Phasors (`EC_phasors_3.sqproj`)

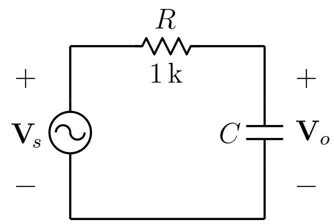

Figure 1: Phasor calculation example.

Question: In the circuit shown in the figure, the frequency is 1 kHz, and the source voltage is $1\angle 0\,\text{V}$.

(a) Find C for which the output voltage has an amplitude of 0.9 V. What is $\angle \mathbf{V}_o$ in this case?

(b) Find C for which the output voltage lags $\mathbf{V}_s$ by 60°. What is $|\mathbf{V}_o|$ in this case?

(c) For (a) and (b), compute the time difference between the zero crossings of $V_s(t)$ and $V_o(t)$.

(d) Draw (to scale) a phasor diagram corresponding to the KVL equation.

Solution:

The output voltage is given by

$$\mathbf{V_o} = \frac{\dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} \times \mathbf{V_s} = \frac{1}{1 + j\omega RC} \times \mathbf{V_s} = \frac{1}{1 + j\omega RC} \times 1\angle 0. \tag{1}$$

(a) For $|\mathbf{V}_o| = 0.9\,\text{V}$, we need

$\dfrac{1}{\sqrt{1 + (\omega RC)^2}} = 0.9 \rightarrow C = 77\,\text{nF}$, and in this situation, $\angle \mathbf{V}_o = -\tan^{-1}\omega RC = -25.8°$.

(b) For $\angle \mathbf{V}_o = -60°$, we need

$-\tan^{-1}\omega RC = -60° \rightarrow \omega RC = \tan 60° = \sqrt{3} \rightarrow C = 276\,\text{nF}$,

and in this situation, $|\mathbf{V}_o| = \dfrac{1}{\sqrt{1 + (\omega RC)^2}} = \dfrac{1}{\sqrt{1+3}} = 0.5$.

(c) Let Δt be the time difference between the zero crossings of $V_s(t)$ and $V_o(t)$. Then we have $\dfrac{|\Delta t|}{T} = \dfrac{|\Delta \phi|}{2\pi} \rightarrow |\Delta t| = \dfrac{25.8°}{360°} \times 1\,\text{ms} = 72\,\mu\text{s}$ in (a), and $|\Delta t| = \dfrac{60°}{360°} \times 1\,\text{ms} = 167\,\mu\text{s}$ in (b).

SequelApp Exercises:

(a) Find C for which the output voltage has an amplitude of $0.7\,\text{V}$. What is $\angle \mathbf{V}_o$ in this case?

(b) Find C for which the output voltage lags $\mathbf{V}_s$ by 30°. What is $|\mathbf{V}_o|$ in this case?

(c) For (a) and (b), compute the time difference between the zero crossings of $V_s(t)$ and $V_o(t)$.

Verify your answers using SequelApp (in frequency domain as well as time domain).