RC **circuits** (`EC_rc_1.sqproj`)

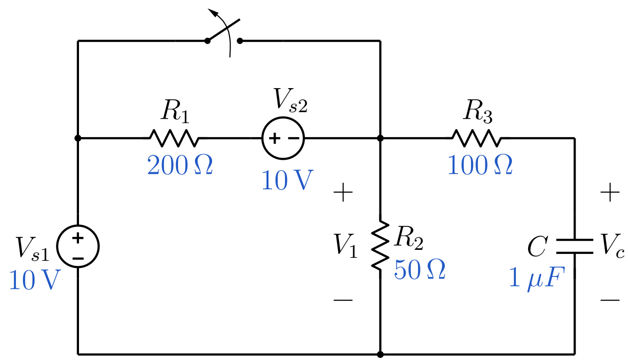


Figure 1: RC circuit example.

Question: In the circuit shown in Fig. 1, the switch has been closed for a long time and opens at $t = 0$.

(a) Find $V_c(0^-)$.

(b) Find $V_c(0^+)$ and $V_1(0^+)$.

(c) Find $V_c(\infty)$ and $V_1(\infty)$.

(d) What is the circuit time constant for $t > 0$?

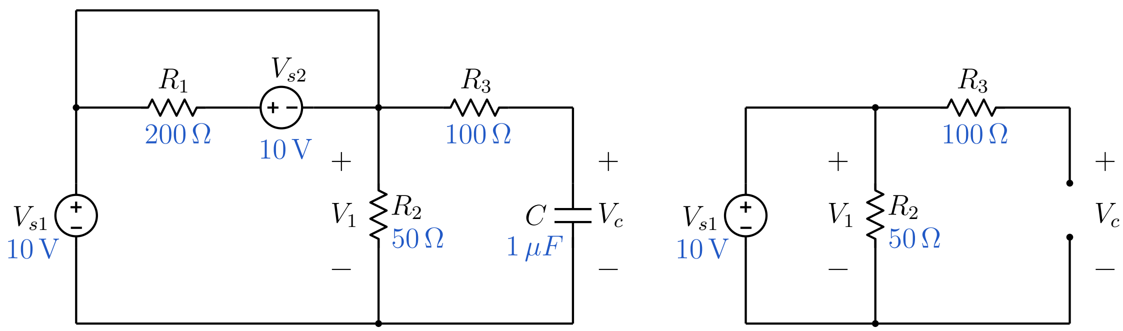
(e) Using the above results, plot $V_c(t)$ and $V_1(t)$.

Solution:

First, let us look at the circuit at $t = 0^-$ (see Fig. 2 (a)). Since the switch has been closed for a long time, the circuit is in steady state, i.e., the currents and voltages are constant. Since $i_c = C\,\dfrac{dV_c}{dt}$ for a capacitor, $i_c(0^-)$ must be zero (since $\dfrac{dV_c}{dt} = 0$) which means that the capacitor behaves like an open circuit, as shown in Fig. 2 (b), and we find that $V_1(0^-) = V_c(0^-) = 10\,\text{V}$.

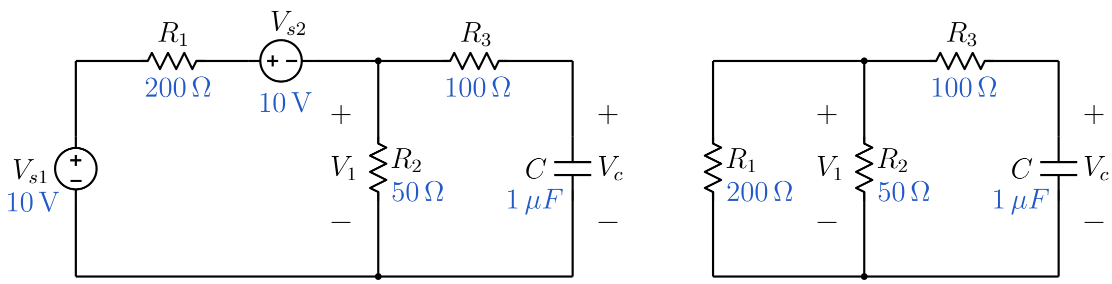
The circuit after the switch opens at $t = 0$ is shown in Fig. 3 (a). The two voltage source are in series and can be replaced with a single source $(V_{s1} + V_{s2})$ which happens to be 0 V, a short circuit. The simplified circuit is shown in Fig. 3 (b).

Now, $V_c(0^+)$ must be the same as $V_c(0^-)$; otherwise, the capacitor current $C\,\dfrac{dV_c}{dt}$ would become infinitely large, thus violating circuit equations. We have therefore



(a) (b)

Figure 2: (a) Circuit of Fig. 1 at $t=0^-$, (b) simplified circuit.



(a) (b)

Figure 3: (a) Circuit of Fig. 1 for $t > 0$, (b) simplified circuit.

$V_c(0^+) = V_c(0^-) = 10\,\text{V}$. From $V_c(0^+)$, we can obtain $V_1(0^+)$ as (see Fig. 3 (b))

$$V_1(0^+) = V_c(0^+) \times \frac{(R_1 \parallel R_2)}{(R_1 \parallel R_2) + R_3} = 10\,\text{V} \times \frac{40\,\Omega}{140\,\Omega} = 2.86\,\text{V}. \tag{1}$$

For $t > 0$, the Thevenin equivalent resistance seen by the capacitor is $(R_1 \parallel R_2) + R_3 = 140\,\Omega$; therefore, the circuit time constant is $\tau = 140\,\Omega \times 1\,\mu\text{F} = 140\,\mu\text{sec}$. After the switch opens at $t = 0$, we expect all transients to vanish in about $5\,\tau$ or $140\,\mu$sec, and all currents and voltages would remain constant thereafter.

From Fig. 3 (b), $V_c(\infty)$ and $V_1(\infty)$ (i.e., after steady state has been attained) are found by inspection to be 0 V each since there is no independent source in the circuit.

Putting together the above observations, we obtain the plots for $V_c(t)$ and $V_1(t)$ as shown in Fig. 4.

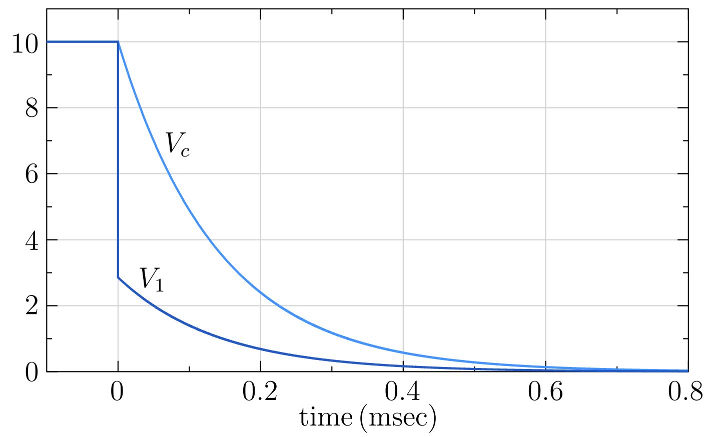


Figure 4: V_c and V_1 versus time for the circuit of Fig. 1.

SequelApp Exercises: Repeat the above steps for the following situations, with other parameters the same as in Fig. 1. Verify your answers using SequelApp.

1. $V_{s2} = -10\,\text{V}$, switch opens at $t = 0$.

2. $V_{s2} = 10\,\text{V}$, switch closes at $t = 0$.

3. $V_{s2} = -10\,\text{V}$, switch closes at $t = 0$.