

For $DT < t < \beta T$, $\quad i_L(t) = \frac{(V_{in}' - V_o)}{L}\,(t - DT) + I_L^{\max}$

At $t = \beta T$, $\quad i_L(t) = 0$

$$\therefore I_L^{\max} = \frac{(V_o - V_{in}')}{L}\,(\beta T - DT) \tag{2}$$

From Eqs. 1 and 2,

$$\frac{V_{in}\,DT}{L} = \frac{(V_o - V_{in}')}{L}\,(\beta T - DT)$$

$$\boxed{\therefore V_o = V_{in}'\left(\frac{\beta}{\beta - D}\right)}$$

The average inductor current is given by

$$\begin{aligned} I_{L2}^{\text{avg}} &= \frac{1}{T}\int_0^T i_{L2}\,dt = \frac{1}{T}\int_0^{\beta T} i_{L2}\,dt \\ &= \frac{1}{T}\left(\frac{1}{2}\,\beta T\,.I_{L2}^{max}\right) \end{aligned}$$

$$\therefore\ I_{L2}^{\text{avg}} = 0.5\beta I_{L2}^{\max} \Rightarrow \beta = 2 \times \frac{I_{L2}^{\text{avg}}}{I_{L2}^{\max}} \Rightarrow \beta = 2 \times \frac{2}{5} = 0.8$$

$$\therefore \mathrm{V}_0 = \frac{100 \times 0.8}{0.8 - 0.5} = 266.67\,\mathrm{V}$$

(ii) The load resistance is given by

$$\begin{aligned} P_{out} = V_o I_o &\Longrightarrow I_o = \frac{P_{out}}{V_o} = \frac{200}{266.67} = 0.75\,\text{mA} \\ V_o = R I_o &\Longrightarrow \quad R = \frac{V_o}{I_o} = 355.56\,\Omega \end{aligned}$$

SequelApp Exercises:

1. If the duty ratio of switches S_1 and S_2 are 0.6 and 0.35, respectively and load power is 300 W, find the new output voltage and load resistance, keeping all other circuit parameters same.

Verify your answer using SequelApp.