Network Theorems-1 (`EC_network_1.sqproj`)

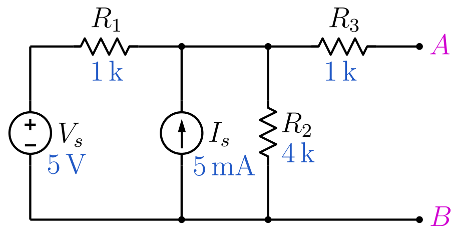


Figure 1: Thevenin theorem example.

Question: For the circuit shown in Fig. 1,

(a) Find the Thevenin equivalent circuit as seen from AB using superposition.

(b) Find the maximum power which can be delivered to a resistor connected between A and B.

(c) Using the results of (a), find the Thevenin equivalent circuit seen from AB for $V_s = 2\,\text{V}$ and $I_s = 2\,\text{mA}$.

Solution:

To find the Thevenin resistance, we deactivate the independent sources by making $V_s = 0\,\text{V}$ and $I_s = 0\,\text{A}$, giving the circuit shown in Fig. 2. By inspection, we can write,

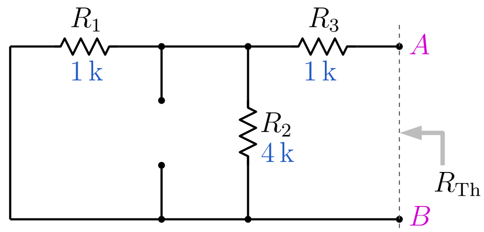


Figure 2: Calculation of Thevenin resistance for the circuit of Fig. 1.

$$R_{\text{Th}} = (R_1 \parallel R_2) + R_3 = 1.8\,\text{k}\Omega. \tag{1}$$

The Thevenin voltage is the same as the open-circuit voltage V_{oc} when port AB is left open. We use superposition to find the contribution $V_{oc}^{(1)}$ due to the voltage source alone, $V_{oc}^{(2)}$ due to the current source alone, and then add the two to get the net V_{oc}.

When only V_s is active (see Fig. 3 (a)), we get

$$V_{oc}^{(1)} = V_s \times \frac{R_2}{R_1 + R_2} = \frac{4}{5} V_s = 4\,\text{V}. \tag{2}$$

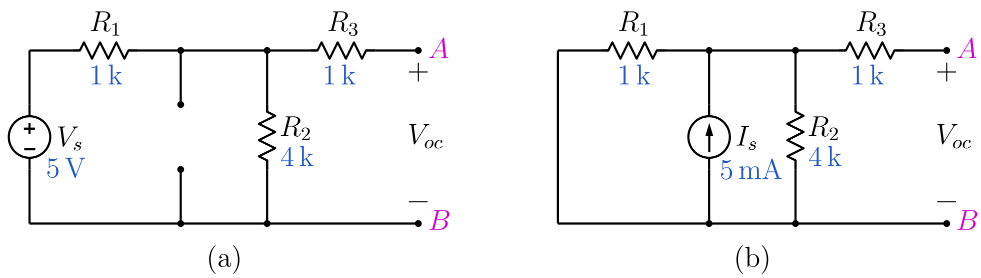


Figure 3: Computation of V_{oc} using superposition for the circuit of Fig. 1. (a) only V_s active, (b) only I_s active.

When only I_s is active (see Fig. 3 (b)), we get

$$V_{oc}^{(2)} = I_s \times (R_1 \parallel R_2) = I_s \times 0.8\,\text{k}\Omega = 4\,\text{V}. \tag{3}$$

The Thevenin voltage is therefore

$$V_{oc}^{\text{net}} = V_{oc}^{(1)} + V_{oc}^{(2)} = 4\,\text{V} + 4\,\text{V} = 8\,\text{V}. \tag{4}$$

Fig. 4 shows the Thevenin equivalent circuit.

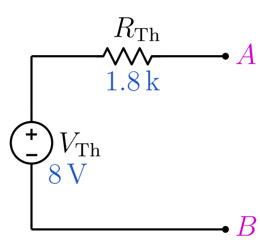


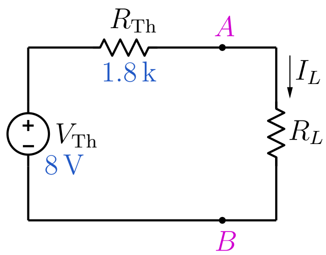
Figure 4: Thevenin equivalent circuit for the network of Fig. 1.

The power delivered by the circuit to a load resistance R_L connected between A and B (see Fig. 5 (a)) is maximum when $R_L = R_{\text{Th}} = 1.8\,\text{k}$ and is given by

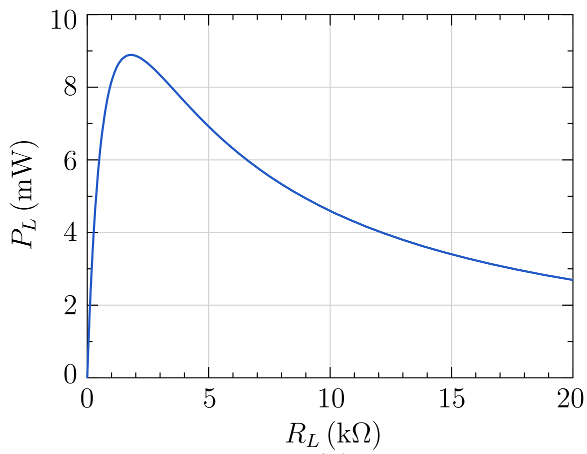
$$P_L^{\max} = I_L^2 R_L = \left(\frac{V_{\text{Th}}}{R_{\text{Th}} + R_{\text{Th}}} \right)^2 \times R_{\text{Th}} = \frac{V_{\text{Th}}^2}{4\,R_{\text{Th}}} = 8.89\,\text{mW}. \tag{5}$$

Fig. 5 (b) shows the power delivered by the circuit as a function of R_L.

To verify if our calculations are correct, we can simulate the circuit of Fig. 1 with a resistance R_L connected between A and B, and observe the following.



(a)



(b)

Figure 5: (a) Thevenin equivalent circuit with R_L connected, (b) P_L (power delivered to R_L) versus R_L.

(a) V_{AB} with $R_L \to \infty$ (say, $R_L = 1\,\text{G}\Omega$). This value should match our V_{Th}.

(b) I_L with $R_L \to 0$ (say, $R_L = 1\,\text{m}\Omega$). This value should match our I_{sc} which is the same as $V_{\text{Th}}/R_{\text{Th}}$.

(c) Vary R_L and plot P_L versus R_L. With this plot, we can check if our P_{max} is correct and whether it does correspond to $R_L = R_{\text{Th}}$.

Finally, for part (c) of the question, we can use the fact that the circuit is linear. Since it has two independent sources V_s and I_s, *any* current or voltage, denoted by x, can be written as

$$x = k_1 V_s + k_2 I_s. \tag{6}$$

For $x = V_{oc}$, we have already found $k_1 = \dfrac{4}{5}$ (see Eq. 2) and $k_2 = 0.8\,\text{k}\Omega$ (see Eq. 3). Using these and the given V_s and I_s values, we obtain

$$V_{oc} = \frac{4}{5} \times 2\,\text{V} + (0.8\,\text{k}\Omega) \times 2\,\text{mA} = 1.6\,\text{V} + 1.6\,\text{V} = 3.2\,\text{V}. \tag{7}$$

Since R_{Th} does not depend on V_s or I_s, we do not need to compute it again. Thus, the answer to (c) is $V_{\text{Th}} = 3.2\,\text{V}$, $R_{\text{Th}} = 1.8\,\text{k}\Omega$.

SequelApp Exercises: Find V_{Th}, R_{Th}, I_{sc}, and $P_{\max}$ for each of the following cases (with other component values as shown in Fig. 1). Verify your answers using SequelApp.

1. R_1 is changed from 1 k to 2 k.
2. R_2 is changed from 4 k to 2 k.
3. R_3 is changed from 1 k to 2 k.
4. Find R_3 such that $R_{Th} = 3\,\mathrm{k}$ (with other component values as shown in Fig. 1). What are V_{oc}, I_{sc}, $P_{\max}$ in this case?