Buck Converter (`PE_buck_5.sqproj`)

Question: In the dc-dc buck converter shown in the figure, $V_{in} = 100\,\text{V}$ and $L = 0.5\,\text{mH}$. The duty cycle of the controllable switch (S) is $50\,\%$ and its switching frequency is $10\,\text{kHz}$. The average value of the load current is $2\,\text{A}$. Assume that all the components are ideal, and the capacitor and inductor are operating in steady state. Determine

(i) the average output voltage.

(ii) the respective time durations for which switch S and diode D conduct in each full cycle.

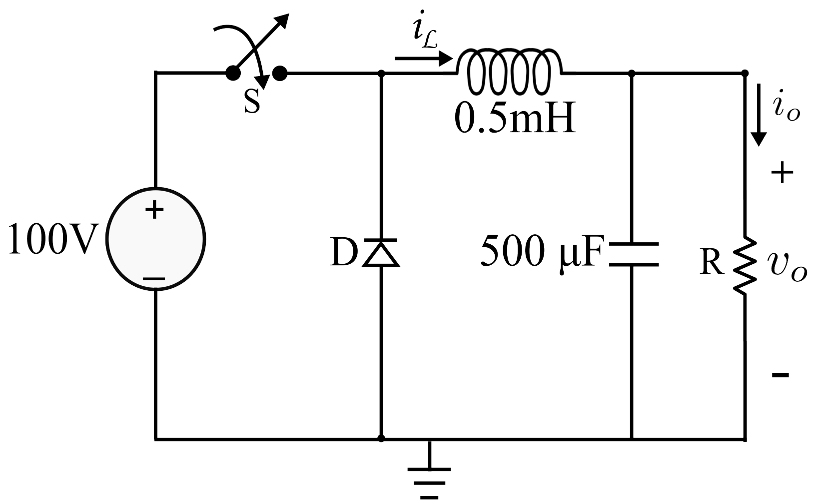

Figure 1: Buck converter

Solution:

Assume that the inductor current is continuous.

When the inductor current is continuous, $I_L^{\text{avg}} > \dfrac{\Delta\, I_L}{2}$, where I_L^{avg} is the average inductor current and $\Delta\, I_L$ is the peak-to-peak ripple inductor current as shown in Fig. 2.

When the switch is ON, applying KVL gives

$$L\,\frac{di_L}{dt} = V_{in} - v_o(t)$$

Similarly, when the switch is OFF, applying KVL gives

$$L\,\frac{di_L}{dt} = -v_o(t)$$

Applying the volt-sec balance equation,

$$\boxed{V_o = DV_{in}}$$

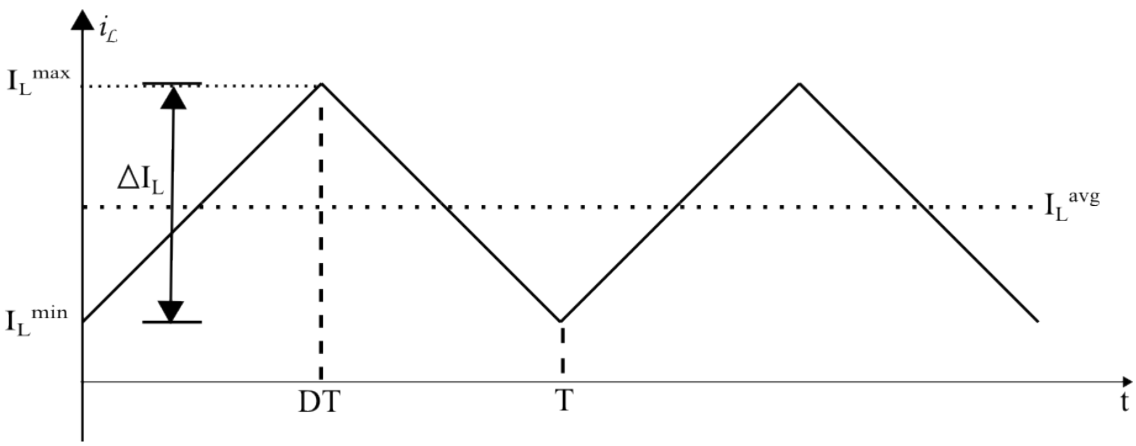

Figure 2: Inductor current waveform for continuous conduction

From Fig. 2, we can see that the peak to peak inductor current is

$$\Delta I_L = I_L^{\max} - I_L^{\min}$$

For $0 < t < DT$, $$i_L(t) = \frac{(V_{in} - V_o)\,t}{L} + I_L^{min}$$

At $t = DT$, $$i_L(t) = I_L^{\max}$$

$$\therefore\; I_L^{\max} = \frac{(V_{in} - V_o)\,DT}{L} + I_L^{\min}$$

$$\Delta I_L = \frac{V_{in}\,D\,(1-D)}{L\,f}$$

$$\therefore \Delta I_L = \frac{100 \times 0.5 \times 0.5}{(0.5 \times 10^{-3}) \times (10 \times 10^3)} = 5\,\mathrm{A}$$

The average current through a capacitor under steady state is zero.

$$\therefore\ I_L^{\text{avg}} = I_o = 2\,\mathrm{A}$$

From the above calculations, $I_L^{\text{avg}} < \dfrac{\Delta I_L}{2}$, i.e., inductor current is discontinuous and hence our assumption is wrong.

Figs. 3 (a) and 3 (b) shows ON and OFF conditions of switch S, respectively.

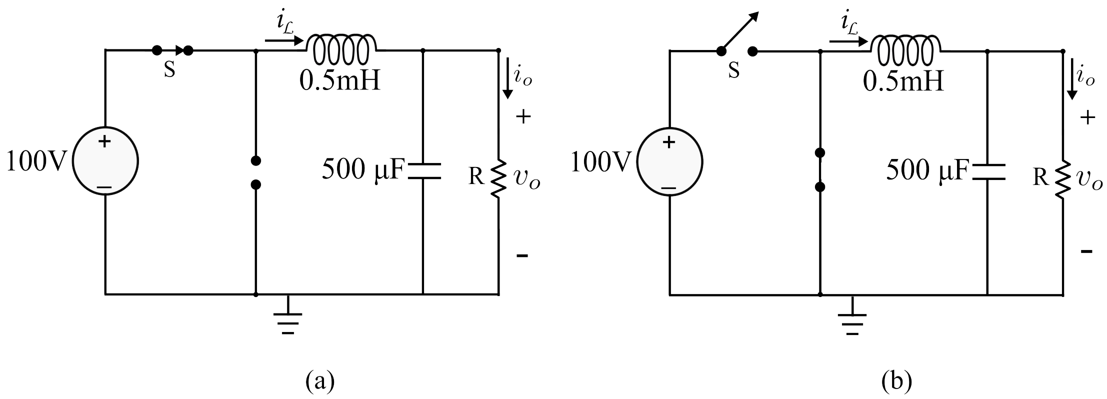

Figure 3: Buck converter circuit. (a) switch is ON, (b) switch is OFF and i_L is non-zero

The circuit is operating under steady state, i.e., the energy stored in the inductor during the ON interval should be released during the OFF interval. Since the inductor current is discontinuous, the inductor releases the complete stored energy before starting the next cycle as shown in Fig. 4.

Operation: When the switch is ON ($0 < t < DT$), the diode is reverse biased and the inductor stores energy. When the switch is OFF and the inductor is releasing energy ($DT < t < \beta T$), the diode is forward biased. When the inductor current is zero ($\beta T < t < T$), the diode is again reverse biased.

For $0 < t < DT$, $\qquad i_L(t) = \dfrac{(V_{in} - V_o)\,t}{L}$

At $t = DT$, $\qquad i_L(t) = I_L^{\max}$

where, V_o is the average voltage across the load. The voltage across the load is assumed to be ripple-free and hence instantaneous value (v_o) and average value (V_o) are considered as equal for the calculations of inductor current.

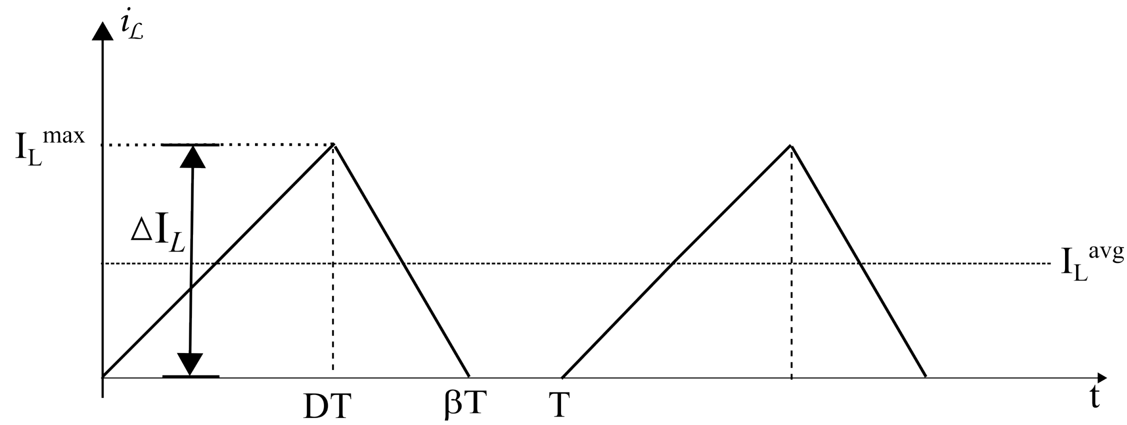

Figure 4: Inductor current waveform for discontinuous conduction

$$\therefore I_L^{\max} = \frac{(V_{in} - V_o)\, DT}{L} \tag{1}$$

For $DT < t < \beta T$, $\qquad i_L(t) = \dfrac{-V_o\,(t - DT)}{L} + I_L^{\max}$

At $t = \beta T$, $\qquad i_L(t) = 0$

$$\therefore I_L^{\max} = \frac{V_o\,(\beta T - DT)}{L} \tag{2}$$

Equating (1) and (2),

$$\frac{(V_{in} - V_o)\, DT}{L} = \frac{V_o\,(\beta T - DT)}{L}$$

$$\boxed{\therefore V_o^{\mathrm{DCM}} = V_{in}\left(\frac{D}{\beta}\right)} \tag{3}$$

where, V_o^{DCM} is the average voltage across load in discontinuous conduction mode. The value of β is less than 1.

$$\boxed{V_o^{\mathrm{CCM}} = DV_{in}} \tag{4}$$

where, V_o^{CCM} is the average voltage across load in continuous conduction mode.
From equations (3) and (4),

$$\boxed{V_o^{\mathrm{DCM}} > V_o^{\mathrm{CCM}}}$$

In buck converter, the output voltage in DCM mode is greater than output voltage in CCM mode.

(i) The average inductor current (I_L^{avg}) is given by

$$\begin{aligned} I_L^{\text{avg}} &= \frac{1}{T}\int_0^T i_L\,dt = \frac{1}{T}\int_0^{\beta T} i_L\,dt \\ &= \frac{1}{T}\left(\frac{1}{2}\,\beta T\,.\,I_L^{max}\right) \end{aligned} \tag{5}$$

From equations (1), (3) and (5),

$$\begin{aligned} 2 = 0.5\times\beta\times I_L^{\max} &= 0.5\times\beta\times\left(100\times\left(1-\frac{0.5}{\beta}\right)\times\frac{0.5}{10^4\times(0.5\times10^{-3})}\right) \\ \implies &\quad \beta = 0.9 \\ \therefore V_o = 100\,.\left(\frac{0.5}{0.9}\right) &= 55.56\,\text{V} \end{aligned}$$

(ii) The ON period of the switch S $= DT = 0.5\,$msec.

For discontinuous conduction, the diode conducts for $\beta T - DT$. Therefore the diode D conducts for $0.4\,$msec.

SequelApp Exercises:

1. A buck converter is feeding a resistive load of $60\,\Omega$. If ΔI_L is $1.5\,$A, average output voltage is $30\,$V, find the source voltage and inductance required. Assume that all the components are ideal, and the duty ratio and switching frequency remain same.

Verify your answer using SequelApp.