DC-DC Converter (`PE_dual.sqproj`)

Question : The converter shown in Fig. 1 (a) is a dual output single input DC – DC converter. Each switch operates with a switching frequency of 500 kHz and their switching pattern is as shown in Fig. 1 (b). All the elements of circuit are assumed to be ideal and capacitors are large enough to assume ripple free output voltages. Find the average value of output voltages, v_o and v_B.

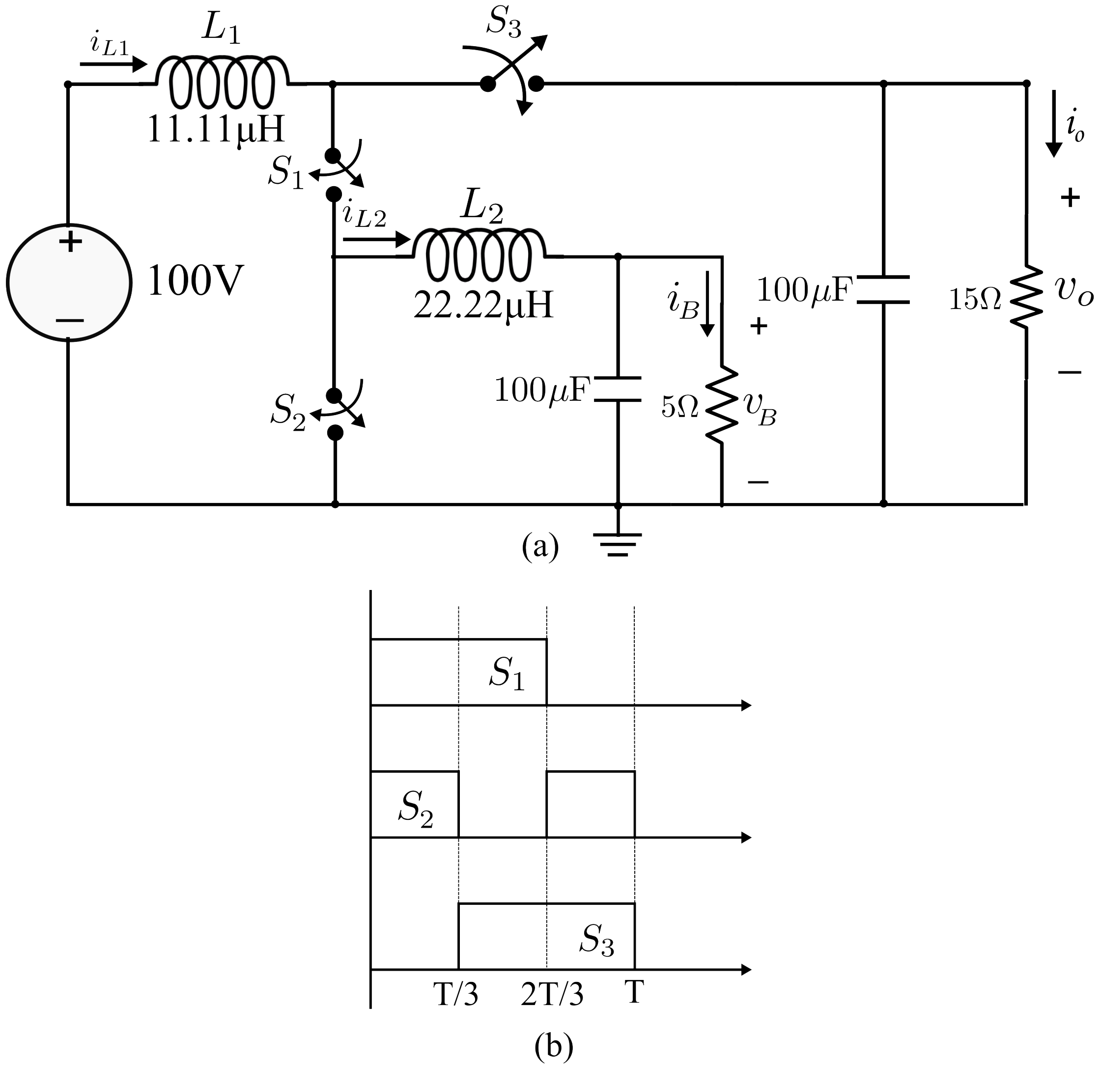

Figure 1: Dual output DC – DC converter

Solution :

Fig. 1(a) shows a dual output single input DC−DC converter with outputs denoted by v_B and v_o. Each switch has a duty ratio of 0.67 and Fig. 2 shows the three switching stages of the converter.

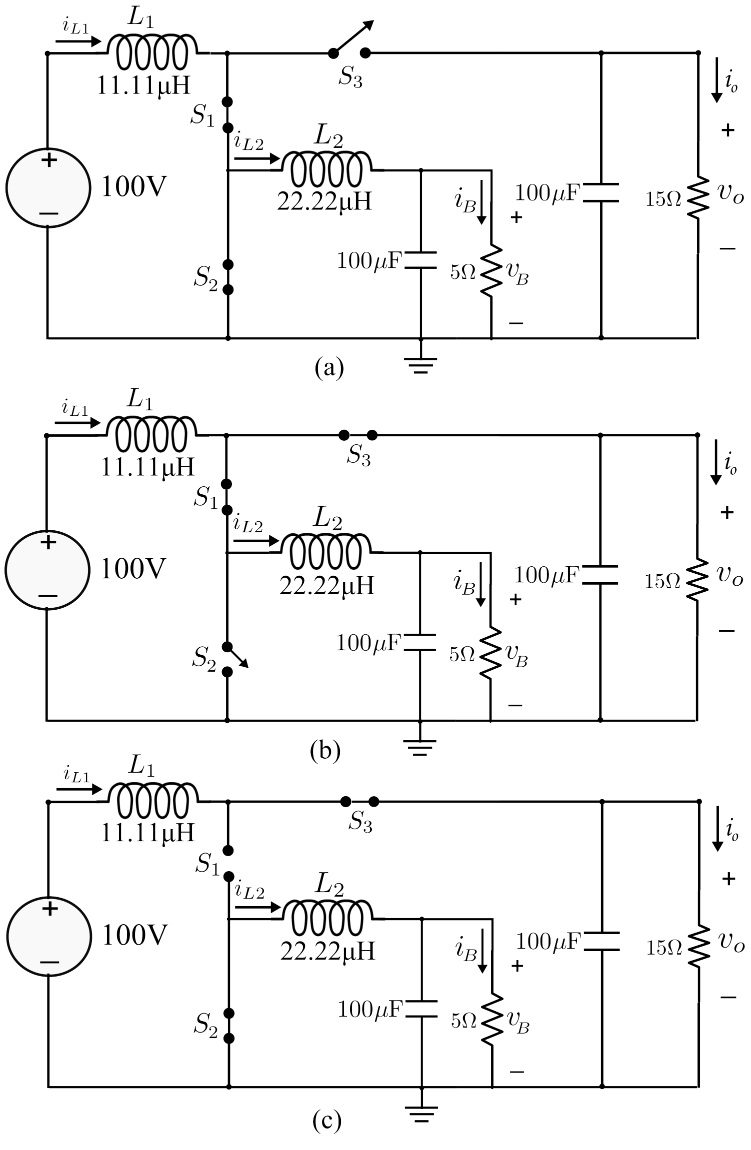

Figure 2: Dual output DC-DC converter (a) S_1 & S_2 are ON, (b) S_1 & S_3 are ON, (b) S_2 & S_3 are ON

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Assume that the inductor currents (i_{L1} and i_{L2}) are continuous.

Operation : When the switches S_1 & S_2 are ON $\left(0 < t < \frac{T}{3}\right)$, inductor L_1 stores energy and L_2 releases energy. When the switches S_1 & S_3 are ON $\left(\frac{T}{3} < t < \frac{2T}{3}\right)$, inductor L_1

releases energy and L_2 stores energy. When the switches S_3 & S_3 are ON $\left(\frac{2T}{3} < t < T\right)$, inductors L_1 and L_2 releases energy.

The circuit is operating under steady state, i.e., the energy stored in the inductor during the ON interval should be released during the OFF interval as shown in Fig. 3.

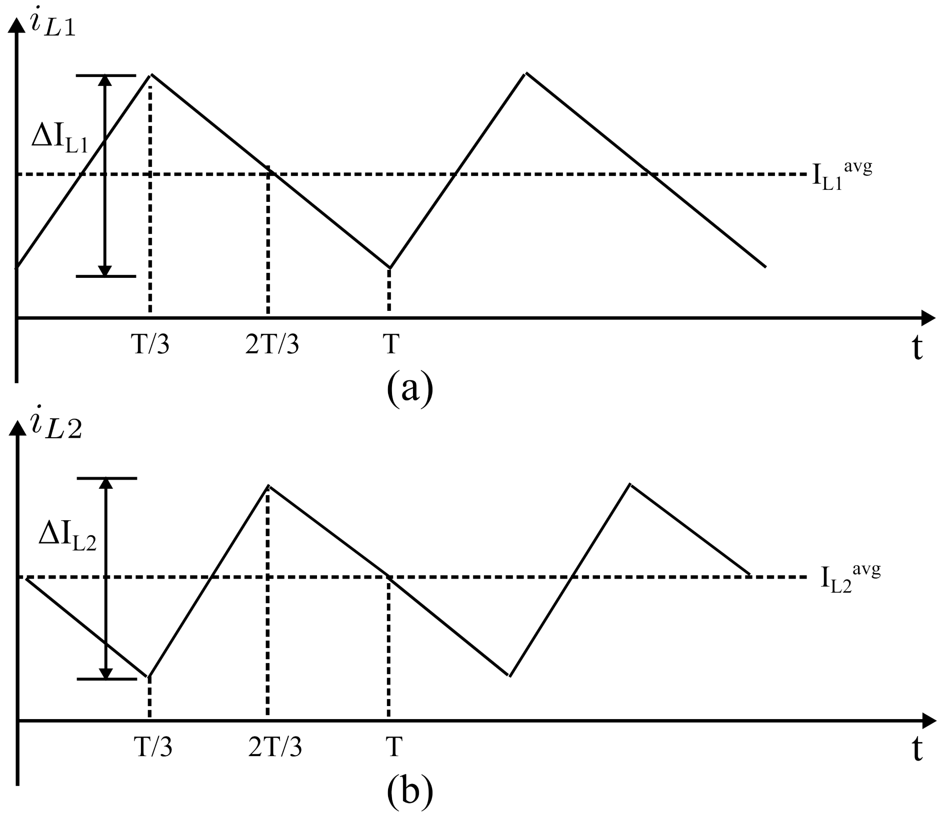

Figure 3: Inductor current waveforms for continuous conduction

The average voltage across the 15 Ω resistor (V_o) can be derived as follows:

When the switches S_1 & S_2 are ON, applying KVL gives

$$V_{in} = L_1 \frac{di_{L1}}{dt}$$

When the switches S_1 & S_3 are ON, applying KVL gives

$$V_{in} = L_1 \frac{di_{L1}}{dt} + v_o(t)$$

When the switches S_2 & S_3 are ON, applying KVL gives

$$V_{in} = L_1 \frac{di_{L1}}{dt} + v_o(t)$$

Applying the volt-sec balance equation across L_1:

$$\boldsymbol{V_o = \frac{3}{2} \cdot V_{in} \qquad \Longrightarrow \qquad V_o = \frac{3}{2} \times 100 = 150\,\mathrm{V}}$$

The average voltage across the $5\,\Omega$ resistor (V_B) can be derived as follows:
When the switches $S_1\,\&\,S_2$ are ON, applying KVL gives

$$v_B + L_2\,\frac{di_{L2}}{dt} = 0$$

When the switches $S_1\,\&\,S_3$ are ON, applying KVL gives

$$v_B + L_2\,\frac{di_{L2}}{dt} = v_o(t)$$

When the switches $S_2\,\&\,S_3$ are ON, applying KVL gives

$$v_B + L_2\,\frac{di_{L2}}{dt} = 0$$

Applying the volt-sec balance equation across L_2:

$$\boldsymbol{V_B = \frac{1}{3}\,.\,V_o \qquad \Longrightarrow \qquad V_B = \frac{1}{3} \times 150 = 50\mathrm{V}}$$

The above calculations were done with the assumption that the inductor currents are continuous. When the inductor current is continuous, $I_L^{\text{avg}} > \dfrac{\Delta\, I_L}{2}$, where I_L^{avg} is the average inductor current and $\Delta\, I_L$ is the peak $-$ to $-$ peak ripple inductor current. To validate our assumption:
The overall circuit is lossless, i.e., $P_{out} = P_{in}$.

$$\therefore\ \frac{V_o^2}{R_o} + \frac{V_B^2}{R_B} = V_{in}I_{in} \Longrightarrow I_{L1}^{avg} = I_{in} = \frac{1}{100}\cdot\left(\frac{150^2}{15} + \frac{50^2}{5}\right) = 20\,\mathrm{A}$$

The average current through capacitor under steady state is zero. Hence the average current through L_2 is equal to the average current through R_B and is given by

$$I_{L2}^{avg} = I_B = \frac{V_B}{R_B} = \frac{50}{5} = 10\,\mathrm{A}$$

The peak $-$ to $-$ peak inductor currents can be calculated as follows:
When the switches $S_1\,\&\,S_2$ are ON $\left(0 < t < \dfrac{T}{3}\right)$, applying KVL gives

$$V_{in} = L_1\,\frac{di_{L1}}{dt} = L_1\,\frac{\Delta I_{L1}}{\Delta t}$$

where ΔI_{L1} is the peak $-$ to $-$ peak inductor current and Δt is $T/3$ as shown in Fig. 3 (a).

$$\therefore\ \Delta I_{L1} = \frac{V_{in} \times \frac{T}{3}}{L_1} = \frac{100}{(500\ \times 10^3 \times 3) \times (11.11\ \times 10^{-6})} = 6\,\mathrm{A}$$

Similarly, when the switches $S_1\,\&\,S_3$ are ON $\left(\dfrac{T}{3} < t < \dfrac{2T}{3}\right)$, applying KVL gives

$$V_B + L_2 \frac{di_{L2}}{dt} = V_o \Longrightarrow V_o - V_B = L_2 \frac{\Delta I_{L2}}{\Delta t}$$

where ΔI_{L2} is the peak − to − peak inductor current and Δt is $T/3$ as shown in Fig. 3 (b).

$$\therefore \ \Delta I_{L2} = \frac{(V_o - V_B) \times \frac{T}{3}}{L_2} = \frac{100}{(500 \times 10^3 \times 3) \times (22.22 \times 10^{-6})} = 3\,\mathrm{A}$$

From the above calculations $I_L^{\mathrm{avg}} > \dfrac{\Delta I_L}{2}$ in both inductors, i.e., the current through the inductors are continuous and hence our assumption is correct.

SequelApp Exercises:

1. Calculate the value of inductances L_1 and L_2 such that the percentage ripple current $\left(\frac{\Delta I_L}{I_L^{\mathrm{avg}}} \times 100\right)$ in both the inductors is $20\,\%$, keeping all other parameters the same as the previous question.

Verify your answer using SequelApp.