Buck Converter `(PE_buck_4.sqproj)`

Question: A buck converter feeding a resistive load is shown in Fig 1. The switching frequency is 50 kHz and the duty ratio is 0.6. The average and maximum current flowing through inductor is 100 mA and 250 mA, respectively. Assume that all the components are ideal, and the capacitor and inductor are operating in steady state. Determine

(i) the average output voltage.

(ii) the inductance and load resistance.

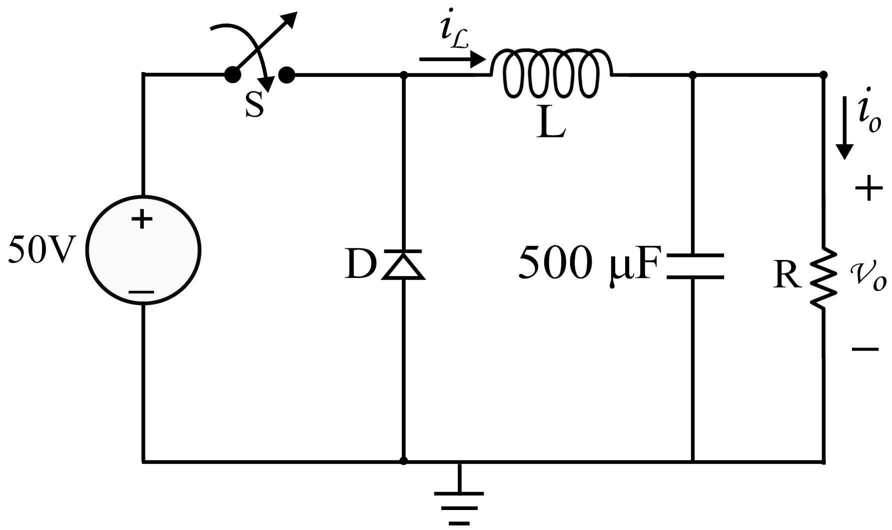

Figure 1: Buck converter

Solution:

Figs. 2 (a) and 2 (b) shows ON and OFF conditions of switch S, respectively.

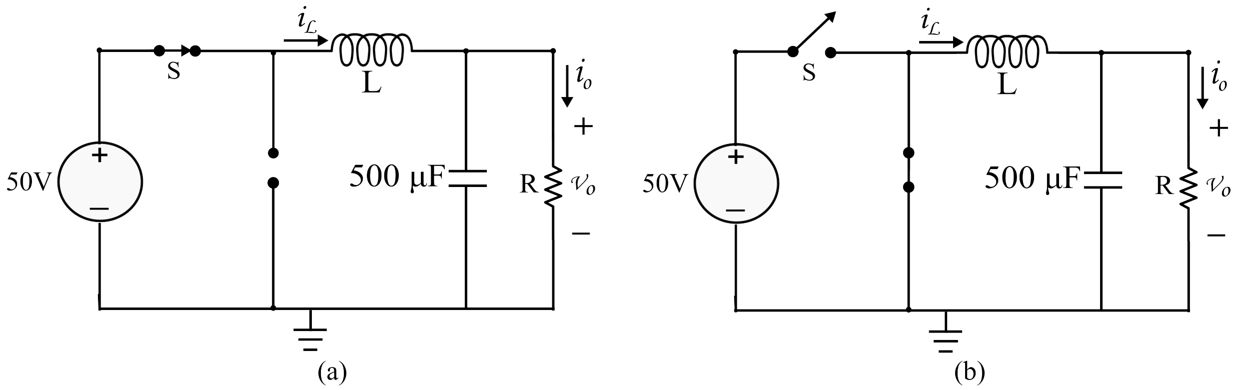

Figure 2: Buck converter circuit. (a) switch is ON, (b) switch is OFF and i_L is non-zero

The circuit is operating under steady state, i.e., the energy stored in the inductor during the ON interval should be released during the OFF interval.
When $I_L^{\text{avg}} < \dfrac{\Delta I_L}{2}$, the inductor current is discontinuous as shown in Fig. 3. Here, the inductor releases the complete stored energy before starting the next cycle.
Operation: When the switch is ON ($0 < t < DT$), the diode is reverse biased and the inductor stores energy. When the switch is OFF and the inductor is releasing energy ($DT < t < \beta T$), the diode is forward biased. When the inductor current is zero ($\beta T < t < T$), the diode is again reverse biased.

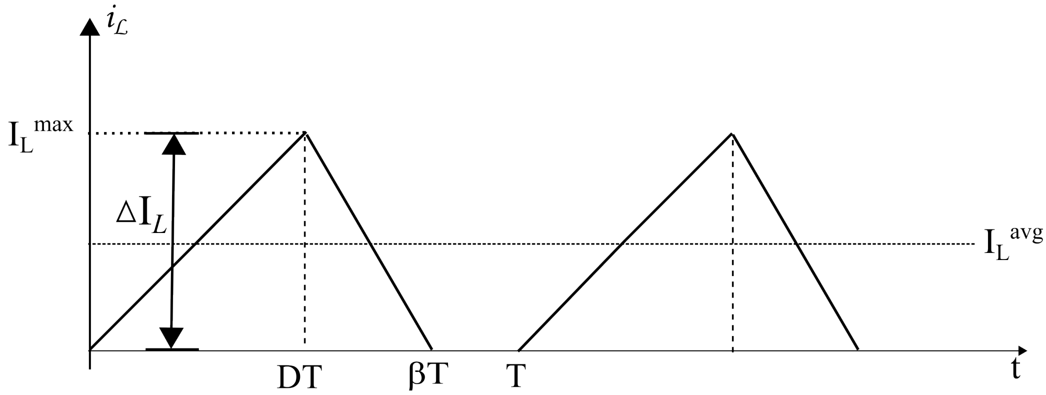

Figure 3: Inductor current waveform for discontinuous conduction

For $0 < t < DT$, $\quad i_L(t) = \dfrac{(V_{in} - V_o)\, t}{L}$

At $t = DT$, $\quad i_L(t) = I_L^{\max}$

where, V_o is the average voltage across the load. The voltage across the load is almost constant due to the smoothing capacitor. Hence instantaneous value (v_o) and average value (V_o) are considered as equal.

$$\therefore I_L^{\max} = \frac{(V_{in} - V_o)\, DT}{L} \tag{1}$$

For $DT < t < \beta T$, $\quad i_L(t) = \dfrac{-V_o\,(t - DT)}{L} + I_L^{\max}$

At $t = \beta T$, $\quad i_L(t) = 0$

$$\therefore I_L^{\max} = \frac{V_o\,(\beta T - DT)}{L} \tag{2}$$

Equating (1) and (2),

$$\frac{(V_{in} - V_o)\, DT}{L} = \frac{V_o\,(\beta T - DT)}{L}$$

$$\boxed{\therefore V_o^{\text{DCM}} = V_{in}\left(\frac{D}{\beta}\right)} \tag{3}$$

where, V_o^{DCM} is the average voltage across load in discontinuous conduction mode (DCM). The value of β is less than 1.

$$\boxed{V_o^{\text{CCM}} = DV_{in}} \tag{4}$$

where, V_o^{CCM} is the average voltage across load in continuous conduction mode (CCM). From equations (3) and (4),

$$\boxed{V_o^{\text{DCM}} > V_o^{\text{CCM}}}$$

In buck converter, the output voltage in DCM mode is greater than output voltage in CCM mode.

(i) The average inductor current (I_L^{avg}) is given by

$$\begin{aligned} I_L^{\text{avg}} &= \frac{1}{T}\int_0^T i_L\, dt = \frac{1}{T}\int_0^{\beta T} i_L\, dt \\ &= \frac{1}{T}\left(\frac{1}{2}\beta T \,.\, I_L^{max}\right) \end{aligned}$$

$$\therefore\ 100 = 0.5\ \times\beta\times I_L^{\max}\ \Rightarrow \beta = 2\times\frac{I_L^{\text{avg}}}{I_L^{\max}} \Rightarrow \beta = 2\times\frac{100}{250} = 0.8$$

$$\therefore \mathrm{V}_0 = \frac{50\times 0.6}{0.8} = 37.5\mathrm{V}$$

(ii) From eqn. 2, $I_L^{\max} = \dfrac{V_o\,(\beta T - DT)}{L}$

$$\Rightarrow L = \frac{37.5\times(0.8-0.6)}{(50\times 10^3)\times(250\times 10^{-3})} = 0.6\,\mathrm{mH}$$

The average current through a capacitor under steady state is zero.

$$\therefore\ I_L^{\text{avg}} = I_o$$

$$V_o = RI_o \qquad \Longrightarrow R = \frac{V_o}{I_o} = 375\,\Omega$$

SequelApp Exercises:

1. Let load resistance $R = 2\,\mathrm{k}\,\Omega$. For an output voltage $V_o = 40\,\mathrm{V}$, what value of inductance is required? (V_{in}, switching frequency and duty ratio remains same as of above question).

Verify your answer using SequelApp.