Phasors (`EC_phasors_4.sqproj`)

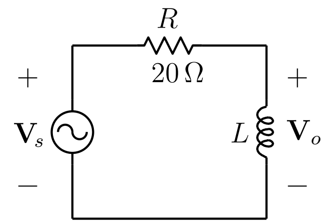

Figure 1: Phasor calculation example.

Question: In the circuit shown in the figure, the frequency is 50 Hz, and $\mathbf{V}_s = V_m \angle 0$.

(a) Find L for which $\dfrac{|\mathbf{V}_o|}{|\mathbf{V}_s|} = 0.8$. What is $\angle \mathbf{V}_o$ in this case?

(b) Find L for which the output voltage leads $\mathbf{V}_s$ by 25°. What is $|\mathbf{V}_o|/|\mathbf{V}_s|$ in this case?

(c) For (a) and (b), compute the time difference between the zero crossings of $V_s(t)$ and $V_o(t)$.

(d) Draw (to scale) a phasor diagram corresponding to the KVL equation.

Solution:

The magnitude and phase of the output voltage are given by

$$|\mathbf{V}_o| = |\mathbf{V}_s| \times \left| \frac{j\omega L}{R + j\omega L} \right| = |\mathbf{V}_s| \times \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}} = |\mathbf{V}_s| \times \frac{\omega L/R}{\sqrt{1 + (\omega L/R)^2}}. \quad (1)$$

$$\angle \mathbf{V}_o = \angle \mathbf{V}_s + \pi/2 - \tan^{-1}\left(\frac{\omega L}{R}\right). \quad (2)$$

(a) Let us denote $\omega L/R$ by x. For $|\mathbf{V}_o| = 0.8 \times |\mathbf{V}_s|$ we need $\dfrac{x}{\sqrt{1+x^2}} = 0.8$

$\rightarrow x = \dfrac{\omega L}{R} = \dfrac{4}{3} \rightarrow L = 84.9\,\text{mH}$, and in this situation,

$\angle \mathbf{V}_o = \angle \mathbf{V}_s + \pi/2 - \tan^{-1} x = \angle \mathbf{V}_s + 36.8°$.

(b) For $\angle \mathbf{V}_o$ to lead $\angle \mathbf{V}_s$ by 25°, we need

$90° - \tan^{-1} x = 25° \rightarrow x = \dfrac{\omega L}{R} = \tan 65° \rightarrow L = 136\,\text{mH}$,

and in this situation, $\dfrac{|\mathbf{V}_o|}{|\mathbf{V}_s|} = \dfrac{1}{\sqrt{1+x^2}} = 0.9$.

(c) Let Δt be the time difference between the zero crossings of $V_s(t)$ and $V_o(t)$. Then we have $\frac{|\Delta t|}{T} = \frac{|\Delta \phi|}{2\pi} \rightarrow |\Delta t| = \frac{36.8°}{360°} \times 20\,\text{ms} = 2\,\text{ms}$ in (a), and $|\Delta t| = \frac{25°}{360°} \times 20\,\text{ms} = 1.4\,\text{ms}$ in (b).

SequelApp Exercises:

(a) Find L for which $\frac{|\mathbf{V}_o|}{|\mathbf{V}_s|} = 0.5$. What is $\angle \mathbf{V}_o$ in this case?

(b) Find L for which the output voltage leads $\mathbf{V}_s$ by 45°. What is $\frac{|\mathbf{V}_o|}{|\mathbf{V}_s|}$ in this case?

(c) For (a) and (b), compute the time difference between the zero crossings of $V_s(t)$ and $V_o(t)$.

Verify your answers using SequelApp (in frequency domain as well as time domain).